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Sectoral inflation persistence and optimal monetary policy

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Abstract

This study examines the optimal monetary policy in a two-sector New Keynesian model with inflation persistence. In contrast to the results suggested by one-sector models, the proposed model illustrates that sectoral inflation persistence significantly changes the property of the optimal monetary policy. Sectoral inflation persistence is likely to generate severe trade-offs between inflation and the output gap as well as between inflation in different sectors. This paper further considers how sectoral heterogeneities of inflation persistence substantially influence the gain from commitment.

Keywords: Optimal monetary policy; Sectoral heterogeneities of inflation persistence; Commitment; Two-sector New Keynesian model;
JEL classification: E52; E58

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1 Introduction

Many studies have adopted the New Keynesian (NK) model as a standard framework for analysing monetary policy (e.g. Clarida et al., 1999; Woodford, 2003; Walsh, 2017; Gali, 2015). In this purely forward-looking NK framework, however, inflation will jump immediately in response to economic shocks. Therefore, a purely forward-looking structure might generate imprecise predictions of the New Keynesian Phillips curve (NKPC). Some studies have incorporated past inflation into the NKPC (Gali and Gertler, 1999; Christiano et al., 2005) and defended such inclusion by arguing that lagged inflation improves the NKPC's fit (Gali and Gertler, 1999; Rudd and Whelan, 2007; Kurmann, 2007).[1] Moreover, theoretical studies have argued that inflation persistence will significantly affect the behaviour of monetary policy (Amato and Laubach, 2003; Steinsson, 2003; Sheedy, 2009; Paez-Farrell, 2012; Ida, 2013). These theoretical studies imply that the central bank faces severe welfare losses when inflation dynamics are mainly depicted by past inflation.

In addition, when examining the effect of aggregate inflation dynamics upon the real economy, it is important to distinguish between the dynamics of core inflation and those of flexible-price inflation.[2] Aoki (2001) demonstrated that stabilising core inflation, which excludes food and energy prices, is the optimal monetary policy in a two-sector NK economy. Airaudo and Zanna (2012) examined whether the central bank should target headline inflation, which includes both core and flexible inflation, rather than core inflation alone. They argued that targeting core inflation can result in reaching the unique rational expectations equilibrium, whereas targeting headline inflation will likely render it indeterminate.

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[1] Cogley and Sbordone (2008) showed that the NKPC with lagged inflation fails to explain actual inflation dynamics if the model considers trend inflation.

[2] Huang and Liu (2005) showed that in an input-output NK economy, the optimal central bank policy should stabilise inflation in both final and intermediate goods. Strum (2009) explored the optimal delegation problem in an input-output economy. Strum (2010) examined the optimal monetary policy in an input-output economy with inflation persistence.

These studies appear to highlight the importance of distinguishing between the dynamics of sticky-price inflation and flexible-price inflation in economies with inflation persistence. Several studies have considered persistence of sector inflation. Recently, Imbs et al. (2011) considered a heterogeneous hybrid NKPC by presuming the existence of sectoral heterogeneities of inflation persistence. Byrne et al. (2013) used a dataset of aggregated and disaggregated sectoral inflation and showed that a sector hybrid NKPC has important consequences for aggregate inflation dynamics.

Irrespective of the importance of sectoral inflation persistence, it remains unclear how sectoral persistence of inflation affects the optimal monetary policy. This paper suggests how the presence of sectoral heterogeneity of inflation persistence changes the properties of the optimal monetary policy suggested in a one-sector model. In this context, the following questions arise: how should the central bank conduct monetary policy in economies with sectoral inflation persistence, and how large are the gains from commitment policies in such an environment? It appears that no studies have yet answered these questions.

This study examines theoretically the optimal monetary policy in a two-sector NK model with inflation persistence. It introduces sectoral heterogeneities of inflation persistence into the model developed by Woodford (2003, 2010). The economy modelled in this paper has two production sectors; the first sector, which models a sticky-price sector, is labelled S, whereas the second sector, which models the flexible goods sector, is labelled F. Following Woodford (2010) and Airaudo and Zanna (2012), this paper assumes that prices in sector F are more flexible than those in sector S. In contrast to Aoki (2001) and Kosem-ALP (2010), however, this paper allows the case wherein prices in sector F are not completely flexible. Therefore, sector S's inflation can be regarded as core inflation.

This paper presents the theoretical foundation of sectoral inflation persistence in a sticky-price model. It utilises the rule-of-thumb hypothesis assumed by Steinsson (2003) to examine inflation persistence in a two-sector NK model. Under the rule-of-thumb hypothesis, firms use information from past aggregate price levels and inflation to set (reset) current prices. According to Steinsson (2003), a generalised rule-of-thumb contains the past price level, past inflation rate and a lagged output gap.[3] This allows a derivation of the generalised NKPC in a two-sector economy with inflation persistence.

Furthermore, this study derives the central bank's loss function, which corresponds to the second-order approximation of the household utility function, in a two-sector NK model with inflation persistence. In this model, inflation persistence in each sector will change the shape of the central bank's objective function. Thus, the shape of the loss function is more complicated than that in a one-sector model. Accordingly, the presence of sectoral heterogeneities of inflation persistence makes the central bank's optimisation problem more complicated.

The theoretical model indicates that inflation persistence in one-sector affects inflation in another sector through a change in the relative price between two sectoral goods, thereby leading to a large fluctuation in the output gap. The trade-offs following cost-push shocks in a sticky-price sector are particularly severe. In contrast, and importantly, trade-offs following cost-push shocks in a flexible-price sector are attenuated. In addition, the central bank faces a trade-off between consumer price index (CPI) inflation and the output gap in a one-sector model, whereas this study considers that sectoral heterogeneities of inflation persistence produce a severe trade-off not only between inflation and the output gap but also between inflation in different sectors in a two-sector model. These results are in contrast to those obtained for the one-sector model. This paper, therefore, emphasises that this mechanism can trigger a large fluctuation in the output gap. Accordingly, the model implies that the central bank needs to consider these mechanisms of sectoral heterogeneities of inflation persistence when implementing the optimal monetary policy.

This result raises questions about how sectoral inflation persistence substantially influences the gain from commitment. The main findings of welfare analyses are as follows. Regardless of the degree of inflation persistence in the flexible-price sector, gain will re-

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[3] Strum (2010) proposed an alternative rule-of-thumb specification that depends on the past price level, past inflation and past real marginal cost.

sult as long as core inflation is depicted by a forward-looking component. Therefore, the gain from commitment almost disappears when core inflation is predominately backward-looking. Accordingly, the gain is small when severe sectoral heterogeneities of inflation persistence exist in both sectors. When moderate inflation persistence is present in both sectors gains occur unless prices in both sectors are more flexible. Importantly, the gain from commitment almost vanishes when prices in both sectors are stickier.

The findings of this paper suggest that persistence of core inflation plays a significant role in a two-sector NK model that contains sectoral heterogeneities of inflation persistence. Other findings regarding welfare analyses are summarised as follows. First, when inflation dynamics are described by the forward-looking component in both sectors, the welfare gain is large if higher price stickiness counteracts the output gap's sensitivity to inflation. As suggested by the standard NK model, this occurs because a central bank that implements a commitment policy can enhance social welfare by manipulating the private sector's expectations in a sticky-price equilibrium. Second, if moderate inflation persistence is present in the core inflation sector, larger gains result from commitment in the case of larger substitutability between two sectoral goods. Third, whether the welfare gain is affected by a change in the elasticity of substitution between individual goods depends on the degree of inflation persistence in the sticky-price sector.

This paper is related to Aoki (2001), Woodford (2003, 2010), Steinsson (2003) and Kosem-ALP (2010). Kosem-ALP (2010), in particular, also examined optimal monetary policy with sectoral inflation persistence. His study shares several similarities with this study. However, the model proposed here differs from his model as follows. First, I explicitly evaluate how the presence of sectoral heterogeneities of inflation persistence affect the gain from commitment. Second, he derived the sectoral NKPC following Gali and Gertler (1999)'s specification of the rule-of-thumb pricing rule; in contrast, I derive it using the specification of Steinsson (2003), who generalised their rule-of-thumb pricing rule. Third, he used a Cobb-Douglas consumption index; this study assumes a consumption index per Woodford (2003, 2010). Fourth, and associated with the two previous differences, this study uses the second-order approximation of the household's utility function to derive the central bank's generalised loss function.

This study is organised as follows. Section 2 describes a two-sector NK model with inflation persistence and explains the optimal monetary policy in economies with sectoral inflation persistence. Section 3 calibrates the deep parameters used in this model. Section 4 presents the results of the study. Section 5 provides a brief conclusion.

2 The model

This section presents the utilised two-sector NK model with inflation persistence. I introduce sectoral inflation persistence into the standard two-sector NK model developed by Woodford (2003, 2010). The model economy has two production sectors, denoted S (first sector) and F (second sector). Both sectors face monopolistic competition and set prices according to Calvo-type (1983) nominal price rigidities.[4] As in Woodford (2003, 2010), this paper assumes that nominal price rigidities are more severe in sector S than in sector F. Thus, sector's prices are assumed to be more flexible than those in sector S. Therefore, sector S can be considered as the sticky (core) price sector, whereas sector F can be regarded as the flexible goods sector.[5]

Following Steinsson (2003), I introduce a rule-of-thumb pricing rule into the model. Firms that follow this rule-of-thumb pricing rule set prices based on past inflation, past price level and past output gap. This pricing rule allows the effect of inflation persistence on the economy to be examined. More specifically, two types of firms operate in each sector. One sets prices optimally, whereas the other sets prices following a rule-of-thumb pricing rule.

The infinitely lived household derives utility from consumption and disutility from supplying labour to each production sector. Finally, the central bank sets its interest

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[4] Airaudo and Zanna (2012) assume price adjustment costs for the two sectors to introduce nominal price rigidities in the economy. They also assume that prices in one sector are more flexible than those in the other sector.

[5] As mentioned earlier, note that unlike Aoki (2001) and Kosem-ALP (2010), this paper does not assume that prices in sector F are completely flexible.

rate to implement a monetary policy by following the targeting rule.

2.1 The household

The inter-temporal utility of an infinitely lived representative household is

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t; \xi_t) - \int_{N_s} v(h_t^s(j); \xi_t^s) dj - \int_{N_f} v(h_t^f(j); \xi_t^f) dj \right], \tag{1}$$

where C_t denotes aggregate consumption. $h_t^s(j)$ and $h_t^f(j)$ denote the labour supply for goods j in sectors S and F, respectively. ξ_t is a preference shock, and ξ_t^i denotes the exogenous disturbance for sector i, where $i = s, f$. The parameter β denotes the discount factor. The utility function $u(\cdot)$ is strictly concave and continuously differentiable. The disutility of labour supply $v(\cdot)$ is strictly convex and continuously differentiable. Furthermore, the intervals of goods in the two sectors are divided by $N_s = [0, n_s]$ and $N_f = [n_f, 1]$. Here, n_i (for $i = s, f$) denotes the share of each sector in the economy.

Aggregate consumption is defined by the constant elasticity of substitution (CES) aggregator of the two consumption goods, C_t^s and C_t^f:

$$C_t = \left[(n_s \varphi_{s,t})^{1/\eta} (C_t^s)^{(\eta-1)/\eta} + (n_f \varphi_{f,t})^{1/\eta} (C_t^f)^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}, \tag{2}$$

where η is the intra-temporal elasticity of substitution between goods S and goods F, and $\varphi_{i,t}$ (for $i = s, f$) is the random coefficient. Parameter n_i (for $i = s, f$) is the share of goods of each type. Assume that $n_s + n_f = 1$. In addition, C_t^i (for $i = s, f$) is given by the following CES aggregate:

$$C_t^i = \left[\int_{N_i} C_t^i(j)^{(\theta_t-1)/\theta_t} dj \right]^{\theta_t/(\theta_t-1)}, \tag{3}$$

$C_t^i(j)$ is consumption for goods j in sector i, for $i = s, f$. The parameter θ_t is the time-varying elasticity of substitution for individual goods, which is common to each sector. To introduce a cost-push shock in each sector, this study assumes time-varying elasticity of substitution. As indicated by Steinsson (2003), this assumption enables us to derive a cost-push shock in the NKPC.[6] Relative demand for goods j of sector i is given by

$$C_t^i(j) = \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\theta_t} C_t^i, \tag{4}$$

for any $j \in [0,1]$ and $i = s, f$. In addition, the demand for the sectoral composite good is given by

$$C_t^i = n_i \varphi_{i,t} \left(\frac{P_t^i}{P_t}\right)^{-\eta} C_t, \tag{5}$$

for any $i = s, f$. Furthermore, the price indices in each sector are defined as follows:

$$P_t^i = \left[\int_{N_i} P_t(j)^{1-\theta_t} dj\right]^{1/(1-\theta_t)}, \tag{6}$$

for $i = s, f$. Finally, when aggregate consumption is defined by Equation (2), the CPI is defined as follows:

$$P_t = \left[n_s \varphi_{s,t} (P_t^s)^{1-\eta} + n_f \varphi_{f,t} (P_t^f)^{1-\eta}\right]^{1/(1-\eta)}. \tag{7}$$

Next, consider the household's utility maximisation problem. The representative household maximises Equation (1) subject to the following budget constraint:

$$P_t C_t + E_t Q_{t,t+1} B_{t+1} = B_t + \int_{N_s} [W_t(j) h_t^s(j) + \Gamma_t^s(j)] dj + \int_{N_f} [W_t(j) h_t^f(j) + \Gamma_t^f(j)] dj - T_t, \tag{8}$$

where B_t denotes the nominal bond, and T_t denotes a lump-sum tax. $W_t(j)$ is the nominal wage and $\Gamma_t^i(j)$ (for $i = s, f$) is dividend of firms in each sector.[7]

The stochastic discount factor $Q_{t,t+1}$ is defined as

$$E_t Q_{t,t+1} = \frac{1}{1 + i_t}, \tag{9}$$

where i_t is the nominal interest rate.

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[6] An alternative is to assume that the wage mark-up is time-varying to derive the term for a cost-push shock in the NKPC. Clarida et al. (2002) discussed this issue in detail.

[7] Note that in this paper, I do not assume the labour mobility between two sectors. Therefore, the wage rate is indifferent between two sectors.

The following conditions are derived from the first-order conditions of the household's utility maximisation problem:

$$u_c(C_t;\xi_t) = \beta E_t(1+i_t)u_c(C_{t+1};\xi_{t+1})\frac{P_t}{P_{t+1}}, \tag{10}$$

and

$$\frac{v_h(h_t^i(j);\xi_t^i)}{u_c(C_t;\xi_t)} = \frac{W_t(j)}{P_t}, \tag{11}$$

for $i = s, f$. Equation (10) represents the Euler equation, which implies that changes in the real interest rate lead to adjustments in inter-temporal consumption. Equation (11) indicates that the marginal rate of substitution between labour supply and consumption equals the real wage for each sector.

2.2 The firm

As in the standard NK model, firms in each sector face Calvo-type (1983) nominal price rigidity. Under the assumption of Calvo pricing, a fraction of firms can set prices optimally, but the remaining firms cannot set their prices. In a two-sector NK model, the probability that firms cannot revise prices in a given period depends on the particular sector. As mentioned earlier, this paper assumes that sector F's prices are more flexible than those of sector S. In addition, some firms that optimally set their prices use a rule-of-thumb pricing rule. Importantly, in contrast to the standard one-sector NK model, the fraction of firms that follow a rule-of-thumb depends on each sector. Finally, as in Woodford (2003), this study assumes a sector-specific disturbance to the preference in terms of household labour supply.

The production function for goods j is given by

$$y_{i,t}(j) = A_{i,t}f(h_t^i), \tag{12}$$

for $i = s, f$, where $y_{i,t}$ (for $i = s, f$) is sector output, $A_{i,t}$ (for $i = s, f$) denotes a sector-specific productivity disturbance, and $f(\cdot)$ is an increasing and concave function.

Under these settings, the real marginal cost of sector i is given by

$$\psi_t^i(j) = \frac{W_t(j)}{P_t}\frac{1}{A_{i,t}f'(f^{-1}(y_{i,t}(j)/A_{i,t}))}, \tag{13}$$

for $i = s, f$, where $\psi_t^i(j)$ is the real marginal cost for goods j for each sector. Using the household's first-order conditions with respect to labour supply for each sector, the above real marginal cost can be rewritten as follows:

$$\psi_t^i(j) = \frac{v_h(h_t^i(j); \xi_t^i)}{u_c(C_t; \xi_t)} \frac{1}{A_{i,t} f'(f^{-1}(y_{i,t}(j)/A_{i,t}))}, \tag{14}$$

for $i = s, f$.

Following Calvo (1983), this study assumes price rigidity in each goods sector. Fraction $1 - \alpha$ of all firms in sector i can adjust prices, whereas the remaining firms (α) cannot. These firms are uncertain regarding when they can change prices in a future period. The firm's optimisation problem is thus given by

$$E_t \sum_{T=t}^{\infty} (\alpha_i \beta)^{T-t} \frac{u_c(C_T; \xi_T)}{u_c(C_t; \xi_t)} \left[P_t^{i,o} \left(\frac{P_t^{i,o}}{P_T^i} \right)^{-\theta_T} Y_{i,T} - W_T f^{-1} \left(\left(\frac{P_t^{i,o}}{P_T^i} \right)^{-\theta_T} \frac{Y_{i,T}}{A_{i,T}} \right) \right], \tag{15}$$

for $i = s, f$, where $P_t^{i,o}$ denotes the optimal price in sector i in period t. The first-order condition of this optimisation problem is

$$E_t \sum_{T=t}^{\infty} (\alpha_i \beta)^{T-t} \frac{u_c(C_T; \xi_T)}{u_c(C_t; \xi_t)} (1 - \theta_T)(P_t^{i,o})^{-1} \left(\frac{P_t^{i,o}}{P_t^i} \right)^{-\theta_T} Y_{i,T} \left[\left(\frac{P_t^{i,o}}{P_T^i} \right) - \mu_T \frac{P_t^{i,o}}{P_T^i} \psi_{t,T}^i \right] = 0, \tag{16}$$

for $i = s, f$. Variable μ_t denotes the time-varying price mark-up, $Y_{i,t}$ denotes sector aggregate output, and $\psi_{t,T}^i$ (for $i = s, f$) denotes the real marginal cost for the firms that last reset their prices in period t. Here, the real marginal costs for each sector under a flexible-price equilibrium are given by

$$\frac{P_t^{i,o}(j)}{P_T^i} = \psi_t^i, \tag{17}$$

for $i = s, f$. The price indices for each sector are defined as follows:

$$P_t^i = \left[(1 - \alpha_i)(P_t^{i,o})^{1-\theta_t} + \alpha_i (P_{t-1}^i)^{1-\theta_t} \right]^{1/(1-\theta_t)}, \tag{18}$$

for $i = s, f$.

Following Steinsson (2003), I presume that among firms that can reset prices in each sector, $1 - \lambda$ number of firms sets prices optimally, whereas the remaining fraction of λ

sets prices according to the following rule-of-thumb:

$$P_t^{i,r} = P_{t-1}^{i,o}\left(\frac{P_{t-1}^i}{P_{t-2}^i}\right)\left(\frac{Y_{t-1}}{Y_{t-1}^n}\right)^{\delta_i}, \tag{19}$$

for $i = s, f$, where $P_t^{i,r}$ (for $i = s, f$) denotes the price of firms that employ rule-of-thumb pricing; Y_{t-1} is aggregate output in period $t-1$, Y_{t-1}^n represents efficient output in period $t-1$, and parameter δ_i (for $i = s, f$) is the weight of the lagged output gap in the rule-of-thumb pricing rule. I assume that the fraction of firms that sets prices optimally differs in sector i in order to examine the effect of sectoral inflation persistence on monetary policy. More concretely, the parameter λ_s is the fraction of firms that follow a rule-of-thumb in sector S. Similarly, the parameter λ_f corresponds to the case of sector F. Following the empirical fact argued in the introduction, this study assumes that backward-looking firms exist in the sticky-price sector, whereas sector F's inflation is depicted by a purely forward-looking component.[8]

In this case, the price index for sector i is significantly affected by price setters who use information in the rule-of-thumb pricing rule. Accordingly, the price index that firms can change their prices becomes

$$P_{i,t}^o = \left[(1-\lambda_i)(P_t^{i,fl})^{1-\theta_t} + \lambda_i(P_t^{i,r})^{1-\theta_t}\right]^{1/(1-\theta_t)}, \tag{20}$$

for $i = s, f$, where $P_t^{i,fl}$ (for $i = s, f$) is the price for firms that optimally set prices. Using this equation, the price indices defined in Equation (18) now change according to

$$P_t^i = \left[(1-\alpha_i)(1-\lambda_i)(P_t^{i,fl})^{1-\theta_t} + (1-\alpha_i)\lambda_i(P_t^{i,r})^{1-\theta_t} + \alpha_i(P_{t-1}^i)^{1-\theta_t}\right]^{1/(1-\theta_t)}. \tag{21}$$

This equation implies that the presence of price setters who follow a rule-of-thumb rule becomes the source of endogenous inflation persistence.

I now present a log-linearisation of structural equations around the steady state with zero inflation. Specifically, a log-linearised variable around the steady state is expressed as $\hat{Z}_t = \log(Z_t/\bar{Z})$, where $\bar{Z}$ represents a steady state value.

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[8] As mentioned earlier, this assumption is relaxed in section 4.

Following Woodford (2003, 2010), all sector-specific disturbances have common values in the two sectors: $\bar{\varphi}_s = \bar{\varphi}_f$ and $\bar{A}_s = \bar{A}_f$. In this setup, a log-linearisation of Equation (16) becomes the following sector NKPC in terms of the real marginal cost:

$$\pi_{i,t} = \gamma_f^i E_t \pi_{i,t+1} + \gamma_b^i \pi_{i,t-1} - \chi_{i1}(\hat{Y}_t - \hat{Y}_t^n) + \chi_{i2}(\hat{Y}_{t-1} - \hat{Y}_{t-1}^n) + \varsigma_i(\hat{\psi}_t^i - \hat{p}_{i,t}) + u_{i,t}, \quad (22)$$

for $i = s, f$. Each coefficient in Equation (22) is given by

$$\gamma_f^i = \frac{\alpha_i \beta}{\alpha_i + \lambda_i(1 - \alpha_i(1-\beta))}, \gamma_b^i = \frac{\lambda_i}{\alpha_i + \lambda_i(1 - \alpha_i(1-\beta))}, \chi_{i1} = \frac{(1-\alpha_i)\alpha_i\lambda_i\beta\delta_i}{\alpha_i + \lambda_i(1 - \alpha_i(1-\beta))}$$
$$\chi_{i2} = \frac{(1-\alpha_i)\lambda_i\delta_i}{\alpha_i + \lambda_i(1 - \alpha_i(1-\beta))}, \varsigma_i = \frac{(1-\alpha_i)(1-\alpha_i\beta)(1-\lambda_i)}{[\alpha_i + \lambda_i(1 - \alpha_i(1-\beta))](1+\omega\bar{\theta})},$$

for $i = s, f$ and $\omega = v_{yy}\bar{y}/v_y$. Here, $\pi_{i,t}$ denotes the sectoral inflation rate for sector i, and $u_{i,t}$ is the sectoral cost-push shock associated with a time-varying price mark-up and $\hat{p}_{i,t}$ (for $i = s, f$) denotes the relative price index for sector i. In contrast to Byrne et al. (2013), the assumption of a rule-of-thumb used by Steinsson (2003) allows the NKPC in each sector to depend on the lagged output gap. In a two-sector model, the log-linearised real marginal costs [Equation (14)] are given as follows:

$$\hat{\psi}_t^i - \hat{p}_{i,t} = (\omega + \sigma^{-1})(\hat{Y}_t - \hat{Y}_t^n) - (1 + \omega\eta)(\hat{p}_{i,t} - \hat{p}_{i,t}^n), \quad (23)$$

for $i = s, f$. Here, $\hat{p}_{i,t}^n$ is the relative price index for sector i in a flexible-price equilibrium. The parameter σ is defined as $\sigma = -u_{yy}\bar{Y}/u_y$.

Substituting Equation (23) into Equation (22) yields the following NKPC expressed by the output gap:

$$\pi_{i,t} = \gamma_f^i E_t \pi_{i,t+1} + \gamma_b^i \pi_{i,t-1} + \kappa_i(\hat{Y}_t - \hat{Y}_t^n) + \chi_{i2}(\hat{Y}_{t-1} - \hat{Y}_{t-1}^n) + \gamma_i(\hat{p}_{R,t} - \hat{p}_{R,t}^n) + u_{i,t}. \quad (24)$$

for $i = s, f$, where $\hat{p}_{R,t}$ represents a measure of the relative price of the two sectoral composite goods, which is defined as $\hat{p}_{R,t} = \hat{P}_t^f - \hat{P}_t^s$. Here, $\hat{p}_{R,t}^n$ is the relative price under the efficient level of output. Each coefficient in Equation (24) is defined as follows:

$$\kappa_i = \varsigma_i(\omega + \sigma^{-1}) - \chi_{i1},$$

for $i = s, f$, and

$$\gamma_s = n_f \varsigma_s (1 + \omega\eta) > 0, \gamma_f = n_s \varsigma_f (1 + \omega\eta) < 0.$$

A one-sector model does not need to consider the effect of relative prices between two sectors on the real economy. However, relative prices play an important role in a two-sector model (Aoki, 2001; Woodford, 2010). In contrast to the purely forward-looking two-sector model, it follows from Equation (24) that the presence of sectoral heterogeneities of inflation persistence significantly affects the real economy through the relative price between two sectors.

The intuition behind this result is as follows. Consider the case where a cost-push shock occurs in sector S. This cost-push shock will lead to an increase in core inflation but a decrease in the output gap. As inflation persistence in sector S becomes more severe, the central bank faces an even starker trade-off between core inflation and the output gap. The persistence of core inflation negatively propagates to sector F's inflation through a change in relative prices between the two sectors. Therefore, the existence of more severe inflation persistence in sector S produces a greater trade-off between core inflation and sector F's inflation. This results in huge output gap drops through both NKPCs. Therefore, in contrast to the situation in a one-sector model, the central bank here should focus on a trade-off between not only inflation and the output gap but also between inflation in each sector.

Log-linearising Equation (10) yields the following dynamic IS equation:

$$x_t = E_t x_{t+1} - \sigma(\hat{R}_t - E_t \pi_{t+1} - \hat{r}_t^n), \tag{25}$$

where $\hat{R}_t$ is the log-linearised gross nominal interest rate, π_t is CPI inflation and $\hat{r}_t^n$ is the natural rate of interest, which holds the real interest rate in the efficient level of output and, which is given by

$$\hat{r}_t^n = \sigma^{-1}[(g_t - \hat{Y}_t^n) - E_t(g_{t+1} - \hat{Y}_{t+1}^n)], \tag{26}$$

where g_t is an exogenous disturbance associated with preference shocks.

Finally, relative prices between two sectors evolve according to

$$\tilde{p}_t^r = \tilde{p}_{t-1}^r + \pi_{f,t} - \pi_{s,t} + e_t, \tag{27}$$

where $\tilde{p}_t^r$ is the relative price gap, which is defined as $\hat{p}_{R,t} - \hat{p}_{R,t}^n$, and e_t denotes the exogenous shock for relative prices.

2.3 Optimal monetary policy

This section describes the optimal monetary policy in a two-sector economy with inflation persistence. Rotemberg and Woodford (1997) and Woodford (2001, 2003) showed that the second-order approximation of the household's utility function corresponds to the central bank's loss function, which features variables denoting the squared inflation rate and squared output gap.

Both Aoki (2001) and Woodford (2003) derived the central bank's loss function in a two-sector economy. In Woodford (2003), the loss function contains both the quadratic term of the relative price gap and the squared inflation rate in each sector. The presence of the relative price gap is associated with an inefficient relative level of the two sectors' goods. The inflation stabilisation is characterised by a weighted average of inflation in both sticky goods and flexible goods. Unlike in the one-sector model, Woodford (2003, 2010) argues that the central bank stabilises the weighted average of sectoral inflation because price stickiness generates price dispersions in each sector.

I address that inflation persistence in each sector changes the shape of the central bank's objective function in a two-sector economy with inflation persistence. Thus, the shape of the loss function is more complicated in a two-sector model than in a one-sector model because in the former, the fraction of firms that follow a rule-of-thumb pricing rule depends on sector i. This study derives the central bank's loss function approximated around a steady state. More specifically, it calculates the second-order approximation of the household's utility function as follows:

$$U_t \simeq \sum_{t=0}^{\infty} \beta^t L_t + t.i.p. + O(||\xi||^3), \tag{28}$$

where *t.i.p.* includes the terms that are independent of monetary policy, and $(||\xi||^3)$ indicates the terms of third or larger orders. Here, the central bank's periodic loss function is given by

$$\begin{aligned} L_t = {} & w_s \pi_{s,t}^2 + w_f \pi_{f,t}^2 + \lambda_x x_t^2 + \lambda_r (\tilde{p}_{R,t} - \tilde{p}_{R,t}^n)^2 + w_1^s \Delta\pi_{s,t}^2 + w_1^f \Delta\pi_{f,t}^2 \\ & + (w_2^s + w_2^f) x_{t-1}^2 + w_3^s \Delta\pi_{s,t} x_{t-1} + w_3^f \Delta\pi_{f,t} x_{t-1}, \end{aligned} \tag{29}$$

where

$$w_i = \frac{n_i \kappa}{\kappa_i}, \kappa_i = \frac{(1-\alpha_i)(1-\alpha_i\beta)(\omega+\sigma^{-1})}{\alpha_i(1+\omega\bar{\theta})}, w_2^i = \frac{n_i(1-\alpha_i)^2\lambda_i\delta_i}{\alpha_i(1-\lambda_i)}, w_1^i = \frac{n_i\lambda_i}{\alpha_i(1-\lambda_i)},$$
$$w_3^i = -\frac{2n_i(1-\alpha_i)\lambda_i\delta_i}{\alpha_i(1-\lambda_i)},$$

for $i = s, f$ and

$$\lambda_x = \frac{\kappa}{\bar{\theta}}, \lambda_r = \frac{n_s n_f (1+\omega\eta)\lambda_x}{1+\omega\bar{\theta}}, \kappa = (n_s \kappa_s^{-1} + n_f \kappa_f^{-1})^{-1}.$$

Variable x_t $(=\hat{Y}_t - \hat{Y}_t^n)$ denotes the output gap. The technical appendix provides a detailed derivation of Equation (29).[9] Although Kosem-ALP (2010) also derives the central bank's loss function in the presence of sectoral heterogeneities of inflation persistence, he assumed both Gali and Gertler (1999)'s specification of the rule-of-thumb pricing rule and a Cobb-Douglas consumption index. In addition, under his specification, sector F's inflation is specified by flexible prices. In contrast, this paper assumes both Steinsson (2003)'s specification of the generalised rule-of-thumb pricing rule and a CES consumption aggregate. Therefore, compared with the results of Aoki (2001) and Kosem-ALP (2010), the shape of the central bank's loss function is more complicated in this paper.

Equation (29) includes the weighted average of inflation in sectors S and F. In addition, in contrast with the two-sector forward-looking model, the central bank's loss function now includes a quadratic term for the change in each sector's inflation rate. Unlike the one-sector NK model with inflation persistence, the central bank should instead stabilise changes in the inflation rate in both sectors. The stabilisation term for a change in inflation for each sector is characterised by the weighted average of inflation

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[9] See Appendix A for a detailed derivation of the loss function.

for goods S and F. Moreover, in contrast with one-sector models, here, the central bank must also stabilise the quadratic term for the output gap in period $t-1$ in both sectors because firms that adopt rule-of-thumb pricing refer to information from the lagged output gap. In contrast to Steinsson's (2003) model, the stabilisation term for the lagged output gap is affected by both price stickiness and degree of inflation persistence in both sectors. In a benchmark economy, I assume that no inflation persistence exists in sector F. Therefore, the terms associated with the parameter λ_f are eliminated in the loss function.[10] When inflation persistence is absent in this economy, the loss function reduces to the one derived by Aoki (2001) and Woodford (2003, 2010).

Accordingly, the presence of sectoral inflation persistence complicates the central bank's optimisation problem. Concretely, the central bank minimises Equation (29) subject to the hybrid NKPC in both sectors and the law of motion for relative prices between the two sectors. In the following central bank's optimisation problem, this study focuses on a commitment policy.[11] The central bank implements its monetary policy under the promise that it credibly commits to future monetary policy at the current period. It regards the commitment policy as an optimal monetary policy from a timeless perspective, as suggested by Woodford (2003).[12] Under this commitment policy, the central bank can conduct monetary policy by manipulating private sector expectations.

Suppose that ϕ_{1t} and ϕ_{2t} are Lagrange multipliers associated with sector NKPCs and that ϕ_{3t} is the Lagrange multiplier in terms of the identity of the relative price. Defining the Lagrangian, the following first order conditions under a commitment policy

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[10] In section 4, I relax this assumption.

[11] In section 4.2, I compare the performance of a commitment solution with that of a discretionary policy. I explain optimal monetary policy with discretion in section 4.2.

[12] Woodford (2003) provides a detailed discussion of a timeless perspective approach.

are derived:

$$\begin{aligned}
&\pi_{s,t}:\ 2w_s\pi_{s,t} + 2w_1^s\Delta\pi_{s,t} - 2\beta w_1^s E_t\Delta\pi_{s,t+1} + 2w_3^s(x_{t-1} - \beta x_t) - 2\beta^{-1}\gamma_f^s E_t\phi_{1t+1} \\
&+ 2\beta\gamma_b^s\phi_{1t-1} + \phi_{1t} + \phi_{3t} = 0, \qquad (30) \\
&\pi_{f,t}:\ 2w_s\pi_{f,t} + 2w_1^f\Delta\pi_{f,t} - 2\beta w_1^f E_t\Delta\pi_{f,t+1} + 2w_3^f(x_{t-1} - \beta x_t) - 2\beta^{-1}\gamma_f^f E_t\phi_{2t+1} \\
&+ 2\beta\gamma_b^f\phi_{2t-1} + \phi_{2t} - \phi_{3t} = 0, \qquad (31) \\
&x_t:\ 2[\lambda_x + \beta(w_2^s + w_2^f)]x_t + w_3^s\beta E_t\Delta\pi_{s,t+1} + w_3^f\beta E_t\Delta\pi_{f,t+1} - 2\kappa_s\phi_{1t} - 2\beta\chi_{s2}E_t\phi_{1t+1} \\
&- 2\kappa_f\phi_{2t} - 2\beta\xi_{f2}E_t\phi_{2t+1} = 0, \qquad (32) \\
&\tilde{p}_{R,t}:\ \lambda_r\tilde{p}_{R,t} - \gamma_s\phi_{1t} - \gamma_f\phi_{2t} + \phi_{3t} - \beta E_t\phi_{3t+1} = 0. \qquad (33)
\end{aligned}$$

Combining these conditions with structural equations, the economic system can be written as follows:

$$A_0X_t = A_1X_{t-1} + B_1R_t + \Gamma_{t+1}, \tag{34}$$

where

$$\begin{aligned}
X_t &= [X_{1t}X_{2t}]' \\
X_{1t} &= [u_{s,t},\ u_{f,t},\ \tilde{p}_{r,t}^n.\ \pi_{s,t-1}\ \pi_{f,t-1},\ x_{t-1},\ \tilde{p}_{r,t}, \phi_{3t}]', \\
X_{2t} &= [\pi_{s,t},\ \pi_{f,t},\ x_t,\ \phi_{1t},\ \phi_{2t}]',
\end{aligned}$$

In addition, A_0, A_1, and B_1 are matrices constructed by deep parameters, R_t denotes a vector of the policy instrument, and Γ_t denotes a shock vector. This study uses the algorithm method developed by Soderlind (1999) to numerically explore the optimal plan of the above commitment policy and illustrate how inflation persistence will change the property of the optimal monetary policy in a two-sector NK model.

3 Calibration

This section describes the deep parameters used in this paper. The discount factor β is set to 0.99. The relative risk-aversion coefficient for consumption σ^{-1} is 5.0 based on the value in Steinsson (2003). Following Steinsson (2003), I also assume that the parameter ω equals 2.0. Following Woodford (2003), I set the elasticity of substitution between goods θ to 7.88. In addition, based on the existing literature, for the value of elasticity of substitution between goods S and F, parameter η is set to 1.5 as a benchmark calibration. With regard to degree of price stickiness, following Woodford (2010), I set a Calvo lottery in sector S to 0.75 and to 0.55 in sector F for the benchmark specification. As mentioned previously, this parameterisation implies that sector F's prices are more flexible than those in sector S. This assumption is consistent with Woodford (2010) and Airaudo and Zanna (2012).[13] After a benchmark analysis, I will relax this assumption.[14]

Following Steinsson (2003), the parameter δ_s is set to 0.052. This paper further sets δ_f to zero.[15] Following Woodford (2010), the share of sticky goods in the CES aggregator of two goods is set to 0.5. Thus, parameter n_1 is set to 0.5. Finally, I assume that a cost-push shock in each sector is temporary, whereas the persistence of a relative price shock is set to 0.9. The standard deviations of a cost-push shock in each sector and a relative price shock are both set to 0.01.

4 Results

This section presents the main results. I present the impulse-response analyses and show how the degree of inflation persistence in sector S changes the property of the optimal monetary policy. In this impulse-response analysis, as mentioned earlier, I assume inflation persistence in sector S, whereas in sector F, the inflation dynamics are purely forward-looking.

In the impulse-response analysis, I follow Steinsson (2003) and I select four values for parameter λ_s. First, I set λ_s to 0.01 in the case of the forward-looking economy. Then, I

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[13] In Airaudo and Zanna (2012), NKPCs are derived from the assumption of the quadratic price adjustment costs. Therefore, they consider that the degree of price adjustment costs are different in the each sector.

[14] In section 4, with respect to these parameters, sensitivity analyses are implemented.

[15] This assumption is natural in the case of a benchmark calibration. I confirm that the results remain unchanged by a change of this parameter.

set it to 0.2, which is empirically supported by Gali and Gertler (1999). The third value of 0.7 is empirically supported by Fuhrer and Moore (1995), who argued that backward-looking inflation is important in the hybrid NKPC. A larger value of the parameter λ_s is supported by previous studies for estimation of the NKPC (e.g., Rudd and Whelan, 2007; Kurmann, 2007). Finally, I set λ_s to 0.90. This value is much larger value, which corresponds to the case in which backward-looking components predominately affect inflation dynamics.

4.1 Impulse response analyses

Figure 1 displays the impulse response to a cost-push shock occurring in sector S. The cost-push shock increases core inflation and leads to a decline in the output gap. Thus, the shock generates a trade-off between inflation and the output gap (Clarida, Gali and Gertler, 1999; Woodford, 2003). The stronger the persistence of core inflation, the greater the trade-off. This result is consistent with the findings of Amato and Laubach (2003) and Steinsson (2003). In a forward-looking economy, the central bank, which has the ability to manipulate private sector expectations, can alleviate trade-off between inflation and the output gap. As inflation persistence becomes more severe, however, the central bank needs to accommodate a substantial decline in the output gap instead of preventing an increase in inflation.

[Figure 1 around here]

In the two-sector model, a trade-off exists between inflation in each sector. Unlike previous studies, this paper considers that the presence of inflation persistence in sector S generates a worse trade-off between sectors' inflation. In this case, a cost-push shock induces an increase in core inflation although it decreases inflation in sector F. Figure 1 reveals that the stronger the inflation persistence in sector S, the worse the trade-off between sectoral inflation. Because sector F's prices are more flexible than those in sector S, fluctuations in sector F's inflation are more volatile than those affecting core inflation.

This result is highly instructive. If the central bank aims to stabilise CPI inflation, it should focus on stabilising headline inflation when a cost-push shock exists in sector

S. However, the central bank might react to movements in each sector's inflation when a cost-push shock occurs in sector S due to concerns of a trade-off between sectors' inflation in its objective function. If inflation persistence is more severe in sector S, therefore, the central bank's attempts to stabilise CPI inflation might induce an imprecise reaction of its policy rate to the shock. This indicates that the central bank's intention to stabilise CPI inflation may be guided by a misleading monetary strategy. In other words, the central bank should be cautious regarding changes in CPI inflation when sectoral heterogeneities of inflation persistence play a significant role in the economy.

Figure 2 illustrates the impulse response to a cost-push shock in sector S following changes in several key parameters. Figure 2(a) depicts the case of an increase in η, which shows that as the value of λ_s increases, a larger value of η amplifies the persistent drop in the relative price gap. The drop in output gap is also amplified. On the other hand, a larger value of η alleviates the trade-off in inflation between sectors. This result is more predominate in the case of $\lambda_s = 0.7$. In contrast, Figure 2(b) shows that a smaller value of η worsens the trade-off between the output gap and relative price gap as inflation persistence in sector S is more severe.

[Figure 2 around here]

Figure 2(c) presents the case wherein prices are more flexible in sector F. In this scenario, the output gap and sector S's inflation remain unaffected except for the case where a larger value of λ_s affects sector F's inflation as well as the relative price gap. This result continues to hold in Figure 2(d), where a larger value of λ_f affects only sector F's inflation and the relative price gap. Thus, the results in Figure 1 are robust to changes in any of the key parameters.

Figure 3 shows the impulse response when the cost-push shock occurs in sector F. Such a shock increases inflation in sector F while leading to a decline in the output gap, thereby reducing core inflation through the heterogeneous NKPC. Compared with the case depicted in Figure 1, however, the shock does not introduce severe inflation persistence in each sector even if inflation persistence exists in sector S. Hence, sector S's inflation persistence does not lead to a persistent decline in the output gap. This result is not surprising. Given that the shock's source is associated with a temporary cost-push shock in sector F, where prices are more flexible, inflation persistence in sector S does not have a long-lasting effect on the economy. Consequently, changes in inflation persistence in sector S appear to affect the trade-off only during the initial period.

[Figure 3 around here]

Importantly, Figure 3 reaffirms the observation that sector S's inflation persistence induces a worse policy trade-off between inflation and the output gap, although it dampens the trade-off in inflation between sectors. Intuitively, because the source of the shock in sector F is characterised by firms' forward-looking behaviour, the central bank can impart policy inertia into the economy by using sector F's forward-looking inflation dynamics. This implies that if the source of the shock is related to sector F, the central bank can still manipulate sector F's inflation expectations even when severe inflation persistence is present in sector S. As a result, in contrast to the case of the cost-push shock occurring in sector S, a higher severity of inflation persistence in sector S does not cause persistent fluctuations in the macro-economy. This mechanism is specific to a two-sector model and is in contrast to a one-sector model.

[Figure 4 around here]

Figure 4 shows the impulse response to a cost-push shock in sector F following changes in several key parameters. First, Figures 4 (a) and 4(b) show that macro-variables are unaffected by any changes in parameter η. However, Figure 4 (c) indicates that when there are more flexible prices in sector F will attenuate considerably the trade-off between sectors' inflation. Moreover, a smaller value of α_f can ease the trade-off between the output gap and relative price gap. On the other hand, moderate inflation persistence exists in sector F causes the trade-off between sector F's inflation and the output gap to become more severe when sector S's inflation is mainly depicted by forward-looking components. Interestingly, such a trade-off is eased in the case of $\lambda_s = 0.7$. This indicates that although a cost-push shock in sector F causes a fluctuations in macro-variables, inflation persistence in sector S helps alleviate such a large fluctuations.

Figure 5 illustrates the impulse response to a relative price shock. Fluctuations in relative prices generate a policy trade-off between each sector's inflation through that sector's NKPCs. As shown in Figure 3, the shock causes the core inflation rate to decline, whereas it increases inflation in sector F.

[Figure 5 around here]

Because sector F's prices are more flexible than those in sector S, the response of inflation in sector F would dominate that of core inflation. Thus, the output gap declines through both NKPCs. Therefore, this relative price shock results in a trade-off between not only sector F's inflation and the output gap but also inflation between sectors. Figure 5 shows that because sector S's inflation persistence is more severe, such inflation inertia weakens the trade-off in inflation between sectors. Instead of attenuating this trade-off between sectors' inflation, however, the central bank must allow a huge decline in the output gap. Intuitively, severe inflation persistence in sector S forces relative prices to move slowly, leading to a smaller adjustment in sector inflation. However, such a sizable decline in the output gap is required in order to accommodate a small adjustment in sectoral inflation during the initial period. Consequently, the central bank accommodate a higher output gap in subsequent periods. In other words, in the case of a larger value of parameter λ_s, the central bank can succeed in restraining fluctuations in sector inflation through a change in relative prices between two sectoral goods, although it will therefore face a large decline in the output gap.

Figure 6 shows the impulse response to a relative price shock following changes to several key parameters. According to Figure 6(a), when λ_s takes a larger value, a larger value of η serves to amplify the trade-off between sector F's inflation and the output gap. In addition, a trade-off in inflation between sectors is also more severe. Finally, a larger value of η dampens a fluctuation in the relative price gap. On the other hand, Figure 6(b) demonstrates that a smaller value of η does not affect any macro-variables. Figure 6(c) shows that, Figure 5's results remain unchanged when sector F's prices are more flexible. Figure 6(d) illustrates that as in the case shown in Figure 4(b), the presence of inflation persistence in sector S helps ease such large fluctuations of macro-variables generated by a relative price shock.

[Figure 6 around here]

The results of the impulse-response analyses can be summarised as follows. Sectoral heterogeneities of inflation persistence play a significant role in the two-sector NK model. Importantly, this paper reveals that inflation persistence in sector S worsens the trade-off not only between inflation and the output gap but also between inflation in each sector when a cost-push shock is generated in sector S. These results also imply that whether or not CPI inflation has a persistent effect crucially depends on the shock's origin. This in turn indicates that the central bank should carefully consider the significance of sectoral inflation persistence if it aims to improve the policy trade-off between CPI inflation and the output gap. In contrast to previous studies, this study demonstrates that a two-sector NK model with inflation persistence has important implications for optimal monetary policy.

4.2 The gain from commitment

This section examines gains from a commitment policy in a two-sector economy with inflation persistence. It compares the performance of a commitment policy with that of a discretionary policy. In contrast to a commitment policy, a discretionary policy enables a central bank to re-optimise its loss function in every period, given future expectations of the private sector. In a purely forward-looking model, the performance of a commitment policy generally dominates that of a discretionary policy, in that the latter cannot use private sector expectations.[16] As Amato and Laubach (2003) and Steinsson (2003) have shown, severe inflation persistence reduces the gain from commitment by counteracting the power to manipulate the private sector's expectations. It remains unclear, however,

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[16] See, Woodford, (2003), Walsh (2003), McCallum and Nelson (2004), Gali (2008), Dennis (2010) and Walsh (2017) for a detailed discussion of the gains from a commitment policy in a purely forward-looking NK model.

whether gains from commitment are negligible when inflation persistence is severe in a two-sector model.

This section aims to answer this question. To do so, I calculate the central bank's unconditional welfare loss for both commitment and discretion. Gains from commitment are defined as follows:

$$Welfare\ gain = \left(\frac{L_{disc}}{L_{com}} - 1 \right) \times 100, \tag{35}$$

where L_{com} denotes the welfare loss under commitment and L_{disc} is the one under discretion. Thus, Equation (35) indicates gains from commitment will occur as long as this value exceeds zero. In this simulation, as in the case of a commitment solution, I solve the optimal monetary policy with discretion using Soderlind (1999)'s methods.

So far, in section 4.1, this paper assumed that inflation is purely forward-looking in sector F. However, sensitivity experiments in impulse response analyses have revealed that a change in parameter λ_f will affect macroeconomic dynamics. One important example is that when inflation persistence is present in sector F, inflation persistence in sector S is able to counteract fluctuations in macro-variables unless the shock was generated by a cost-push shock associated with sector S. Thus, the presence of inflation persistence in both sectors might affect the performance of a commitment policy. Therefore, I examine the case in which inflation is also characterised by inflation persistence in sector F. This examination is a quite natural extension simply by assuming that flexible goods inflation can also become inertial behaviour.

Figure 7 shows the result of this examination. Interestingly, when core inflation is depicted by a purely forward-looking component, the maximum gain is attained in the case of $\lambda_f = 0.99$. Indeed, Figure 7 indicates that the maximum gain in that case is approximately 4%. It seems that this value is smaller than ones reported for a one-sector model (cf. Steinsson, 2003; Amato and Laubach, 2003). It can also be seen that a larger value of parameter λ_s reduces the welfare gain from commitment in the case where sector F does not contain inflation persistence. Even if inflation persistence is severe in sector F, no gain will occur as long as severe inflation persistence exists in sector S. This outcome implies that whether or not the gain from commitment is large crucially depends on the degree of inflation persistence present in sector S.

[Figure 7 around here]

We can interpret this result as indicating that inflation persistence for core inflation plays a significant role in two-sector NK models. In fact, Figure 1 indicates that a larger value of parameter λ_s produces a persistent effect on macro-variables in the case of a cost-push shock for core inflation. Furthermore, even when shocks originate with a sector F's cost-push shock or relative price shock, sector S's inflation persistence creates a non-negligible effect on endogenous variables (cf., Figures 2 to 6). This result is supported by the fact that these figures demonstrate that a larger value of λ_s generates a substantial decline in both the output gap and relative price gap, whereas it can ease a trade-off in inflation between sectors. As a consequence, regardless of the value of λ_f, almost no gain occurs when λ_s takes a larger value. Overall, the results in Figure 7 are consistent with those of the impulse-response analyses.

[Figure 8 around here]

Next, this paper checks whether welfare gains arise from commitment when both α_s and α_f change under several parameterisations of λ_s and λ_f.[17] Figure 8 reports the welfare gain when moderate inflation persistence exists in both sectors. This figure shows that the gain disappears when prices in both sectors are more flexible. Both sectors see their price dispersions disappear when their prices become more flexible. Therefore, the burdens on price stability are reduced for the central bank in this case. In other words, the welfare gain from price stability declines. The gain is also negligible when prices are sticky in both sectors. As in the one-sector model, this occur because in such a case, the central bank cannot fully manipulate the private sector's expectation. Because the NKPCs for each sector flatten, the gain from commitment disappears. This result is consistent with those reported by Walsh (2003) and Steinsson (2003).

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[17] I do not report the case of $\lambda_s = \lambda_f = 0.01$. I did check that the result of Figures 8 and 9 remains unchanged in that case.

Interestingly, unlike in the one-sector model, gains arise from commitment when prices are sticky in either sector S or F. Consider the case of price stickiness in sector S. Although sector F's inflation immediately jumps in response to shocks, core inflation does not. In addition, inflation persistence in sector S forces core inflation' to make more persistent movements. These mechanisms, in turn, indicate that a central bank that stabilises sectoral inflation can smooth inflation dynamics through appropriate management of expectations; therefore, it can achieve the welfare gain from commitment. It follows from this figure that gains arise for the case of price stickiness in sector F. The intuition of this result is the same as the interpretation of the case for sector S.

Figure 9 shows the welfare gain from commitment when α_s and λ_s change in the case where firms in both sectors completely follow a backward-looking pricing rule (i.e. $\lambda_s = \lambda_f = 0.99$). In this case, regardless of the degree of price stickiness in both sectors, almost no gain from commitment arises. Specifically, the gain from commitment is only 1% when prices are sticky in both sectors. As discussed repeatedly, this occurs because the central bank loses its ability to manage the private sector's expectations.

[Figure 9 around here]

The results for the sensitivity analyses when several parameters change are as follows. Here, I focus on a change in the value of λ_s based on the results from Figures 7-9. Figure 10 calculates the welfare gain from commitment when parameters α_s and λ_s change. In general, higher nominal price rigidities cause welfare loss through bigger price dispersions. In addition, as shown in previous studies, the more severe inflation persistence becomes, the smaller the gain from a commitment becomes because the central bank cannot fully manage private sector expectations when inflation persistence is severe.

[Figure 10 around here]

According to Figure 10, when sector F's inflation dynamics are determined by the purely forward-looking component, no gain from commitment arises when the degree of nominal price rigidities is considerably small in sector S. This result is not affected by the degree of parameter λ_s. Intuitively, price dispersion in sector S would be eliminated because sector S's prices are close to flexible ones, which means that the central bank can enhance social welfare by putting a smaller weight on inflation stabilisation. Similarly, if prices are completely flexible, almost no gain from commitment would arise. In other words, this case implies that the difference between discretion and commitment is negligible.

In contrast, when the degree of inflation persistence is considerably small in sector S (i.e. $\lambda_s = 0.01$), the central bank can attain a larger gain from commitment because prices are stickier. As explained earlier, in a sticky-price equilibrium, the central bank can ease the policy trade-off between inflation and output gap by managing the private sector's expectations. This result is thus consistent with the argument of the standard NK model. For instance, the welfare gain from commitment exceeds 10% if most firms cannot set their prices optimally.

However, the gain from commitment decreases if inflation persistence becomes more severe in sector S. Because the central bank cannot fully manipulate inflation expectations in sector S, it cannot prevent a huge decline in the output gap. The gain from commitment considerably decreases when inflation dynamics are predominately backward-looking in sector S (i.e. $\lambda_s = 0.99$). Notably, when price stickiness is extremely high, the central bank still produces a smaller welfare gain of approximately 4% if inflation persistence is severe. This result contrasts the one obtained by Steinsson (2003).

According to Figure 11, for a smaller value of parameter λ_s, the welfare gain is smaller when parameter η takes a larger value. A larger value of parameter η means higher substitutability between goods S and F. Thus, there is almost no difference between the two sectors. This means that the central bank does not need to focus on the difference in sectoral inflation levels. Hence, higher substitutability leads to no difference between commitment and discretion because higher substitutability increases the flexibility of sector S's prices. Interestingly, opposite effects emerge if parameter λ_s takes a larger value. In that case, higher substitutability between goods S and F produces a larger welfare gain from commitment. Indeed, in the case of a larger value of parameter η, the maximum gains from commitment are achieved when parameter λ_s takes a value from

0.6 to 0.8. For instance, in the case of $\eta = 10$, the welfare gain is approximately 8% when $\lambda_s = 0.8$. Interestingly, this result is also in contrast to the result of Steinsson (2003).

[Figure 11 around here]

The intuition of this result is as follows. Sector F's inflation jumps immediately after the shock occurs because this inflation is more flexible and forward-looking. However, core inflation changes sluggishly in response to the shock. The difference in inflation between sectors is much smaller when parameter η takes a larger value. As the impulse-response analyses showed, sector F's inflation dominates core inflation. In this case, if the central bank can use the private sector's expectations through sector F's inflation, such a policy may induce an imprecise reaction to the shock by the nominal interest rate, therefore destabilising the economy. If goods S and F are not completely substitutable, however, the presence of inflation persistence in sector S might prevent inflation in sector S from jumping immediately in response to the shock. Hence, the central bank can appropriately impart policy inertia into the economy. This is the source of the gain from commitment.

Figure 12 demonstrates the welfare gain from a commitment policy when parameters λ_s and θ change. Parameter θ represents the elasticity of substitution between individual goods in each sector. This value affects the structural parameters in NKPCs for each sector. A larger value of this parameter lowers the sensitivity of both relative prices between two sectoral goods and the output gap to inflation in each sector. In other words, a higher-value parameter θ flattens the NKPC, thereby implying that the central bank needs to accommodate a large fluctuation in relative prices as well as the output gap to stabilise sectoral inflation.

[Figure 12 around here]

Furthermore, unless inflation persistence is severe in sector S, a larger value of parameter θ decreases the welfare gain from commitment because the central bank faces a severe trade-off among relative prices, the output gap and sectoral inflation in the case of a flattened NKPC in each sector. In fact, Walsh (2003) showed that discretion performs better than commitment in an extremely flattened NKPC. This paper confirms these findings of Walsh (2003) for a two-sector NK model with inflation persistence. Furthermore, it is interesting that in the case of a larger value of the parameter λ_s, the central bank obtains the gain from commitment increases as the value of parameter θ increases. More concretely, in the case of $\theta = 14$, the maximum gain is around 5% when $\lambda_s = 0.8$. This might be in contrast to findings of previous studies that used a one-sector NK model.

Intuitively, even in the two-sector model, a larger value of parameter θ counteracts the market power of firms in each sector. In addition, inflation responds strongly to a shock in sector F, where prices are flexible and price setting is forward-looking. Even if sector S loses market power in its own sector, core inflation might not jump immediately in response to the shock due to the presence of severe inflation persistence. Therefore, the presence of sector S's inflation persistence drives a wedge between inflation in each sector. This mechanism can create scope for the gain from commitment to emerge. Hence, a larger value of parameter θ indicates that a central bank aiming to stabilise sectoral inflation can contain a large fluctuation in both relative prices and output gap. Therefore, it is possible that commitment leads to a superior performance compared with discretion.

5 Conclusions

It has debated whether the presence of inflation persistence will significantly affect the gain from a commitment policy in an NK model. Some have argued that as long as inflation persistence is not predominately backward-looking, gains will arise from a commitment policy. Recent studies have also addressed the role of sectoral heterogeneities of inflation persistence. Although the gain from commitment in the presence of inflation persistence has been reported for a one-sector economy, the gains from commitment in a two-sector model remained ambiguous.

This paper suggested that inflation persistence in one-sector affects inflation in another sector through a change in the relative price between two sectoral goods, leading to a large fluctuation in the output gap. Trade-offs following cost-push shocks in a sticky-price sector are particularly severe. However, trade-offs following cost-push shocks in a flexible-price sector are attenuated. In addition, the central bank faces a trade-off between CPI inflation and the output gap in a one-sector model, whereas this study addresses the scenario where sectoral heterogeneities of inflation persistence produces a severe trade-off not only between inflation and the output gap but also between each sector's inflation in a two-sector model. These results contrast the ones obtained for the one-sector model. This paper, therefore, emphasises that the central bank needs to take into account these mechanisms of sectoral heterogeneities of inflation persistence when implementing the optimal monetary policy.

This study focused on the mechanisms through which sectoral inflation persistence substantially influences the gain from commitment. The main findings of welfare analyses are as follows. Regardless of degree of inflation persistence in the flexible-price sector, the gain will arise as long as core inflation is depicted by a forward-looking component. Therefore, the gain from commitment decreases when core inflation is backward-looking. Naturally, the gain is small when sectoral heterogeneities of inflation persistence are considerably more severe in both sectors. When moderate inflation persistence is present in both sectors, gains occur unless prices are more flexible in both sectors. However, the gain from commitment vanishes again when prices are stickier in both sectors.

This study focused on the property of a commitment policy from a timeless perspective in a standard NK model with sectoral heterogeneities of inflation persistence. How should the central bank implement its monetary policy if it cannot commit to the implementation of its future monetary policy? In this case, a possible solution is that the government delegates the loss function with policy inertia, which is different from a social loss function, to the central bank (see Jensen, 2002; Walsh, 2003; Bilbiie, 2014). It will be interesting for future research to examine whether gains from several targeting regimes is large. In addition, as Cogley and Sbordone (2008) and Ascari and Sbordone (2014) argue for the importance of trend inflation to explain actual inflation dynamics, we will examine the optimal monetary policy in an economy with sectoral trend inflation. Particularly building upon the findings of this study, it may be interesting to explore how inclusion of inflation persistence changes the optimal monetary policy in such an economy.

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A Appendix: Derivation of the central bank's loss function (Not for publication)

This appendix provides a detailed derivation of the central bank's loss function in a two-sector model. As shown in studies by Woodford (2001, 2003), the loss function corresponds to the second-order approximation of the household utility function. Derivation of the central bank's loss function in this model is mainly based on Woodford (2003), but the approximation of a price dispersion term differs. In a two-sector NK model, inflation persistence in each sector significantly changes the shape of the central bank's loss function. Compared to one-sector models, the shape of the loss function is more complicated in a two-sector model because the proportion of firms that follow rule-of-thumb pricing depends on sector i.

This appendix derives the loss function approximated around a steady state. Before doing so, I define the relevant notations. First, $\bar{Z}$ denotes the value of the steady state, and Z_t^n is the value of the efficient level. Second, as in the main text, $\hat{Z}_t = \log(Z_t/\bar{Z})$ is the deviation of Z_t from the steady state.

As in Woodford (2003), I provide the second-order approximation of the household's utility function, which is given by:

$$V_t = u(C_t;\xi_t) - \int_{N_s} v(h_t^s(j);\xi_t^s)dj - \int_{N_f} v(h_t^f(j);\xi_t^f)dj, \tag{A.1}$$

The second-order approximation of the first term on the right-hand side is derived by

$$\tilde{u}(C_t;\xi_t) \simeq u_c\bar{Y}\left(\hat{Y}_t + \frac{1}{2}(1-\sigma^{-1})\hat{Y}_t^2 + \sigma^{-1}g_t\hat{Y}_t\right) + t.i.p. + O(||\xi||^3), \tag{A.2}$$

where $t.i.p.$ represents the terms that are independent of monetary policy, $O(||\xi||^3)$ indicates the terms of third or higher orders, and $\tilde{u} = u(Y;\xi)$.

Next, I derive the second-order approximation of the household's utility function. To do so, I introduce some useful equations:

$$\hat{Y}_t = \sum_{i\in\{s,f\}} n_j(1+\eta^{-1}\hat{\varphi}_{i,t})\hat{Y}_{i,t} + \frac{1}{2}n_1 n_2(1-\eta^{-1})(\hat{Y}_{s,t}-\hat{Y}_{f,t})^2 + t.i.p. + O(||\xi||^3), \quad (A.3)$$

$$\hat{Y}_{i,t} = E^i_j\hat{y}_{i,t}(j) + \frac{1}{2}(1-\bar{\theta}^{-1})var^i_j y_{i,t}(j) + O(||\xi||^3), \quad (A.4)$$

for $i = s, f$.[18] Also, $var^i_j y_{i,t}(j)$ denotes the variance of $y_{i,t}(j)$ over j.

The second-order approximation of the second and third terms on the right-hand side is derived as follows:

$$\int_{N_i}\tilde{v}(y^i_{i,t}(j);\xi^i_t)dj \simeq n_i\bar{Y}u_c\left\{(1-\Phi)E^i_j\hat{y}_{i,t}(j) + \frac{1}{2}(1+\omega)[E^i_j\hat{y}^2_{i,t}(j) + var^i_j y_{i,t}(j)] - \omega q_{i,t}\hat{y}_{i,t}(j)\right\}$$
$$+ t.i.p. + O(||\xi||^3), \quad (A.5)$$

for $i = s, f$. Also, $q_{i,t}$ denotes the firm-specific exogenous disturbance required to maintain a constant marginal disutility of the labour supply and $\tilde{v} = v(f^{-1}(y/A);\xi)$. I used the following fact held in the steady state:

$$\frac{\tilde{u}_c(\bar{Y};0)}{\tilde{v}_y(\bar{y};0)} = \frac{\bar{\theta}}{\bar{\theta}-1} \equiv 1-\Phi,$$

where $\Phi = 1/\bar{\theta}$.

Summing the second-order approximation of the disutility of labour supply in each sector and using Equations (A.3) and (A.4), the following equation is derived:

$$\int_{N_s} v(h^s_t(j);\xi^s_t)dj + \int_{N_f} v(h^f_t(j);\xi^f_t)dj$$
$$\simeq u_c\bar{Y}\left\{(1-\Phi)\hat{Y}_t + \frac{1}{2}(1+\omega)\hat{Y}^2_t - \sum_{i\in\{s,f\}} n_i(\omega q_{i,t} + \eta^{-1}\hat{\varphi}_{i,t})\hat{Y}_{i,t}\right.$$
$$\left. + \frac{1}{2}n_1 n_2(\eta^{-1}+\omega)(\hat{Y}_{s,t}-\hat{Y}_{f,t})^2 + \frac{1}{2}(\bar{\theta}^{-1}+\omega)\sum_{i\in\{s,f\}} n_i var^i_j y_{i,t}(j)\right\} + t.i.p. + O(||\xi||^3),$$
$$(A.6)$$

Combining Equations (A.2) and (A.6) yields

$$V_t \simeq -\frac{u_c\bar{Y}}{2}\left\{(\sigma^{-1}+\omega)(x_t-x^*)^2 + n_1 n_2(\eta^{-1}+\omega)x^2_{r,t} + (\bar{\theta}^{-1}+\omega)\sum_{i\in\{s,f\}} n_i var^i_j y_{i,t}(j)\right\}$$
$$+ t.i.p. + O(||\xi||^3), \quad (A.7)$$

---

[18] See Woodford (2003) for a detailed explanation of these equations.

where $x_{r,t} = x_{s,t} - x_{f,t}$ and $x_{i,t} = \hat{Y}_{i,t} - \hat{Y}^n_{i,t}$, for $i = s, f$. Also,

$$
\begin{aligned}
&(\sigma^{-1} + \omega)\hat{Y}^n_t = \sigma^{-1} g_t + \sum_{i \in \{s,f\}} n_i q_{i,t}, \\
&(\eta^{-1} + \omega)(\hat{Y}^n_{s,t} - \hat{Y}^n_{f,t}) = \eta^{-1}(\hat{\varphi}_{f,t} - \hat{\varphi}_{s,t}) + \omega(q_{f,t} - q_{s,t}), \\
&x^* = \frac{\Phi}{\sigma^{-1} + \omega}.
\end{aligned}
$$

Using the definition of the relative price gap, Equation (A.7) can be rewritten as follows:

$$
\begin{aligned}
V_t \simeq -\frac{u_c \bar{Y}}{2} \Big\{ &(\sigma^{-1} + \omega)(x_t - x^*)^2 + n_1 n_2 \eta (1 + \eta\omega)(\hat{p}_{r,t} - \hat{p}^n_{r,t})^2 + (\bar{\theta}^{-1} + \omega) \sum_{i \in \{s,f\}} n_i var^i_j y_{i,t}(j) \Big\} \\
&+ t.i.p. + O(||\xi||^3), \qquad \text{(A.8)}
\end{aligned}
$$

Furthermore, from firm j's demand in each sector, we can obtain the following equations:

$$
var^i_j \log y_{i,t}(j) = \bar{\theta}^2 var^i_j \log P_{i,t}(j),
$$

for $i = s, f$. Then, we now define

$$
\bar{P}_{i,t} = E^i_j \log P_{i,t}(j), \; \Delta^j_{i,t} = var^i_j \log P_{i,t}(j).
$$

Based on the derivation of Steinsson (2003), we obtain the following second-order approximation of the sectoral price dispersion:

$$
\begin{aligned}
\sum_{t=0}^{\infty} \beta^t \Delta^j_{i,t} \simeq \frac{1}{1 - \alpha_i \beta} \sum_{t=0}^{\infty} \beta^t \Big[&\frac{\alpha_i}{1 - \alpha_i} \pi^2_{i,t} + \frac{\lambda_i}{(1 - \alpha_i)(1 - \lambda_i)} \Delta\pi^2_{i,t} + \frac{(1 - \alpha_i)\lambda_i \delta_i}{(1 - \lambda_i)} x^2_{t-1} \\
&- 2 \frac{\lambda_i \delta_i}{1 - \lambda_i} \Delta\pi_{i,t} x_{t-1} \Big] + t.i.p. + O(||\xi||^3), \qquad \text{(A.9)}
\end{aligned}
$$

for $i = S, F$. Substituting Equation (A.9) into Equation (A.8) yields Equation (29) in the main text.

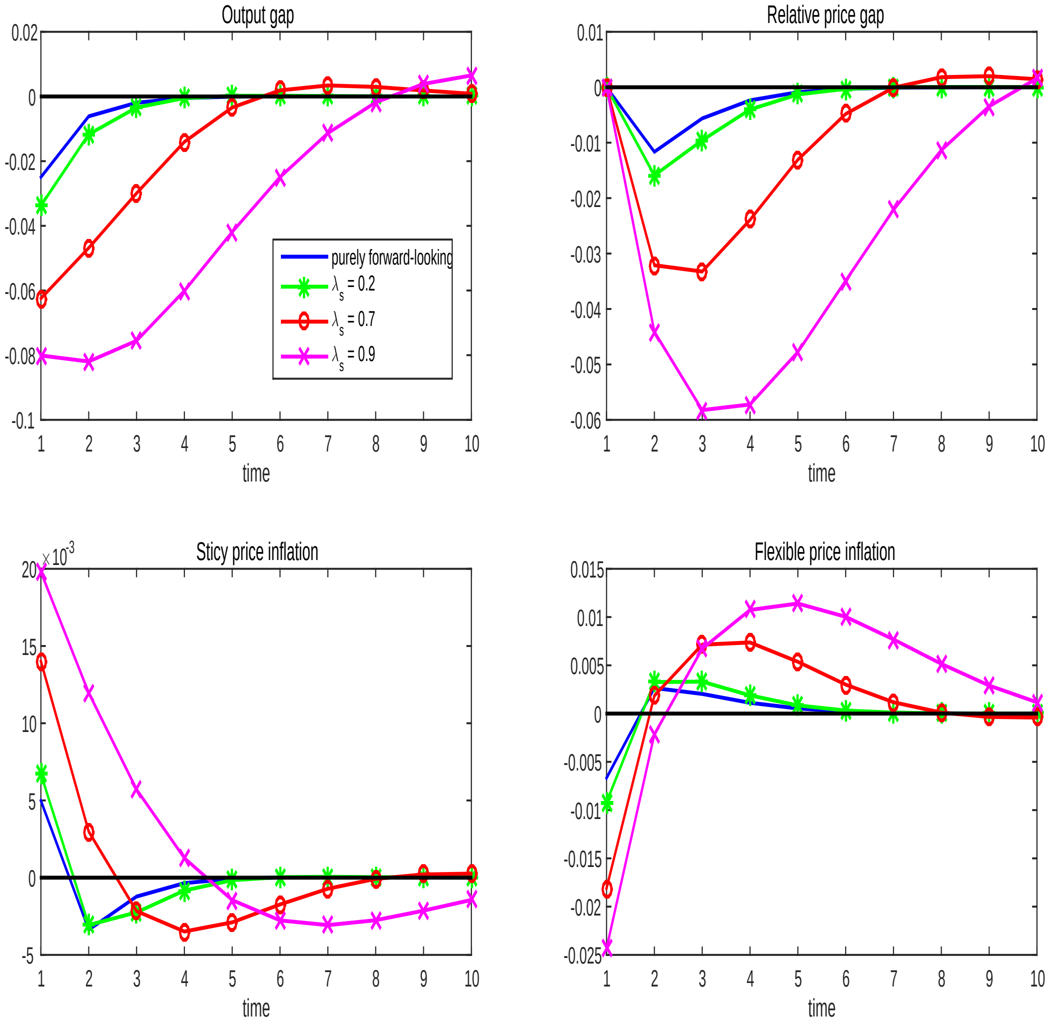

Figure 1: Impulse response of optimal monetary policy to a cost-push shock in sector S

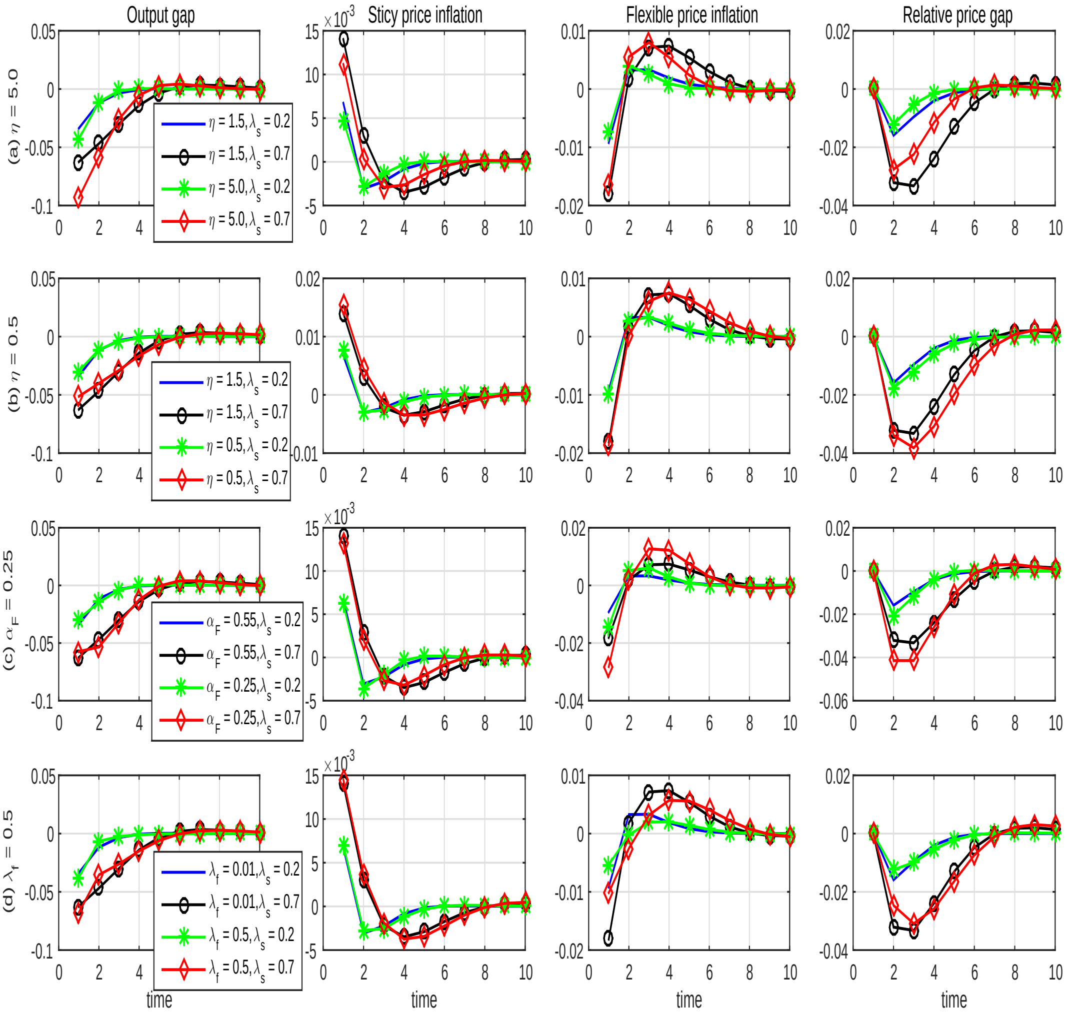

Figure 2: Impulse response of optimal monetary policy to a cost-push shock in sector S: Sensitivity experiments

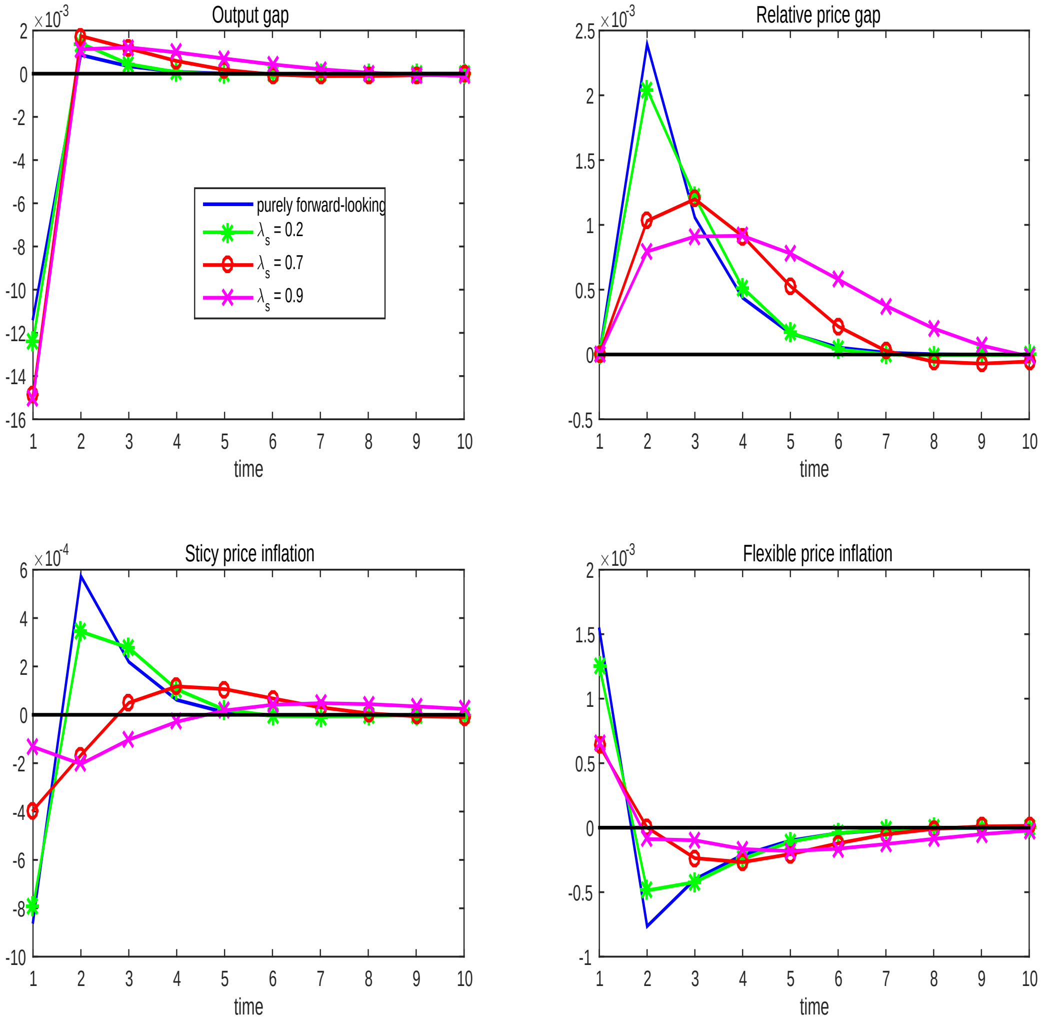

Figure 3: Impulse response of optimal monetary policy to a cost-push shock in sector F

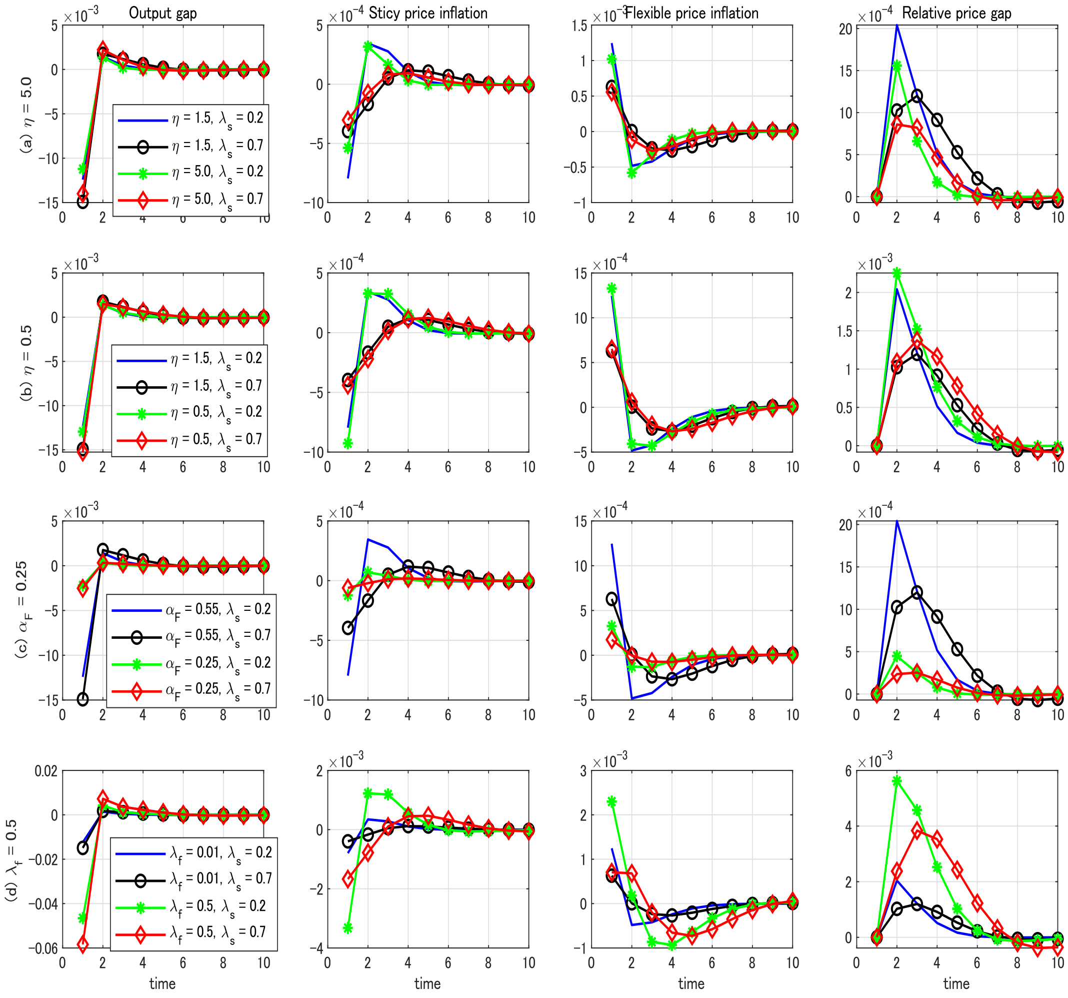

Figure 4: Impulse response of optimal monetary policy to a cost-push shock in sector F: Sensitivity experiments

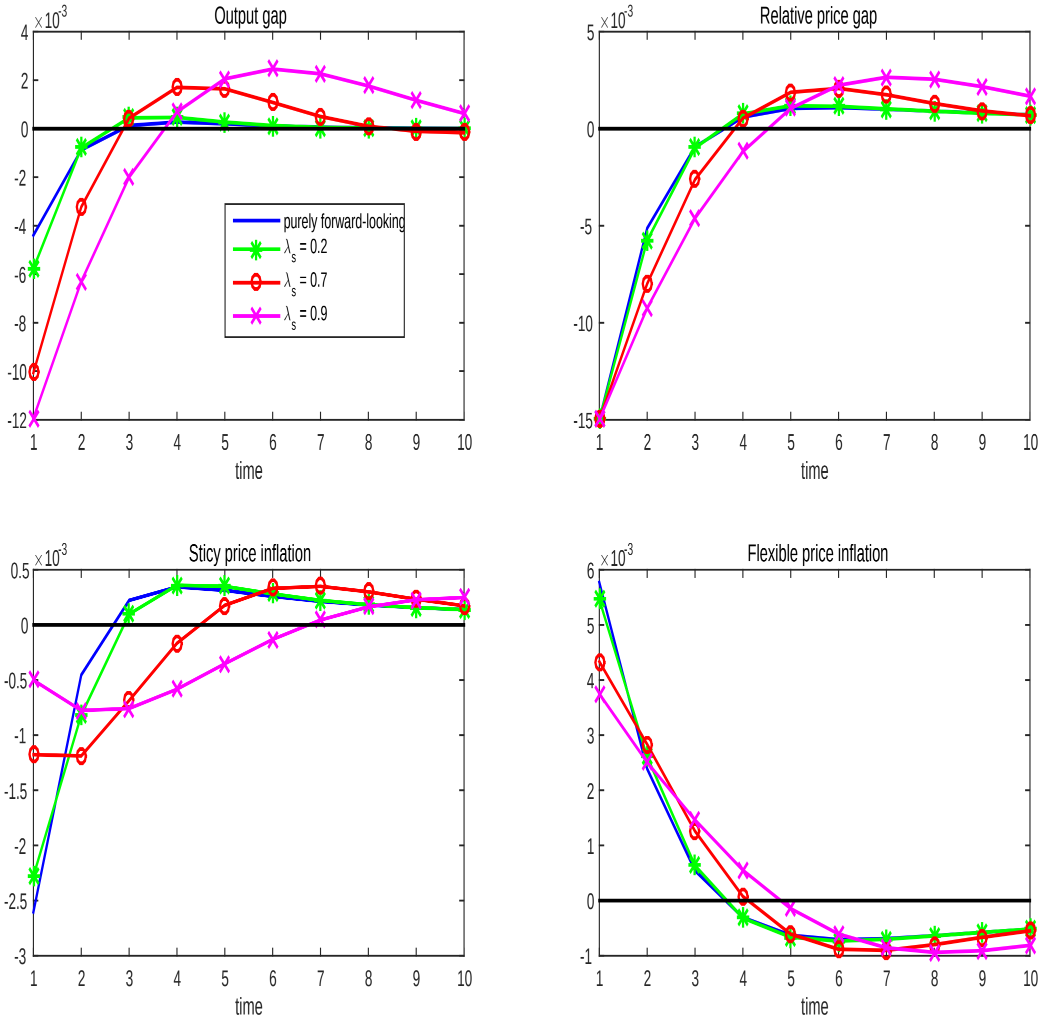

Figure 5: Impulse response of optimal monetary policy to a relative price shock

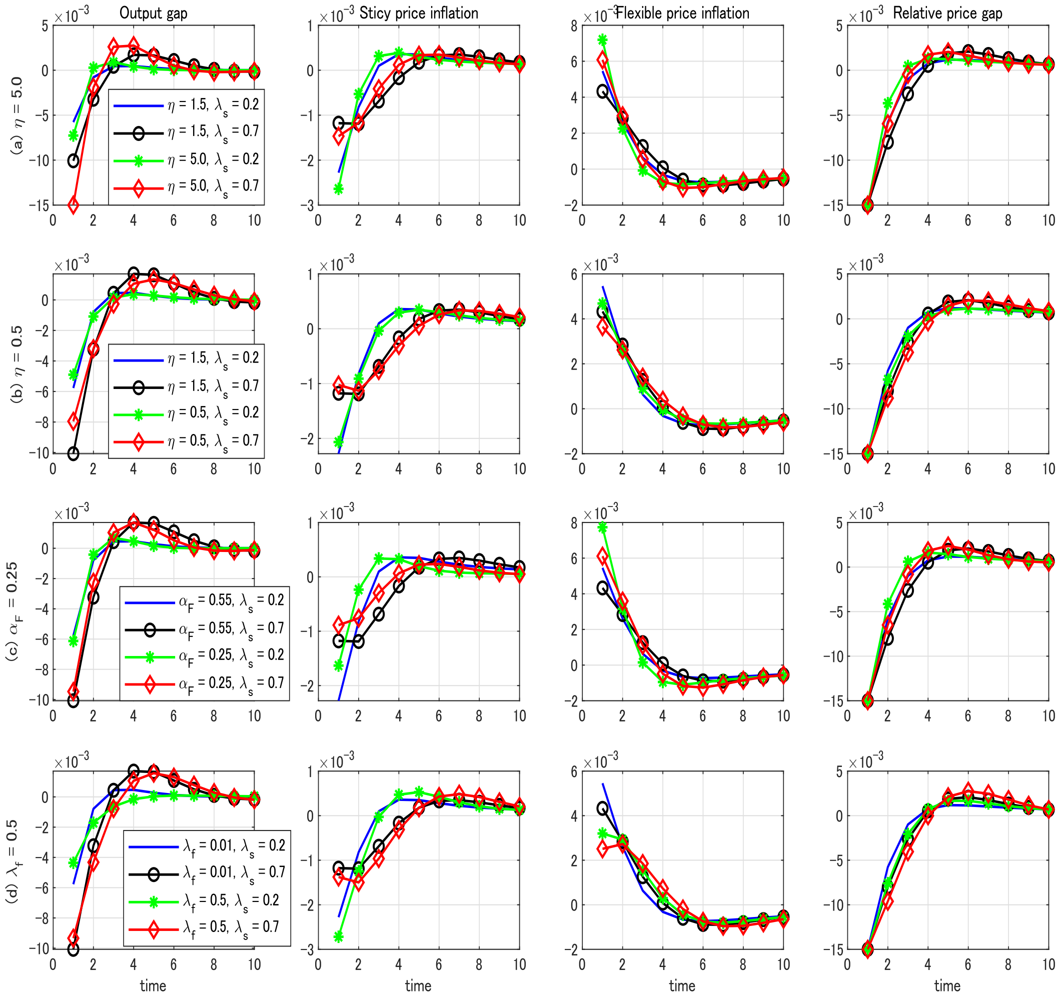

Figure 6: Impulse response of optimal monetary policy to a relative price shock: Sensitivity experiments

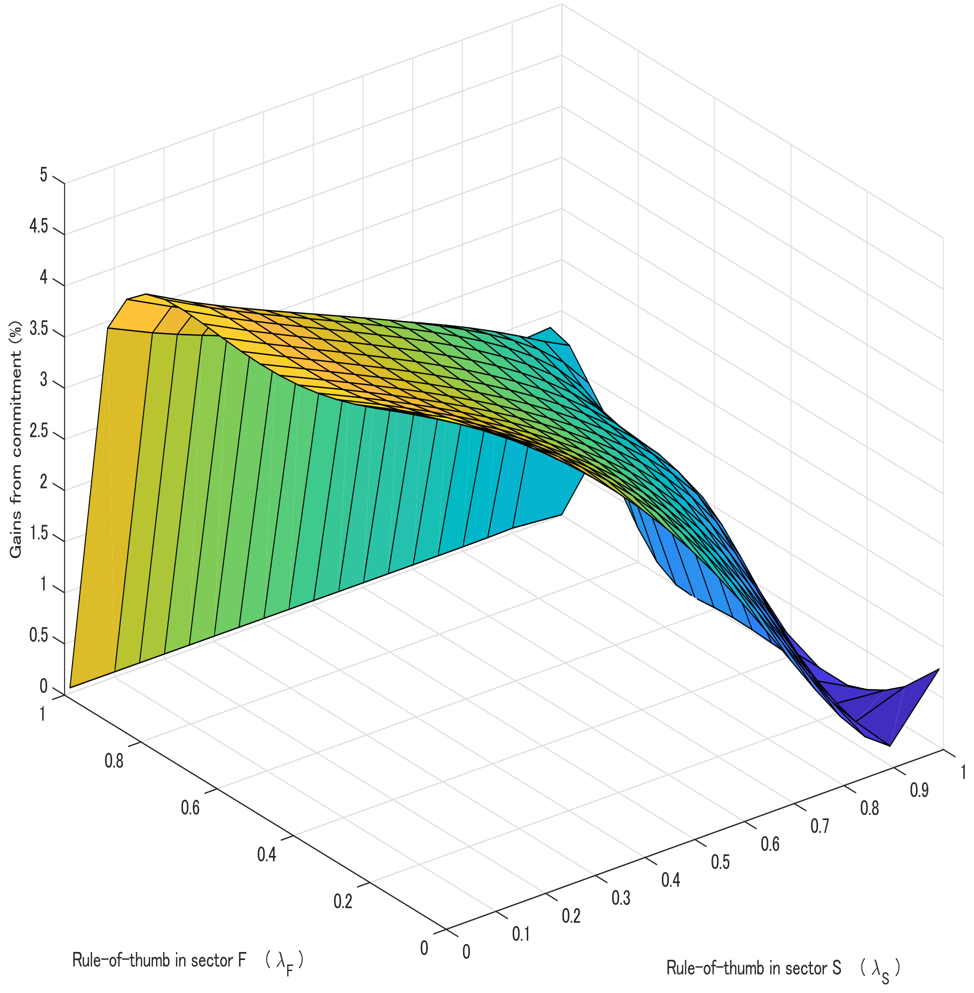

Figure 7: Welfare gain when λ_s and λ_f change

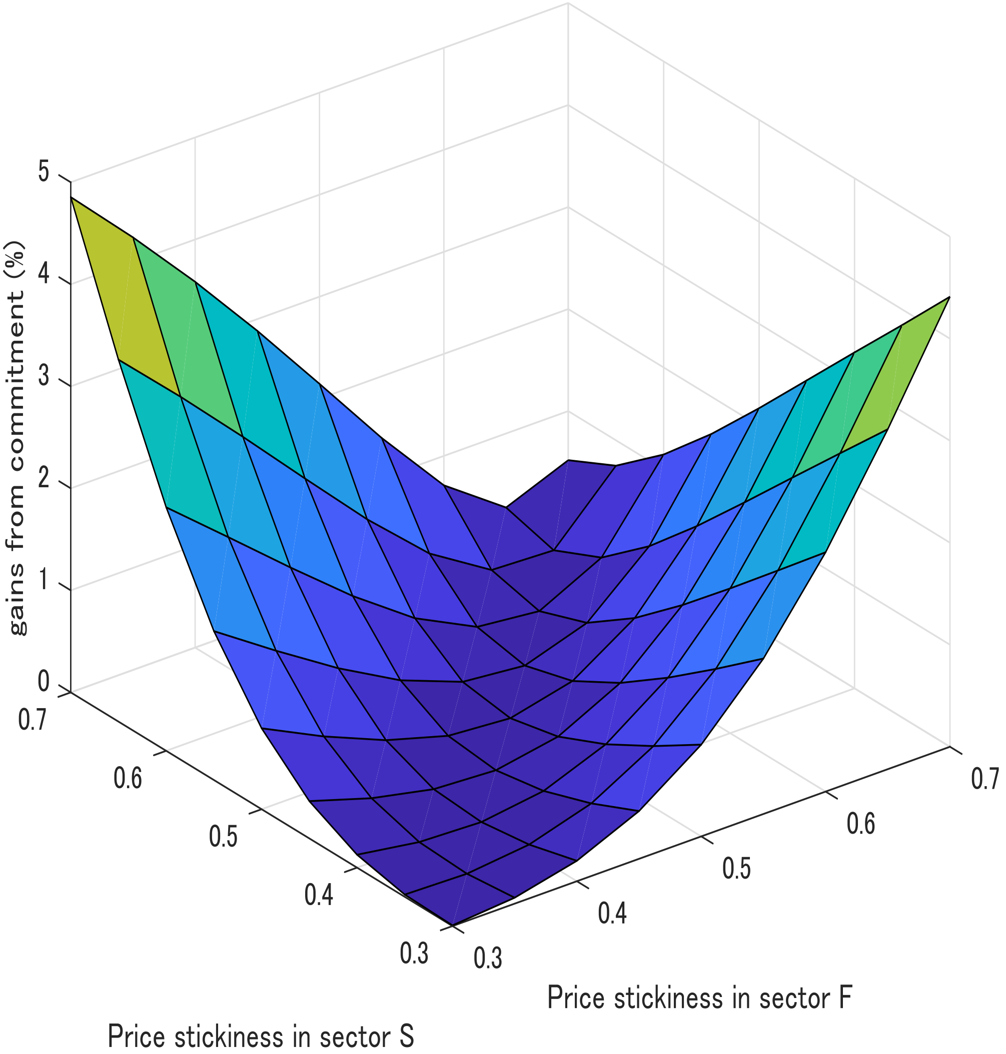

Figure 8: Welfare gain when α_s and α_f change in the case of $\lambda_s = \lambda_f = 0.5$

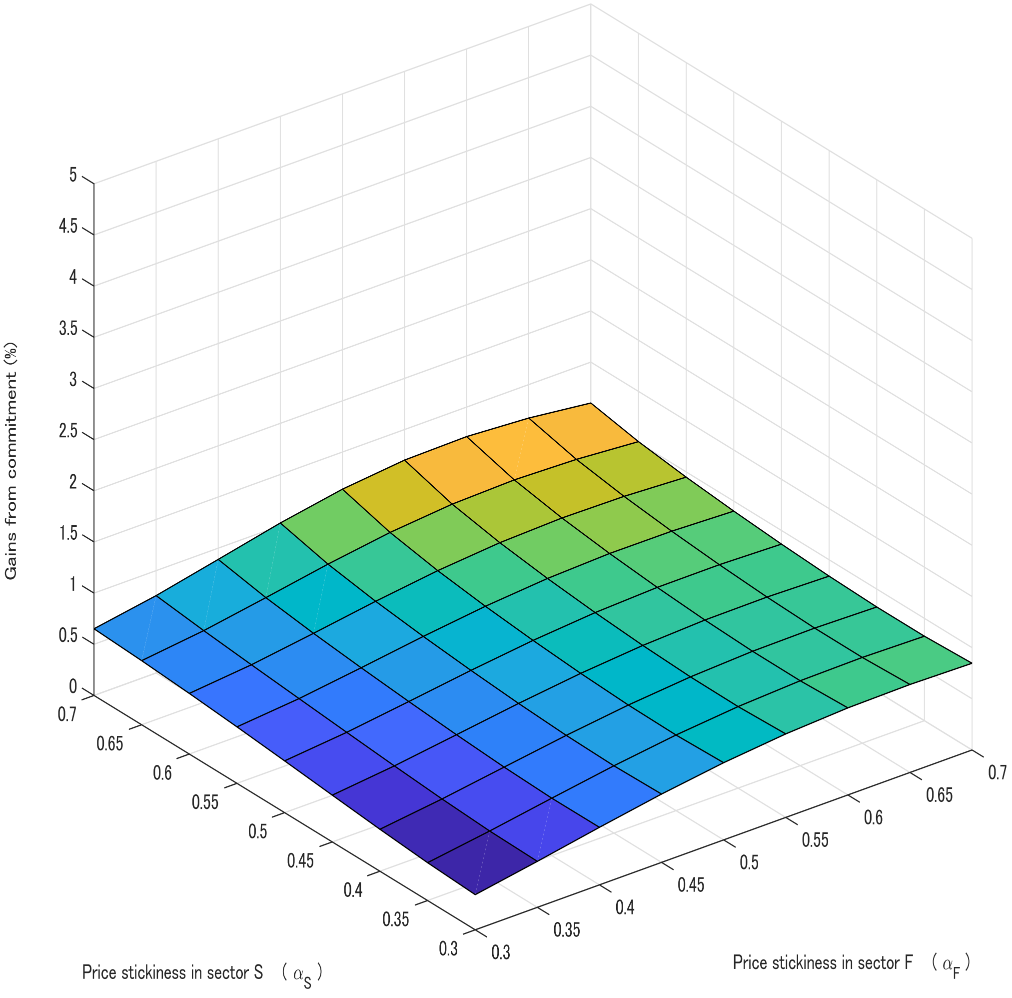

Figure 9: Welfare gain when α_s and α_f change in the case of $\lambda_s = \lambda_f = 0.99$

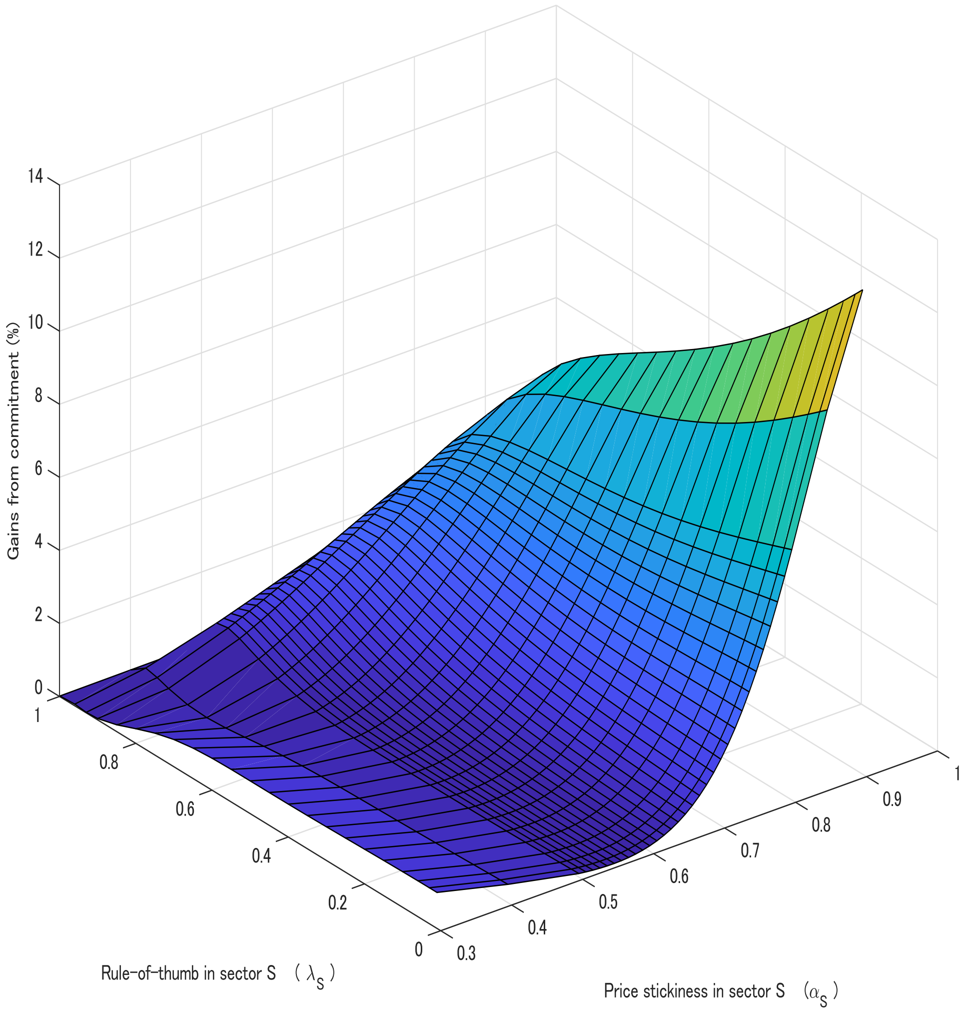

Figure 10: Welfare gain when λ_s and α_s change

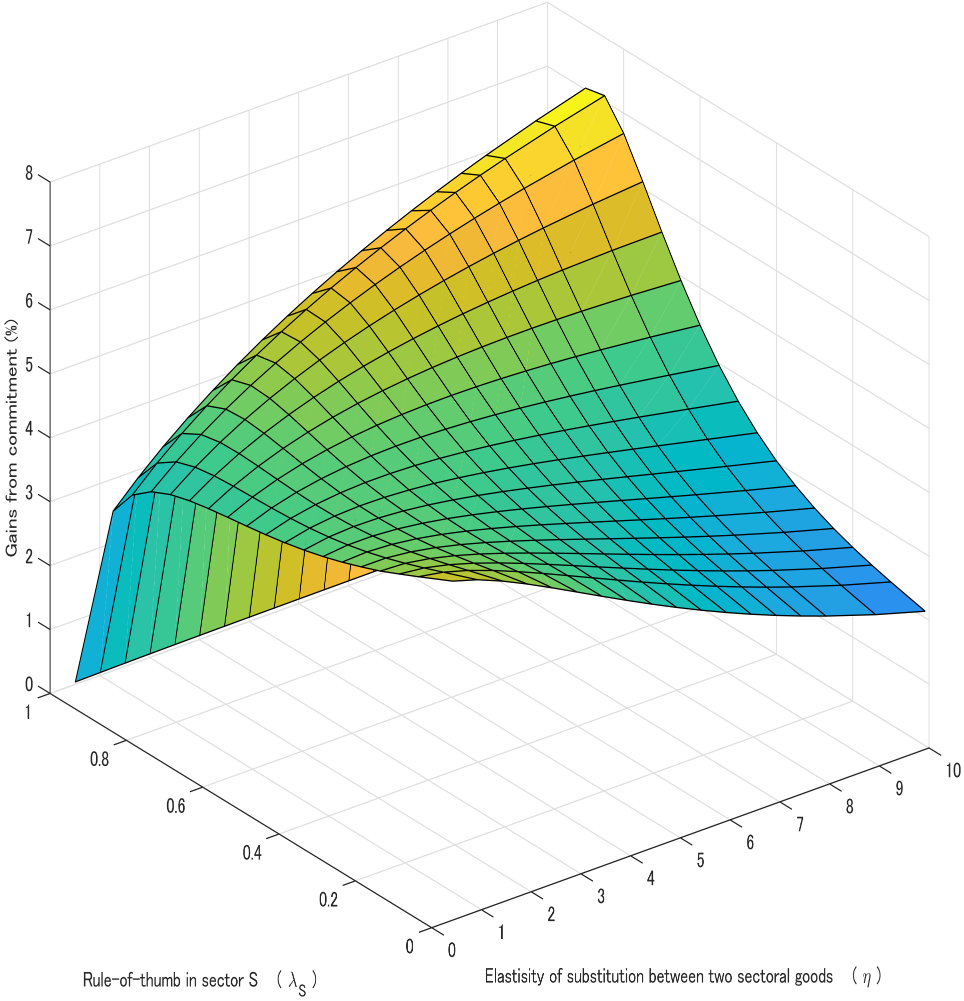

Figure 11: Welfare gain when λ_s and η change

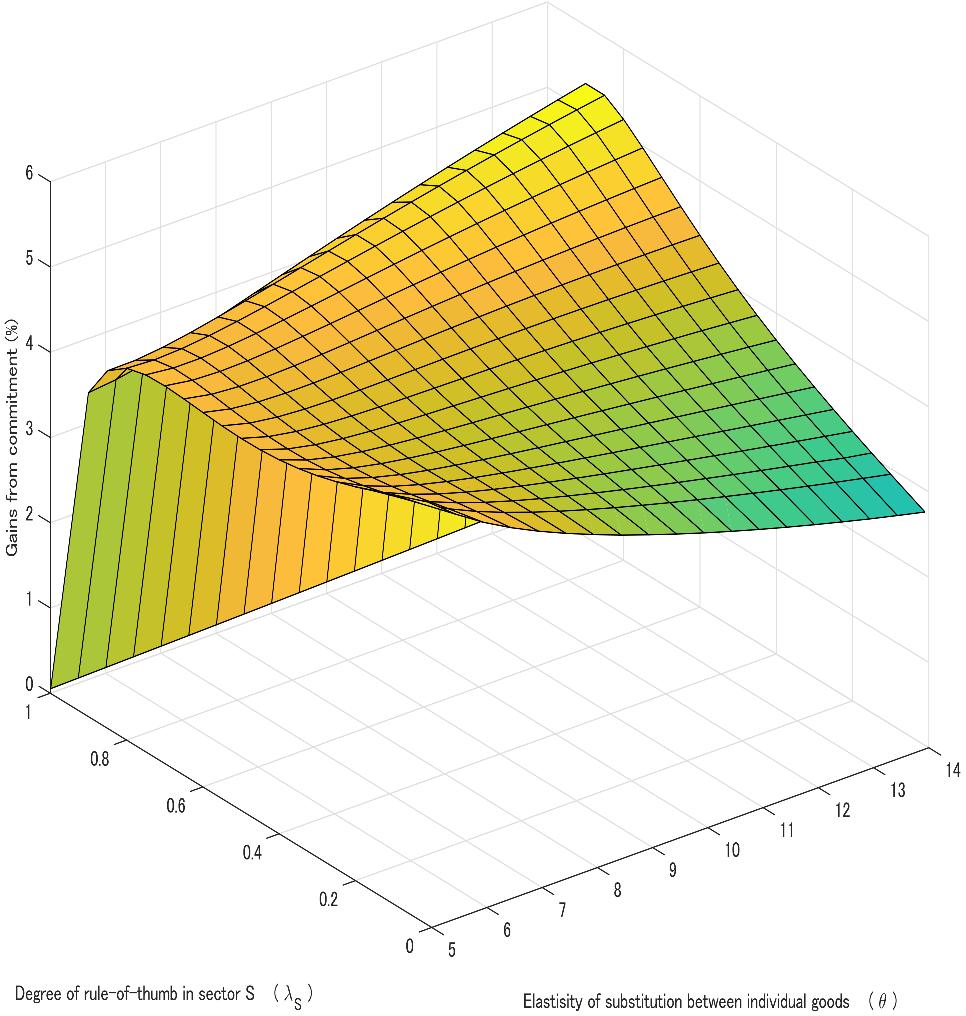

Figure 12: Welfare gain when λ_s and θ change