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Delegating optimal monetary policy inertia in a small open economy*

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Abstract

This paper explores the delegation of targeting regimes in a small open economy model. It examines the effects of an open economy on the coefficients of stabilization weights in the delegated objective function of the central bank. We address the role of the exchange rate in delegating optimal monetary policy. We analytically derive the condition whereby each of the considered targeting regimes coincides with a commitment policy. The results suggest the presence of non-negligible open economy effects on each coefficient in delegated targeting regimes.

JEL codes: E52; E58; F41

Keywords : Optimal monetary policy; Targeting regime; Commitment; Discretion

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1 Introduction

It has been argued that a commitment policy leads to preferable outcomes compared with a discretionary policy in the standard new Keynesian model.[1] As noted by Woodford (1999, 2003), the central bank that commits its future policy stances to the public sector can conduct its monetary policy through the management of expectations of the private sector. Such an approach can lead to economic policy inertia because a commitment policy is characterized by a gradual change of policy stances in response to economic shocks. In contrast, a discretionary policy that treats the future expectations of macro-variables as given cannot induce policy inertia in the economy. This difference is referred to as stabilization bias, which is the main source by which commitment policy can be superior to discretionary policy.[2]

Several studies have shown that in order for a discretionary policy to overcome the problem of stabilization bias, the government delegates the objective function, which is different from the true social objective function, to the central bank (Jensen, 2002; Walsh; 2003; Nessen and Vestin, 2005; Vestin, 2006).[3] Recently, Bilbiie (2014) analytically derived the condition whereby a delegated policy regime coincides with a commitment policy and showed that each coefficient for the stabilization terms in the delegated loss function is characterized by deep parameters in an economy.

This paper analytically examines the role of delegated policy regimes in a small-open economy. Previous studies focused on delegated policy schemes in a closed-economy setting, but as noted by various authors, optimal monetary policy should consider open-economy effects. Clarida, Gali and Gertler (2001) and Gali and Monacelli (2005) investigated optimal monetary policy in a small open economy. [4] Furthermore, several studies examined optimal monetary policy in a two-country economy framework (Clarida, Gali and Gertler, 2002; Pappa, 2004; Benigno and Benigno, 2006). However, to the best of our knowledge, no studies appear to focus on delegating optimal monetary policy regimes in an open economy framework regardless of the importance of the

[1]This paper regards a commitment policy as a timeless perspective approach; see Woodford (2003) and McCallum and Nelson (2004) for further details.

[2]See Loisel (2008) and Walsh (2010) for a detailed discussion of stabilization bias in a forward-looking model.

[3]In addition, to eliminate inflation bias, several studies have argued for the superiority of delegated policy schemes over discretionary alternatives in backward-looking model. These studies numerically illustrate the superiority of delegated targeting regimes over purely discretionary policy (Rogoff, 1985; Walsh, 1995; Svensson, 1997)

[4]Their approach is based on a producer currency pricing (PCP) model. Monacelli (2005) explored optimal monetary policy when exchange rate pass-through is incomplete.

open-economy effect on macro variables.

The objective of this paper is to analytically show the condition that delegated monetary policy regimes coincide with a commitment solution in a small-open new Keynesian model. In such a sense, our study extends Bilbiie (2014)'s idea to a small-open new Keynesian model. To do this, we use the framework of a small open economy on the basis of Gali (2015). In contrast to a closed economy model, the degree of openness and the intratemporal elasticity of substitution between home and foreign goods plays an important role in a small-open economy.

The findings of this paper are summarized as follows. First, in contrast to previous studies that focus on delegating policy regimes in a closed economy model, we consider the role of the real exchange rate in delegating monetary policy schemes. This paper analytically derives the condition that the optimal policy under the central bank's loss function that incorporates the stabilization of the real exchange rate corresponds to that under a commitment policy. We show that the presence of the real exchange rate is crucial when we consider delegated monetary policy schemes in a small-open economy.

Second, this paper explores whether several policy regimes correspond to the solution of a commitment policy. More specifically, we consider the following policy regimes: speed limit policy, nominal income growth targeting, and consumer price index (CPI) inflation targeting. The results suggest that speed limit policy can also achieve the same outcomes as those under a commitment policy in a small-open economies. The difference between closed and small open economy is captured by an indirect real exchange rate channel, in addition to the conventional role of the speed limit target for the output gap. According to the standard new Keynesian model, the output gap affects the inflation rate through the new Keynesian Phillips Curve (NKPC). In addition to traditional channel, the output gap affects the inflation rate through a change the terms of trade, resulting in a movement in the real exchange rate. Hence, the speed limit policy contains an indirect mechanism for stabilizing the output gap through the real exchange rate.

Third, in contrast, targeting regimes such as CPI inflation and nominal income growth could lead to the same outcomes as an optimal commitment policy if the relative weight of the output gap to inflation takes a considerably smaller value in the social loss function.

The reminder of the paper is structured as follows. Section 2 briefly describes the characteristics of a small open economy new Keynesian model. We specify targeting regimes in a small open economy in Section 3, considering the following regimes: real exchange rate targeting, speed limit policy, CPI inflation targeting, and nominal income growth targeting. Section 4 analytically derives the condition under which optimal delegation schemes coincide with a commitment policy and shows how the open economy affects each delegated monetary policy regime. Section 5 briefly concludes. Appendix A provides proofs of the propositions derived in Section 4 and Appendix B derives the proof of the proposition based on Bilbiie's (2014) general quadratic method.

2 Model

The model used herein is based on Gali (2015). Section 2.1 provides a log-linearized system of structural equations in a small open new Keynesian model. We explain the central bank's loss function in Section 2.2 and discuss the characteristics of a commitment policy. In sum, we put forward the effect an open economy has on optimal monetary policy.

2.1 Linearized system of a small open new Keynesian model

This section briefly describes the small open economy new Keynesian model based on Gali (2015). The home country is infinitesimally small relative to the rest of the world. Representative households in the home country purchase domestic and foreign goods. Households in the home country can have access to a complete set of state-contingent securities that are traded both domestically and internationally. Firms face both monopolistically competitive environments and nominal staggered-price rigidities as specified by Calvo (1983). As shown in Gali (2015), based on the foregoing, a log-linearized system is given as follows:[5]

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_\nu x_t + u_t, \tag{1}$$
$$x_t = E_t x_{t+1} - \sigma_\nu^{-1}(i_t - E_t \pi_{t+1} - \bar{r}r_t), \tag{2}$$
$$\pi_t^c = \pi_t + \nu(s_t - s_{t-1}), \tag{3}$$
$$q_t = (1-\nu)s_t \tag{4}$$
$$s_t = \sigma_\nu x_t. \tag{5}$$

where π_t is producer currency inflation, x_t is the output gap, i_t is the nominal interest rate, π_t^c is CPI inflation, s_t is the terms of trade gap, and q_t is the real exchange rate gap. Gap variables are expressed by the log deviation of the

[5]See Gali (2015) for a detailed derivation of a small open new Keynesian model.

endogenous variables from the efficient level of their variables. In addition, $\bar{rr}_t$ denotes the natural rate of interest, which holds the real interest rate under flexible price equilibrium, and u_t is the exogenous cost-push shock, which follows an AR(1) process. Finally, the coefficients for each structural equation are defined as follows:

$$\begin{aligned}
\sigma_\nu &= \frac{\sigma}{1+\nu[\sigma\eta+(1-\nu)(\sigma\eta-1)-1]}, \\
\kappa_\nu &= \delta\left(\sigma_\nu + \frac{1+\psi}{1-\alpha}\right) \\
\delta &= \frac{(1-\omega)(1-\omega\beta)}{\omega}\Theta. \\
\Theta &= \frac{1-\alpha}{1-\alpha+\alpha\epsilon}
\end{aligned}$$

The parameters β, σ, η, and ψ represent the discount factor, the relative risk aversion coefficient, the substitutability between domestic and foreign goods, and the inverse labor supply elasticity, respectively. The parameter ν is the degree of openness and α is the degree of diminishing returns to scale for labor supply. The parameter ω characterizes the degree of nominal price rigidities (i.e. Calvo's lottery), and ϵ denotes the elasticity of substitution for individual goods.

Equation (1) represents the small open economy NKPC, which is derived from the firm's profit maximization problem subject to Calvo pricing. Equation (2) is a dynamic IS equation, which results from solving the household's intertemporal optimization problem. Equation (3) specifies the relationship between CPI and producer price index (PPI) inflation. Equation (4) states that the real exchange rate proportionally changes in response to a change in the terms of trade. Finally, Equation (5) represents the relationship between the terms of trade and the output gap.

The effect of the open economy is characterized by changes by both ν and $\sigma\eta$. Here, the parameter ν is the degree of openness. These parameters affect the coefficient of the σ_ν, which is included in κ_ν and σ_ν. Where $\sigma\eta > 1$, an increase in ν induces a decline in σ_ν. Thus, changes in ν and $\sigma\eta$ alter the slopes of the IS curve and the NKPC. In this case, through the risk-sharing condition, the domestic output gap changes in accordance with changes in the terms of trade, which implies a fluctuating real exchange rate. This mechanism can be depicted in the IS equation.

In addition, changes in the terms of trade affect the sensitivity of inflation to the real marginal cost in the NKPC through two channels. First, domestic inflation reacts positively to a change in the terms of trade by a change in the real exchange rate through international consumption risk sharing. Second, changes in the terms of trade produce fluctuations in domestic inflation because they induce changes in the real marginal cost. Whether domestic inflation increases depends on the movement of $\sigma\eta$. In the case of $\sigma = \eta = 1$, both κ_ν and σ_ν decrease to $\delta(1 + \frac{1+\psi}{1-\alpha}) \equiv \kappa$ and 1, respectively. Thus, the open economy effect disappears in this case.

Finally, because we assume that the home country is infinitesimally small relative to the rest of the world, note that the other variables for the rest of the world (π_t^*, i_t^*, x_t^*) are exogenous in terms of the home country. A variable with $(*)$ denotes a foreign country.

2.2 Objective of the central bank and optimal monetary policy

The central bank conducts its monetary policy following a targeting rule derived from the minimization problem of the central bank's loss function. As shown in Woodford (2003), the central bank's loss function is derived from the second-order approximation of the household's utility function. In the standard new Keynesian model, the central bank's loss function includes the stabilization of inflation and the output gap.

However, we often have difficulty deriving well-defined loss function in an open economy model. In particular, we face this problem in the case of $\sigma\eta \neq 1$. To overcome this shortcoming, Gali (2015) derived the central bank's loss function in the special case of $\sigma = \eta = 1$.

This paper assumes the standard loss function that stabilizes both inflation and the output gap in the case of $\sigma\eta > 1$. We address the advantage of using this loss function. The reason is twofold. First, we can focus on the open economy effect when considering delegating monetary policy regimes in a small open economy. Monacelli (2005) used the standard loss function to consider optimal monetary policy in cases where difficulties exist in deriving the loss function in a small open economy with LCP. Second, this enables us to analyze optimal delegation regimes in the case where $\sigma\eta$ is greater than unity.

More specifically, the central bank minimizes the following social loss function subject to structural equations:

$$L_t = E_t \sum_{j=0}^{\infty} \beta^j (\pi_t^2 + \lambda x_t^2). \tag{6}$$

where λ denotes the stabilization weight on the output gap relative to inflation stabilization.

Herein, the central bank minimizes the loss function (6) subject to the NKPC under the presence of the cost-push shock. A commitment solution produces the following targeting rule:

$$\pi_t = -\frac{\lambda}{\kappa_\nu}(x_t - x_{t-1}) \tag{7}$$

Except for the structural equation coefficients, a targeting rule with commitment is the same shape as the variant obtained in a closed economy. The central bank that conducts a commitment policy can manipulate the expectations of the private sector by gradually changing its policy variable. Thus, the presence of a lagged output gap in the targeting rule allows the central bank to employ an inertial behavior vis-à-vis its policy stance. Therefore a commitment policy is superior to a purely discretionary policy (McCallum and Nelson, 2004). Finally, note that the meaning of commitment policy as applied in this paper implies a timeless perspective as proposed by Woodford (2003).[6]

From the optimization problem of the central bank, we can obtain the reduced form of endogenous variables, solved by the standard factorization method. More specifically, the reduced forms of both inflation and the output gap are solved as follows:

$$\pi_t = \psi^c_\pi x_{t-1} + \psi^c_{u\pi} u_t, \tag{8}$$
$$x_t = \psi^c_x x_{t-1} + \psi^c_{ux} u_t, \tag{9}$$

where

$$\psi_{\pi^c} = \frac{\lambda}{\kappa_\nu}(1-\mu_1); \psi^c_{u\pi} = -\frac{\lambda}{\kappa_\nu}\psi^c_{xu},$$
$$\psi^c_x = \mu_1; \psi^c_{ux} = -\frac{\kappa_\nu}{\beta\lambda}\frac{1}{\mu_2 - \rho_u}.$$

Here, μ_1 and μ_2 are eigenvalues obtained from the solution of the factorization method in a commitment policy.[7] The parameter ρ_u denotes the degree of persistency of the cost-push shock.

[6]See Woodford (2003) for a detailed discussion of a timeless perspective on optimal monetary policy

[7]See Bilbiie (2014) for a detailed calculation of μ_1 and μ_2.

As we can see in Equations (8) and (9), notwithstanding the forward-looking model, the reduced forms of endogenous variables depends on the lagged variable x_{t-1}. Thus, if the central bank can proffer a credible commitment to the private sector, such a commitment policy enables endogenous variables to gradually change in response to a cost-push shock. In other words, this implies that the central bank can alleviate a policy trade-off generated by a cost-push shock through management of private sector expectations.

Importantly, the optimal targeting rule (7) includes the stabilization of both the terms of trade effect and the risk-sharing effect. Thus, in contrast to the closed economy model, the optimal targeting rule replaces κ with κ_ν. This modification indicates that the central bank considers not only the stabilization of the domestic macroeconomy also the effect of the open economy on the home country. When the parameter ν takes zero, the optimal targeting rule (7) decreases to that of a closed economy.

3 Targeting regimes in a small open economy

This section examines delegating optimal monetary policy inertia in a small-open forward-looking model. As mentioned in many previous studies that invoke the standard new Keynesian model, as long as the central bank can commit its future monetary policy stance and never renege on this commitment policy, the commitment solution is superior to the alternative under a discretionary policy[8].

Assuming this is the case and as noted in the extant literature, the central bank, which conducts a discretionary policy, can enhance social welfare when the government delegates the loss function with lagged endogenous variables to that bank. Jensen (2002) suggests the policy regime that the government delegates the objective function that includes stabilization of nominal income growth to the central bank. Obviously, nominal income growth possesses policy inertia through lagged output. Further, Walsh (2003) asserted the effectiveness of the speed limit policy that stabilizes a change in the output gap.[9]

These studies point out that the presence of the lagged variable in the central bank's objective function allows the central bank to impart policy inertia into the economy even when the central bank discretionally conducts its monetary policy. Therefore, the central bank that employs these targeting

[8]For instance, see Kydland and Prescott (1977) for a detailed discussion of the time-inconsistency problem in optimal policy.

[9]Nessen and Vestin (2005) and Vestin (2006) considers the effectiveness considered average inflation and price level targeting, respectively, as delegating monetary policy regimes.

regimes can improve social welfare to a greater extent compared with the case of pure discretionary monetary policy.

As mentioned earlier, this paper analytically explores whether delegating optimal monetary policy inertia can coincide with a commitment solution in a small-open economy model. In contrast to a closed economy, we conjecture that the presence of an open economy effect should significantly change the condition that delegating optimal monetary policy corresponds to a commitment policy. Bilbiie (2014) derived the condition that delegating monetary policy regimes coincides with a commitment solution, whereas this paper extends that work to the small open new Keynesian model. Accordingly, the shape of the delegated objective function is based on Bilbiie (2014).

The merit of considering the optimal delegation problem in a small-open economy is to explore whether the policy inertia from the real exchange rate can achieve the same outcome as a commitment policy. Therefore, we consider real exchange rate targeting as the first policy regime.[10] More specifically, the real exchange rate targeting regime is defined as follows:

$$\pi_t^2 + \lambda_x x_t^2 + \lambda_q \Delta q_t^2 + 2c_\pi x_{t-1}\pi_t. \tag{10}$$

As explained in Bilbiie (2014), as with all policy regimes in this paper, technically we add a fourth term to the objective function to obtain the analytical solution that corresponds to the solution under a commitment policy. In other words, this term reflects the linear inflation contract term proposed by Walsh (1995) and Svensson (1997).

Clearly, the policy inertia is introduced through a change in the real exchange rate. According to Equation (5), since the output gap is proportional to the terms of trade, we can replace the term for the real exchange rate with that for the output gap in the targeting regime (10):

$$\pi_t^2 + \lambda_x x_t^2 + \sigma_\nu^2 \lambda_q \Delta x_t^2 + 2c_\pi x_{t-1}\pi_t. \tag{11}$$

One may suppose that as suggested by Walsh (2003), the central bank can attain the same outcomes as a commitment policy if the government delegates the objective function to the central bank that directly includes a change in the output gap. Introducing the lagged output gap into the objective function allows the central bank to gradually respond to economic shocks even if the central bank conducts a discretionary policy. We consider the speed limit policy, which is defined as follows:

$$\pi_t^2 + \lambda x_t^2 + \lambda_S \Delta x_t^2 + \lambda_\pi (\pi_t^c)^2 + 2c_\pi x_{t-1}\pi_t. \tag{12}$$

[10] See also Taylor (2001) for a detailed discussion of the role of real exchange rate targeting in an open economy.

As explained in Walsh (2003), the central bank can achieve preferable outcomes over and above a pure discretionary policy.

Moreover, we consider whether the delegated objective function with policy inertia through CPI inflation coincides with a commitment solution. In contrast to a closed economy model, we can differentiate CPI inflation and PPI inflation. The inflationary difference between both metrics is reflected in a change in the terms of trade gap. More specifically, we define the CPI inflation targeting regime as follows:

$$\pi_t^2 + \lambda_x x_t^2 + \lambda_\pi (\pi_t^c)^2 + 2c_\pi x_{t-1}\pi_t \tag{13}$$

Assuming this formulation, one might speculate that this policy regime does not include policy inertia. According to Equation (3), however, the CPI inflation rate is affected by a change in the terms of trade. Thus, this policy scheme can introduce policy inertia into the economy through a change in the terms of trade, which implies a change in the output gap. Hence, it is possible that the performance of CPI inflation targeting in a small open economy is superior to that in a closed economy model.

Finally, as suggested by Jensen (2002), we consider the targeting regime including a nominal income growth, which is defined as follows:

$$\pi_t^2 + \lambda_x x_t^2 + \lambda_N (\pi_t^c + \Delta x_t)^2 + 2c_\pi x_{t-1}\pi_t. \tag{14}$$

We assume that nominal income growth is defined in terms of CPI because the private sector may be interested in CPI fluctuations in terms of social welfare.[11] It follows from Equation (14) that policy inertia is incorporated into the objective function through the nominal income growth rate. In addition, nominal income growth includes policy inertia through the CPI inflation rate.

For these four regimes, we allow λ_x to diverge from λ.

4 Optimal delegation problem in a small open economy

This section details our results. To begin, we derive the condition whereby a discretionary policy under real exchange rate targeting corresponds to a commitment solution. Then, we compare the results of real exchange rate targeting

[11] Ida and Okano (2017) examines the role of nominal income growth targeting in a small-open economy. They explore how the difference in between PPI and CPI inflation affects the performance of nominal income growth targeting.

with those of a speed limit policy. Moreover, we examine the condition under nominal income growth targeting and CPI inflation targeting.

First, consider the optimal delegation parameters under real exchange rate targeting. Solving the optimal monetary policy with discretion under this regime leads to the following proposition:

Proposition 1. *(Real exchange rate targeting) The Markov-perfect equilibrium value of domestic inflation and output gap that occur if the central bank minimizes the delegated loss function (10) are identical to the timeless-optimal commitment solutions (8) and (9) if and only if the delegation parameters are given by the following:*

$$c_\pi^* = -\frac{\lambda\gamma}{\kappa_\nu\gamma + \lambda} \tag{15}$$

$$\lambda_q^* = \frac{1}{(1-\nu^2\sigma_\nu^2)}\left(\frac{\lambda\gamma}{\kappa_\nu\gamma + \lambda}\right)\left(\frac{\lambda}{\kappa_\nu}\right) \tag{16}$$

$$\lambda_x^* = \frac{\lambda\gamma^2}{\kappa_\nu\gamma + \lambda} \tag{17}$$

where $\gamma = \kappa_\nu + \beta\psi_\pi^c$.

The proof is shown in Appendix A. As we will show, under real exchange rate targeting and the speed limit policy, the policy implications are different although both solutions coincide with the solution under commitment.

An important characteristic of real exchange rate targeting is the direct stabilization of the exchange rate. Since an increase in ν induces exchange rate fluctuations, the central bank needs to respond by militating against instability. This is reflected as $1/(1-\nu^2\sigma_\nu^2)$ in (16). Note that $\lambda_q \neq 0$ even when $\nu = 0$. Not surprisingly, this is because the central bank employs the delegated loss function that directly includes the stabilization term for the real exchange rate.

[Figure 1 around here]

Figure 1 illustrates the optimal delegation parameters when the degree of openness ν moves from 0.0 (which is reduced to the closed economy case) to 0.4 (which represents a typical small open economy,) under alternative regimes: real exchange rate targeting and the speed limit policy. Other deep parameters are based on the calibration used in the standard new Keynesian model. Other calibrated deep parameters are displayed in Table 1. Note that an increase in ν lowers σ_ν since $\sigma\eta = 2.0 > 1.0$.

[Table 1 around here]

The parameters under real exchange rate targeting are shown in Figure 1(a). λ_q is a real exchange rate stabilization weight, and it plays a key role in the regime: hence, the magnitude of λ_q dominates the other parameters unless the degree of openness is too restricted ($\nu < 0.03$, approximately). The reason is that λ_q not only stabilizes the output gap through its influence on the real exchange rate but also generates inertia from the real exchange rate. Thus, a weight on the stabilization of the output gap is smaller, whereas optimal inertia can be replicated by placing a larger weight on real exchange rate stabilization. Note that λ_q is increasing in openness ν because an increase in openness lowers σ_ν, thus increasing output gap volatility due to the larger effect of the real exchange rate on the output gap. The other parameters λ_x and c_π, however, cannot fully stabilize this volatility since they cannot capture the effect of the real exchange rate on the output gap. Therefore, λ_q has an advantage of compensating for the inferior stabilization properties of both λ_x and c_π.

The contract term c_π represents a penalty/reward for a change in inflation, and the marginal penalty depends upon the past values of the output gap.[12] This parameter appears under all regimes as an assumption. c_π is negative for all ν; hence, for the same given prior contraction in output gap, future contractions should be rewarded under these regimes. These results are consistent with those in previous studies. As shown in Figure 1, the absolute value of c_π is smaller under the real exchange rate targeting compared with its value under the speed limit policy; the weight is smaller (larger) under the former (latter). This result suggests that under real exchange rate targeting, the required inertia has been provided by the real exchange rate, as measured by large λ_q, resulting in the smaller dependency on c_π. The reward for additional inflation increases in ν under both regimes, although the change in c_π is small.

λ_x is the output gap stabilization weight. Note that these regimes allow λ_x to differ from λ, which represents the relative weight of output gap stabilization to inflation in the social loss function. λ_x is positive for all ν. This means stabilizing the output gap is rewarded for replicating the optimal outcome. As noted earlier, the parameter λ_x is slightly smaller than that in the social loss function because the positive weight on λ_q means the central bank directly stabilizes the output gap through the real exchange rate channel. The penalty for output gap volatility decreases in ν, although the degree of change is negligible.

[12] See Bilbiie (2014) for a detailed discussion about this issue.

Next, we derive the optimal delegation parameters under a speed limit policy. Then, we compare the properties of the delegation parameters under real exchange rate targeting with that under the speed limit policy. Before doing this, we summarize the properties of the optimal delegation parameters under the speed limit policy.

Proposition 2. *(Speed limit policy) The Markov-perfect equilibrium values of domestic inflation and output gap, which occur if the central bank minimizes the delegated loss function (12), are identical to the timeless-optimal commitment solutions (8) and (9) if and only if the delegation parameters are given by the following:*

$$\lambda_S^* = \frac{\lambda}{\gamma^2 \kappa_\nu}(\nu\sigma_\nu(\gamma\kappa_\nu + \lambda) + \gamma\lambda) \tag{18}$$

$$c_\pi^* = -\frac{\lambda}{\gamma} \tag{19}$$

$$\lambda_\pi^* = -\frac{\lambda(\gamma\kappa_\nu + \lambda)}{\gamma^2(\kappa_\nu\nu\sigma_\nu - \lambda)} \tag{20}$$

The proof is shown in Appendix A.

The difference between closed and small open economy appears in ν, σ_ν and κ_ν. In addition to the speed limit channel, the terms of trade (the real exchange rate) channel is also used for stabilization. In other words, the real exchange rate is indirectly stabilized under the speed limit policy. The intuition of this result is as follows. The output gap affects the inflation rate through the NKPC in the standard new Keynesian model. In addition to this traditional channel, the output gap affects the inflation rate through a change in the terms of trade, resulting in a movement in the real exchange rate. Therefore, the speed limit policy contains an indirect stabilization of the output gap through the real exchange rate. Note that when openness ν is increasing, the increase in λ_S^* is smaller than that in λ_q^*.

Parameters under the speed limit policy are shown in Figure 1(b). λ_S plays a key role in the regime, and this parameter value never takes negative values for all ν. Comparing λ_S in the speed limit policy with λ_q under real exchange rate targeting, both parameters increase in openness, whereas those levels and the degree of changes are different as mentioned above. When degree of the openness is small, λ_q is smaller than λ_S since for the purpose of stabilizing the output gap λ_q need not to be weighted heavily. A smaller value of ν weakens the direct effect of the real exchange rate of the output gap. Since the speed limit target directly aims at stabilizing a change of the output gap.

On the contrary, λ_S must be weighted heavily. Moreover, while the marginal change of λ_q increases with respect to openness, the marginal change of λ_S decreases. This implies that under real exchange rate targeting there are no other parameters generating real exchange rate inertia except for λ_q; under the speed limit policy, λ_S indirectly alleviates a fluctuation of the output gap produced by a change in the real exchange rate through its effects on the CPI.[13]

The optimal delegation parameter λ_π is the CPI inflation stabilization weight. Note that the figure indicates $\lambda_\pi > 1$ since λ_π deals with both domestic inflation stabilization—on which the weight equals 1 in general—and real exchange rate stabilization through its influence on CPI inflation. The absolute value of the contract term c_π is larger under the speed limit policy than under real exchange rate targeting. This result implies that the parameter c_π under the speed limit policy fills the gap between optimal inertia and the inertia provided by λ_S and λ_π.

As already noted, we can also consider alternative targeting regimes. First, we consider the optimal delegation problem under CPI inflation targeting. Then, we consider the role of nominal income growth targeting. The result of solving a discretionary policy under CPI inflation targeting is summarized in Proposition 3.

Proposition 3. *(CPI inflation targeting) The Markov-perfect equilibrium values of domestic inflation and output gap which occur if the central bank minimizes the delegated loss function (13), are identical to the timeless-optimal commitment solutions (8) and (9) if and only if the delegation parameters are given by the following:*

$$\lambda_\pi^* = \frac{\gamma}{\chi_\pi} \tag{21}$$

$$c_\pi^* = -\Psi_\pi \left(\Psi_\pi \frac{\kappa_\nu}{\lambda} - 1\right) \left(\frac{\gamma}{\chi_\pi}\right) \tag{22}$$

$$\lambda_x^* = \frac{\gamma^2}{\chi_\pi} \left(\Psi_\pi \frac{\kappa_\nu}{\lambda} - 1\right) \tag{23}$$

where $\Psi_\pi = \nu\sigma_\nu$ *and* $\chi_\pi = \frac{\kappa_\nu}{\lambda}\Psi_\pi^2 \left(\frac{\kappa_\nu}{\lambda}\gamma + 1\right) - \gamma - \Psi_\pi$*.In particular, these delegation parameters under CPI inflation targeting lead to the same outcome as commitment if and only if the parameter* λ *takes a smaller value.*

[13]If $\sigma\eta > 1$, openness influences the sensitivity of the output gap to a change in the real exchange rate, as measured by $1/(\sigma_\nu(1-\nu))$; the marginal weight on λ_q also increases with respect to openness.

The proof is shown in Appendix A. Like in the case of both real exchange rate targeting and speed limit targeting, one may conjecture that this targeting regime also corresponds to the commitment equilibrium. However, we show that this regime cannot lead to the same outcome as commitment because the second-order condition of this problem is not satisfied unless λ takes a smaller value. As shown in Bilbiie (2014), while the parameter $\theta_{\pi s}$ must take a negative value to satisfy the second-order condition,[14] that condition is not satisfied if λ takes a higher value in this study.

Figure 3 shows $\theta_{\pi s}$ satisfying the second-order conditions of the optimization problem under the delegated regime. Figure 3(a) illustrates the case of CPI inflation targeting. We observe that the second-order condition is not satisfied when $\lambda > 0.3$. This implies that the CPI inflation targeting regimes cannot lead to the same solution as commitment when the private sector requires the policymaker to put a higher weight on the stabilization of the output gap relative to inflation.

(B.11) in Appendix B shows the loss functions of the nominal income growth targeting. Consider the coefficients of x_t^2, which is given by $(\lambda_x - \lambda + \lambda_\pi \nu^2 \sigma_{nu}^2)$. As shown in Fig 2(a), the delegation parameter λ_x never takes negative values, whereas λ_π necessarily takes negative values. Therefore, if λ takes a higher value, the stabilization term of the output gap might be negative. This implies that the welfare would be improved when the output gap fluctuates. However this contradicts a traditional assertion that the central bank faces the trade-off between output gap stabilization and inflation stabilization in the presence a the cost-push shock.

[Figure 2 around here]

Now, we derive the optimal delegation parameters under nominal income growth targeting, which are summarized in the following proposition:

Proposition 4. *(Nominal income growth targeting) The Markov-perfect equilibrium value of domestic inflation and output gap, which occur if the central bank minimizes the delegated loss function (14) are identical to the timeless-optimal commitment solutions (8) and (9) if and only if the delegation param-*

[14] See Appendix B for the definition of parameter $\theta_{\pi s}$.

eters are given by the following:

$$\lambda_N^* = \frac{\gamma}{\chi_N} \tag{24}$$

$$c_\pi^* = -\Psi_N \left(\Psi_N \frac{\kappa_\nu}{\lambda} - 1\right) \left(\frac{\gamma}{\chi_N}\right) \tag{25}$$

$$\lambda_x^* = \frac{\gamma^2}{\chi_N} \Psi_N \left(\Psi_N \frac{\kappa_\nu}{\lambda} - 1\right) \tag{26}$$

where $\Psi_N = 1 + \nu\sigma_\nu$ *and* $\chi_N = \frac{\kappa_\nu}{\lambda}\Psi_N^2\left(\frac{\kappa_\nu}{\lambda}\gamma + 1\right) - \gamma - \Psi_N$. *In particular, these delegation parameters under nominal income targeting lead to the same outcome as commitment if and only if* λ *takes a smaller value.*

The proof is shown in Appendix A. As in the case of CPI inflation targeting, this regime fails to satisfy the condition that $\theta_{\pi s}$ is negative. Nominal income growth targeting requires a smaller value of λ for the same reason as in the CPI inflation targeting case. Thus, (B.11) in Appendix B presents loss functions of nominal income growth targeting. Consider the coefficients of x_t^2, which is given by $(\lambda_x - \lambda + (1+\nu\sigma_\nu)^2\lambda_N)$. As shown in Figure 2(b), the delegation parameter λ_x never takes negative values, whereas λ_N necessarily takes negative values. Therefore, if λ takes a higher value, the stabilization term of the output gap might be negative. This implies that welfare would be improved when the output gap fluctuates. Again, this contradicts the traditional assertion that the central bank faces a trade-off between output and inflation stabilization in the presence of a cost-push shock.

Figure 3(b) illustrates the case of nominal income growth targeting. We observe that the second-order condition is satisfied provided $\lambda < 0.1$. In contrast to the case of CPI inflation targeting, this requires considerably smaller values compared with the calibrated value of λ in the standard new Keynesian model. For instance, Jensen (2002) and Walsh (2003) considered the range of λ between 0.1 and 1.0. However, we show that this parameter value is invalid in that the targeting regime that stabilizes nominal income growth cannot achieve the same solution as a commitment policy.

5 Conclusions

This paper explores the delegation of targeting regimes in a small open economy model. We examined how the presence of an open economy affects the coefficients in the delegated objective function of the central bank. We analytically derived the condition whereby each targeting regime considered herein coincides with a commitment policy.

The findings of this paper are summarized as follows. First, in contrast to previous studies that focused on delegating policy regimes in a closed economy model, we consider the role of the real exchange rate in delegating monetary policy schemes. We analytically derived the condition whereby the optimal policy under the central bank's loss function that incorporates the stabilization of the real exchange rate corresponds to that under a commitment policy. We showed that the presence of the real exchange rate is crucial when considering delegated monetary policy schemes in a small open economy.

Second, we explored whether several policy regimes corresponds to the solution of a commitment policy. More specifically, we considered the following policy regimes: speed limit policy, nominal income growth targeting, and CPI inflation targeting. Based on this, the speed limit policy can also achieve the same outcomes as that of a commitment policy in a small-open economy. The difference between a closed and small open economy is captured by an indirect real exchange rate channel in addition to the conventional role of the speed limit target for the output gap. According to the standard new Keynesian model, the output gap affects the inflation rate through the NKPC. In addition to the traditional channel, the output gap affects the inflation rate through a change in the terms of trade, resulting in a movement in the real exchange rate. Hence, the speed limit policy contains an indirect stabilization of the output gap through the real exchange rate.

Third, in contrast, targeting regimes such as CPI inflation and nominal income growth could lead to the same outcomes as an optimal commitment policy if the relative weight on the output gap to inflation takes a considerably smaller value in the social loss function.

Finally, we suggest possible extensions to our study. First, we examined the optimal delegation problem in the case of perfect exchange rate pass-through. As pointed out by Monacelli (2005), that assumption may not be realistic. Therefore, extending our study to the case of incomplete exchange rate pass-through could generate results with greater empirical credibility. Second, our study is also applicable to the case of a two-large country model (Clarida, Gali and Gertler, 2002; Benigno and Benigno, 2006): as such, this provides another avenue that could be explored in the future.

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A Appendix A: Proof of Proposition

A.1 Proof of Proposition 1

We define the Bellman equation as follows:

$$V(x_{t-1}; u_t) = \min \frac{1}{2}\left[\lambda_x x_t^2 + \pi_t^2 + \lambda_q \Delta q_t^2 + 2c_\pi \pi_t x_t + \beta E_t V(x_t; u_{t+1})\right].$$

Substituting the NKPC into the above Bellman equation, we obtain

$$V(x_{t-1}; u_t) = \min \frac{1}{2}\Big\{\lambda_x x_t^2 + [\beta E_t \pi_{t+1} + \kappa_\nu x_t + u_t]^2 + \lambda_q \Delta q_t^2 + 2c_\pi \pi_t x_t + \beta E_t V(x_t; u_{t+1})\Big\}.$$

The first-order condition with respect to x_t is as follows:

$$[\lambda_x + \lambda_q (1-\nu)^2 \sigma_\nu^2] x_t + \gamma \pi_t + c_\pi \gamma x_{t-1} - \lambda_q (1-\nu)^2 \sigma_\nu x_{t-1} + \beta E_t \frac{\partial V(x_t; u_{t+1})}{\partial x_t} = 0. \tag{A.1}$$

In addition, from the envelop theorem, we obtain

$$\frac{\partial V(x_{t-1;} u_t)}{\partial x_{t-1}} = -\lambda_q (1-\nu)^2 \sigma_\nu \Delta x_t + c_\pi \pi_t. \tag{A.2}$$

Substituting Equation (A.2) and Equations (8) and (9) into Equation (A.1) and then considering the optimal targeting rule, we obtain the following second-order difference equation:

$$\beta[-c_\pi \frac{\lambda}{\kappa_\nu} - \lambda_q (1-\nu)^2 \sigma_\nu] E_t x_{t+1} + [\lambda_x + (1+\beta)\lambda_q (1-\nu)^2 \sigma_\nu^2 + \beta c_\pi \frac{\lambda}{\kappa_\nu} - \frac{\lambda\gamma}{\kappa_\nu}] x_t + [c_\pi \gamma + \frac{\gamma\lambda}{\kappa_\nu} - \lambda_q (1-\nu)^2 \sigma_\nu] = 0. \tag{A.3}$$

As explained in Bilbiie (2014), in this equation, the optimal delegation parameter is found by noticing that Equation (A.3) evaluated at the conjectured solution (x_t^c, π_t^c) should be identity. Therefore, to identify the optimal delegation parameter, all coefficients should be zero. This leads to the following relationship:

$$c_\pi \frac{\lambda}{\kappa_\nu} + \lambda_q (1-\nu)^2 \sigma_\nu = 0, \tag{A.4}$$

$$\lambda_x + (1+\beta)\lambda_q (1-\nu)^2 \sigma_\nu^2 + \beta c_\pi \frac{\lambda}{\kappa_\nu} - \frac{\lambda\gamma}{\kappa_\nu} = 0, \tag{A.5}$$

$$c_\pi \gamma + \frac{\gamma\lambda}{\kappa_\nu} - \lambda_q (1-\nu)^2 \sigma_\nu = 0. \tag{A.6}$$

Solving Equations (A.4)–(A.6), we obtain the optimal delegation parameters in Proposition 1.

A.2 Proof of Proposition 2

Before defining the Bellman equation, we arrange the objective function. Substituting the definition of CPI inflation into the objective function, we obtain the following objective function:

$$\begin{aligned} &(\lambda + \lambda_S + \lambda_\pi \nu^2 \sigma_\nu^2) x_t^2 + (\lambda_S + \lambda_\pi \nu^2 \sigma_\nu^2) x_{t-1}^2 - 2(\lambda_S + \lambda_\pi \nu^2 \sigma_\nu^2) x_t x_{t-1} \\ &+ 2\lambda_\pi \nu \sigma_\nu \pi_t \Delta x_t + 2 c_\pi x_{t-1} \pi_t + \lambda_\pi \pi_t^2. \end{aligned} \tag{A.7}$$

Then we define the Bellman equation as follows:

$$\begin{aligned} V(x_{t-1}; u_t) = \min \frac{1}{2} \Big\{ &(\lambda + \lambda_S + \lambda_\pi \nu^2 \sigma_\nu^2) x_t^2 + (\lambda_S + \lambda_\pi \nu^2 \sigma_\nu^2) x_{t-1}^2 \\ - 2(\lambda_S + \lambda_\pi \nu^2 \sigma_\nu^2) x_t x_{t-1} &+ 2\lambda_\pi \nu \sigma_\nu \pi_t \Delta x_t + 2 c_\pi x_{t-1} \pi_t + \lambda_\pi \pi_t^2 + \beta E_t V(x_t; u_{t+1}) \Big\}. \end{aligned}$$

The first-order condition with respect to x_t is as follows:

$$\begin{aligned} &(\lambda + \lambda_S + \lambda_\pi \nu^2 \sigma_\nu^2) x_t - (\lambda_S + \lambda_\pi \nu^2 \sigma_\nu^2 + \lambda_\pi \nu \sigma_\nu \gamma - c_\pi \gamma) x_{t-1} + \lambda_\pi (\gamma + \nu \sigma_\nu) \pi_t \\ &+ \beta E_t \frac{\partial V(x_t; u_{t+1})}{\partial x_t} = 0. \end{aligned} \tag{A.8}$$

In addition, from the envelop theorem, we obtain

$$\frac{\partial V(x_{t-1;}u_t)}{\partial x_{t-1}} = (\lambda_S + \lambda_\pi \nu^2 \sigma_\nu^2)\Delta x_t + (c_\pi - \lambda_\pi \nu \sigma_\nu)\pi_t. \tag{A.9}$$

Substituting (A.9) into (A.8) and considering the optimal targeting rule, we obtain the following second-order difference equation:

$$\begin{aligned}
&\beta\left[\left(\lambda_\pi \nu \sigma_\nu - c_\pi\right)\frac{\lambda}{\kappa_\nu} - (\lambda_S + \lambda_\pi \nu^2 \sigma_\nu^2)\right] E_t x_{t+1} \\
&+ \left[\lambda + \lambda_S + \lambda_\pi \nu^2 \sigma_\nu^2 + \lambda_\pi \nu \sigma_\nu \gamma - \frac{\lambda_\pi(\gamma + \nu\sigma_\nu)\lambda}{\kappa_\nu} + \beta(\lambda_S + \lambda_\pi \nu^2 \sigma_\nu^2) + \frac{\beta(c_\pi - \lambda_\pi \nu \sigma_\nu)\lambda}{\kappa_\nu}\right] x_t \\
&+ \left[\frac{\lambda_\pi(\gamma + \nu\sigma_\nu)\lambda}{\kappa_\nu} - (\lambda + \lambda_S + \lambda_\pi \nu^2 \sigma_\nu^2 + \lambda_\pi \nu \sigma_\nu \gamma) + c_\pi \gamma\right] x_{t-1} = 0.
\end{aligned} \tag{A.10}$$

As explained in Proposition 1, in this equation, the optimal delegation parameter is founded by noticing that Equation (A.10) evaluated at the conjectured solution (x_t^c, π_t^c) should be identity. Therefore, to observe the optimal delegation parameter, all coefficients should be zero. This leads to the following relationship:

$$(\lambda_\pi \nu \sigma_\nu - c_\pi)\frac{\lambda}{\kappa_\nu} - (\lambda_S + \lambda_\pi \nu^2 \sigma_\nu^2) = 0 \tag{A.11}$$

$$\lambda + \lambda_S + \lambda_\pi \nu^2 \sigma_\nu^2 + \lambda_\pi \nu \sigma_\nu \gamma - \frac{\lambda_\pi(\gamma + \nu\sigma_\nu)\lambda}{\kappa_\nu} + \beta(\lambda_S + \lambda_\pi \nu^2 \sigma_\nu^2) + \frac{\beta(c_\pi - \lambda_\pi \nu \sigma_\nu)\lambda}{\kappa_\nu} = 0 \tag{A.12}$$

$$\frac{\lambda_\pi(\gamma + \nu\sigma_\nu)\lambda}{\kappa_\nu} - (\lambda + \lambda_S + \lambda_\pi \nu^2 \sigma_\nu^2 + \lambda_\pi \nu \sigma_\nu \gamma) + c_\pi \gamma = 0 \tag{A.13}$$

Solving Equations (A.11)–(A.13), we obtain the optimal delegation coefficients under the speed limit policy.

A.3 Proof of Proposition 3

Before defining the Bellman equation for solving this optimization problem, we calculate the objective function under CPI inflation targeting as follows:

$$\lambda_x x_t^2 + (1 + \lambda_\pi)\pi_t^2 + 2\lambda_\pi \nu \sigma_\nu \pi_t \Delta x_t + \lambda_\pi \nu^2 \sigma_\nu^2 \Delta x_t^2 + 2c_\pi x_{t-1}\pi_t. \tag{A.14}$$

Using this objective function, we can define the Bellman equation as follows:

$$V(x_{t-1};u_t) = \min \frac{1}{2}\Big\{\lambda_x x_t^2 + (1+\lambda_\pi)\pi_t^2 + 2\lambda_\pi \nu\sigma_\nu \pi_t \Delta x_t + \lambda_\pi \nu^2 \sigma_\nu^2 \Delta x_t^2 + 2c_\pi x_{t-1}\pi_t$$
$$+ \beta E_t V(x_t;u_{t+1})\Big\}.$$

The first-order condition of this optimization is given as follows:

$$\lambda_x x_t + (1+\lambda_\pi)\gamma\pi_t + \lambda\nu\sigma_\nu\gamma\Delta x_t + \lambda_\pi\nu\sigma_\nu\pi_t + \lambda_\pi\nu^2\sigma_\nu^2 x_t - \lambda_\pi\nu^2\sigma_\nu^2 x_{t-1}$$
$$c_\pi\gamma x_{t-1} + \beta E_t \frac{\partial V(x_t;u_{t+1})}{\partial x_t} = 0. \tag{A.15}$$

From the envelope theorem, we obtain the following equation:

$$\frac{\partial V(x_{t-1;}u_t)}{\partial x_{t-1}} = -\lambda_\pi\nu\sigma_\nu\pi_t - \lambda_\pi\nu^2\sigma_\nu^2\Delta x_t + c_\pi\pi_t. \tag{A.16}$$

Using this envelope theorem and the optimal targeting rule with commitment leads to the following second-order difference equation:

$$\beta[(\lambda_\pi\nu\sigma_\nu - c_\pi)\frac{\lambda}{\kappa_\nu} - \lambda_\pi\nu^2\sigma_\nu^2]E_t x_{t+1} + [\lambda_x - \frac{\lambda\gamma}{\kappa_\nu}(1+\lambda_\pi)$$
$$+ \lambda_\pi\nu\sigma_\nu\gamma - \frac{\lambda\lambda_\pi\nu\sigma_\nu}{\kappa_\nu} + \lambda_\pi\nu^2\sigma_\nu^2 + \beta(c_\pi - \lambda_\pi\nu\sigma_\nu)\frac{\lambda}{\kappa_\nu} + \beta\lambda_\pi\nu^2\sigma_\nu^2]x_t$$
$$[\frac{\lambda\gamma}{\kappa_\nu}(1+\lambda_\pi - \lambda_\pi\nu\sigma_\nu\gamma + \lambda_\pi\nu\sigma_\nu - \lambda_\pi\nu^2\sigma_\nu^2 + c_\pi\gamma]x_{t-1} = 0. \tag{A.17}$$

Again, as per the foregoing, the optimal delegation parameter is found by noticing that Equation (A.24) evaluated at the conjectured solution (x_t^c, π_t^c) should be identity. To identify the optimal delegation parameter, all coefficients should be zero. This leads to the following relationship:

$$(\lambda_\pi\nu\sigma_\nu - c_\pi)\frac{\lambda}{\kappa_\nu} - \lambda_\pi\nu^2\sigma_\nu^2 = 0, \tag{A.18}$$

$$\lambda_x - \frac{\lambda\gamma}{\kappa_\nu}(1+\lambda_\pi) + \lambda_\pi\nu\sigma_\nu\gamma - \frac{\lambda\lambda_\pi\nu\sigma_\nu}{\kappa_\nu} + \lambda_\pi\nu^2\sigma_\nu^2 + \beta(c_\pi - \lambda_\pi\nu\sigma_\nu)\frac{\lambda}{\kappa_\nu} + \beta\lambda_\pi\nu^2\sigma_\nu^2 = 0, \tag{A.19}$$

$$\frac{\lambda\gamma}{\kappa_\nu}(1+\lambda_\pi) - \lambda_\pi\nu\sigma_\nu\gamma + \lambda_\pi\nu\sigma_\nu - \lambda_\pi\nu^2\sigma_\nu^2 + c_\pi\gamma = 0, \tag{A.20}$$

Solving Equations (A.11)–(A.13) leads to the result of Proposition 3.

We now show that the delegation parameters under CPI inflation targeting lead to the same outcome as commitment if and only if parameter λ takes a smaller value. As shown in Appendix B, $\theta_{\pi s}$ under nominal income targeting is given by

$$\theta_{\pi s} = -\lambda_\pi \nu \sigma_\nu + c_\pi.$$

Using the delegation parameters, this equation can be written as

$$\theta_{\pi s} = -\Psi_\pi \frac{\gamma}{\chi_\pi}\left(\Psi_\pi \frac{\kappa_\nu}{\lambda} - 1\right).$$

As argued in Bilbiie (2014), the second-order condition is satisfied if $\theta_{\pi s} < 0$. Because Ψ_π, γ, and γ are positive, the condition requires χ_π and $(\Psi_\pi \frac{\kappa_\nu}{\lambda} - 1)$ are positive. These two conditions can be rewritten as

$$\begin{aligned}&\frac{\kappa_\nu}{\lambda}\Psi_\pi^2\left(\frac{\kappa_\nu}{\lambda}\gamma + 1\right) > \gamma + \Psi_\pi \\ &\Psi_\pi \frac{\kappa_\nu}{\lambda} > 1.\end{aligned}$$

Both conditions are satisfied if and only if λ takes a smaller value.

A.4 Proof of Proposition 4

Before defining the Bellman equation, we arrange the objective function. Substituting the definition of CPI inflation into the objective function, we obtain the following objective function:

$$\begin{aligned}&(1+\lambda_N)\pi_t^2 + [\lambda_x + \lambda_N(1+\nu\sigma_\nu)^2]x_t^2 + \lambda_N(1+\nu\sigma_\nu)^2 x_{t-1}^2 + 2\lambda_N(1+\nu\sigma_\nu)\pi_t x_t \\ &+ 2[c_\pi - \lambda_N(1+\nu\sigma_\nu)]\pi_t x_{t-1} - 2\lambda_N(1+\nu\sigma_\nu)^2 x_t x_{t-1} \qquad \text{(A.21)}\end{aligned}$$

Then, we define the Bellman equation as follows:

$$\begin{aligned}V(x_{t-1}; u_t) = \min \frac{1}{2}\Big\{&(1+\lambda_N)\pi_t^2 + [\lambda_x + \lambda_N(1+\nu\sigma_\nu)^2]x_t^2 + \lambda_N(1+\nu\sigma_\nu)^2 x_{t-1}^2 \\ &+ 2\lambda_N(1+\nu\sigma_\nu)\pi_t x_t + 2[c_\pi - \lambda_N(1+\nu\sigma_\nu)]\pi_t x_{t-1} - 2\lambda_N(1+\nu\sigma_\nu)^2 x_t x_{t-1} \\ &+ \beta E_t V(x_t; u_{t+1})\Big\}.\end{aligned}$$

The first-order condition of this optimization is given as follows:

$$(1+\lambda_N)\gamma\pi_t + [\lambda_x + \lambda_N(1+\nu\sigma_\nu)^2]x_t + \lambda_N(1+\nu\sigma_\nu)\gamma x_t + \lambda_N(1+\nu\sigma_\nu)\pi_t$$
$$+ [c_\pi - \lambda_N(1+\nu\sigma_\nu)] + \beta E_t \frac{\partial V(x_t; u_{t+1})}{\partial x_t} = 0. \tag{A.22}$$

It follows from the envelope theorem that

$$\frac{\partial V(x_{t-1;}u_t)}{\partial x_{t-1}} = -\lambda_N(1+\nu\sigma_\nu)^2\Delta x_t + [c_\pi - \lambda_N(1+\nu\sigma_\nu)]\pi_t. \tag{A.23}$$

Using the envelope theorem and the optimal targeting rule, we obtain the following second-order difference equation:

$$\beta\left[\left(\lambda_N(1+\nu\sigma_\nu) - c_\pi\right)\frac{\lambda}{\kappa_\nu} - \lambda_N(1+\nu\sigma_\nu)^2\right]E_t x_{t+1}$$
$$+ \left[\lambda_x + \lambda_N(1+\nu\sigma_\nu)^2 - (1+\lambda_N)\frac{\gamma\lambda}{\kappa_\nu} + \lambda_N(1+\nu\sigma_\nu)\gamma - \frac{\lambda\lambda_N}{\kappa_\nu}(1+\nu\sigma_\nu)\right.$$
$$\left. + \beta\lambda_N(1+\nu\sigma_\nu) + \beta\left(c_\pi - \lambda_N(1+\nu\sigma_\nu)\right)\frac{\lambda}{\kappa_\nu}\right]x_t$$
$$+ \left[(1+\lambda_N)\frac{\gamma\lambda}{\kappa_\nu} + \frac{\lambda\lambda_N}{\kappa_\nu}(1+\nu\sigma_\nu) + (c_\pi - \lambda_N(1+\nu\sigma_\nu))\gamma - \lambda_N(1+\nu\sigma_\nu)^2\right]x_{t-1} = 0 \tag{A.24}$$

Once more, this equation the optimal delegation parameter requires noticing that Equation (A.24) evaluated at the conjectured solution (x_t^c, π_t^c) should be identity, and all coefficients should be zero. This leads to the following relationship:

$$[\lambda_N(1+\nu\sigma_\nu) - c_\pi]\frac{\lambda}{\kappa_\nu} - \lambda_N(1+\nu\sigma_\nu)^2 = 0, \tag{A.25}$$

$$\lambda_x + \lambda_N(1+\nu\sigma_\nu)^2 - (1+\lambda_N)\frac{\gamma\lambda}{\kappa_\nu} + \lambda_N(1+\nu\sigma_\nu)\gamma - \frac{\lambda\lambda_N}{\kappa_\nu}(1+\nu\sigma_\nu)$$
$$+ \beta\lambda_N(1+\nu\sigma_\nu) + \beta[c_\pi - \lambda_N(1+\nu\sigma_\nu)]\frac{\lambda}{\kappa_\nu} = 0, \tag{A.26}$$

$$(1+\lambda_N)\frac{\gamma\lambda}{\kappa_\nu} + \frac{\lambda\lambda_N}{\kappa_\nu}(1+\nu\sigma_\nu) + (c_\pi - \lambda_N(1+\nu\sigma_\nu))\gamma - \lambda_N(1+\nu\sigma_\nu)^2 = 0. \tag{A.27}$$

Solving Equations (A.11)–(A.13) provides the result of Proposition 4.

We now show that the delegation parameters under nominal income growth targeting lead to the same outcome as commitment if and only if λ takes a smaller value. As shown in Appendix B, $\theta_{\pi s}$ under nominal income targeting is given by

$$\theta_{\pi s} = -(1 + \nu\sigma_\nu)\lambda_N + c_\pi.$$

Using the delegation parameters, this equation can be written as

$$\theta_{\pi s} = -\frac{\gamma}{\chi_N}\Psi_N\frac{\kappa_\nu}{\lambda}.$$

As argued in Bilbiie (2014), the second-order condition is satisfied if $\theta_{\pi S} < 0$. Because $\Psi_N\kappa_\nu/\lambda$ and γ are positive, the condition requires that χ_N is positive. Thus, the following condition must be satisfied to guarantee $\chi_N > 0$:

$$\frac{\kappa_\nu}{\lambda}\Psi_N\left(\frac{\kappa_\nu\gamma}{\lambda} + 1\right) > \gamma + \Psi_N.$$

This condition is satisfied if and only if λ is very small.

B Appendix B: Proof of Proposition by General-Quadratic Method

This appendix shows that the proof provided in Appendix A can also be arrived by general quadratic method. According to Bilbiie (2014), we consider the general form of delegation by adding a quadratic form $Z_t'\Theta Z_t$ over all relevant variables $Z_t = (\pi_t, x_t, x_{t-1})'$ to the social loss function:

$$L_t^b = \frac{1}{2}\left[L_t + Z_t'\Theta Z_t\right], \tag{B.1}$$

where Θ is the symmetric matrix of all delegation parameters, $\Theta = \{\theta_{ij}\}, i, j = \pi, x, s, \theta_{ij} = \theta_{ji}$. The script s denotes the term for lagged output gap.

Applying Bilbiie's(2014) proposition to an open economy, the conditions that need to be fulfilled by the delegation parameters to induce the optimal amount of inertia areas follows:

$$\begin{aligned}\theta^*_{xs} &= \frac{\lambda}{\kappa}\theta^*_{\pi s} \\ \lambda + \theta^*_{xx} + \gamma\theta^*_{\pi x} + \beta\theta^*_{ss} &= -\left(\gamma + \frac{\lambda}{\kappa} + \frac{\lambda}{\kappa}\beta\right)\theta^*_{\pi s} \\ \gamma(1+\theta_{\pi\pi})^* + \theta^*_{\pi x} &= -\left(\frac{\kappa}{\lambda}\gamma + 1\right)\theta^*_{\pi s}\end{aligned} \tag{B.2}$$

and

$$\theta^*_{\pi s} < 0. \tag{B.3}$$

B.1 Real Exchange Rate Targeting

The policy regime is described by the following loss function:

$$L^q_t = \frac{1}{2}[\pi^2_t + \lambda_x x^2_t + \lambda_q \Delta q^2_t + 2c_\pi \pi_t x_{t-1}] \tag{B.4}$$

Substituting (4) and (5), we obtain

$$\begin{aligned} L^q_t &= \frac{1}{2}[\pi^2_t + \lambda_x x^2_t + (1-\nu)^2\sigma^2_\nu \lambda_q \Delta x^2_t + 2c_\pi \pi_t x_{t-1}] \\ \Rightarrow L^q_t &= \frac{1}{2}[\pi^2_t + \lambda x^2_t + (\lambda_x - \lambda + \lambda_q(1-\nu)^2\sigma^2_\nu)x^2_t \\ &\qquad + (1-\nu)^2\sigma^2_\nu\lambda_q x^2_{t-1} + 2c_\pi\pi_t x_{t-1} - 2\lambda_q(1-\nu)^2\sigma^2_\nu x_t x_{t-1}]. \end{aligned} \tag{B.5}$$

For the loss function (B.5), we have the following mapping between its delegating parameters in the general case [the Θ matrix in (B.1)]:

$$\begin{array}{lll} \theta_{\pi\pi} = 0 & \theta_{xx} = \lambda_x - \lambda + \lambda_q(1-\nu)^2\sigma^2_\nu & \theta_{ss} = (1-\nu)^2\sigma^2_\nu\lambda_q \\ \theta_{\pi x} = 0 & \theta_{\pi s} = c_\pi & \theta_{xs} = -\lambda_q(1-\nu)^2\sigma^2_\nu \end{array}$$

Substituting this in the parameter restriction for the optimal delegation (B.2), we obtain the optimal delegation parameters for our targeting regime as in Proposition 1.

B.2 Speed Limit Policy

The policy regime is described by the following loss function:

$$L_t^S = \frac{1}{2}[\lambda x_t^2 + \lambda_S \Delta x_t^2 + \lambda_\pi (\pi_t^c)^2 + 2c_\pi \pi_t x_{t-1}] \tag{B.6}$$

Substituting (3) and (5) into the loss function, we obtain

$$\begin{aligned} L_t^S &= \frac{1}{2}[\pi_t^2 - \pi_t^2 + \lambda x_t^2 + \lambda_S \Delta x_t^2 + \lambda_\pi (\nu\sigma_\nu \Delta x_t - \pi_t)^2 + 2c_\pi \pi_t x_{t-1}] \\ \Rightarrow L_t^S &= \frac{1}{2}[\pi_t^2 + \lambda x_t^2 + (-1+\lambda_\pi)\pi_t^2 + \lambda_\pi \nu^2 \sigma_\nu^2 x_t^2 + (\lambda_S + \lambda_\pi \nu^2 \sigma_\nu^2) x_{t-1}^2 \\ &\quad + 2\lambda_\pi \nu\sigma_\nu \pi_t x_t + 2(-\lambda_\pi \nu\sigma_\nu + c_\pi)\pi_t x_{t-1} + 2(-\lambda_S - \lambda_\pi \nu^2 \sigma_\nu^2) x_t x_{t-1}]. \end{aligned} \tag{B.7}$$

For the loss function (B.7), we have the following mapping between it's delegating parameters in the general case [the Θ matrix in (B.1)]:

$$\begin{array}{lll} \theta_{\pi\pi} = -1 + \lambda_\pi & \theta_{xx} = \lambda_\pi \nu^2 \sigma_\nu^2 & \theta_{ss} = \lambda_S + \lambda_\pi \nu^2 \sigma_\nu^2 \\ \theta_{\pi x} = \lambda_\pi \nu \sigma_\nu & \theta_{\pi s} = -\lambda_\pi \nu\sigma_\nu + c_\pi & \theta_{xs} = -\lambda_S - \lambda_\pi \nu^2 \sigma_\nu^2 \end{array}$$

Substituting this in the parameter restriction for the optimal delegation (B.2), we obtain the optimal delegation parameters for our targeting regime as in Proposition 2.

B.3 CPI Inflation Targeting

The policy regime is described by the following loss function:

$$L_t^\pi = \frac{1}{2}[\pi_t^2 + \lambda_x x_t^2 + \lambda_\pi (\pi_t^c)^2 + 2c_\pi \pi_t x_{t-1}] \tag{B.8}$$

Substituting (3) and (5) into the loss function, we obtain

$$\begin{aligned} L_t^\pi &= \frac{1}{2}[\pi_t^2 + \lambda x_t^2 + (\lambda_x - \lambda) x_t^2 + \lambda_\pi (\nu\sigma_\nu \Delta x_t + \pi_t)^2 + 2c_\pi \pi_t x_{t-1}] \\ \Rightarrow L_t^\pi &= \frac{1}{2}[\pi_t^2 + \lambda x_t^2 + \lambda_\pi \pi_t^2 + (\lambda_x - \lambda + \lambda_\pi \nu^2 \sigma_\nu^2) x_t^2 + \lambda_\pi \nu^2 \sigma_\nu^2 x_{t-1}^2 \\ &\quad + 2\lambda_\pi \nu\sigma_\nu \pi_t x_t + 2(-\lambda_\pi \nu\sigma_\nu + c_\pi)\pi_t x_{t-1} - 2\lambda_\pi \nu\sigma_\nu x_t x_{t-1}]. \end{aligned} \tag{B.9}$$

For the loss function (B.9), we have the following mapping between it's delegating parameters in the general case [the Θ matrix in (B.1)]:

$$\theta_{\pi\pi} = \lambda_\pi \qquad \theta_{xx} = \lambda_x - \lambda + \lambda_\pi \nu^2 \sigma_\nu^2 \qquad \theta_{ss} = \lambda_\pi \nu^2 \sigma_\nu^2$$
$$\theta_{\pi x} = \lambda_\pi \nu \sigma_\nu \qquad \theta_{\pi s} = -\lambda_\pi \nu \sigma_\nu + c_\pi \qquad \theta_{xs} = -\lambda_\pi \nu \sigma_\nu$$

Substituting this in the parameter restriction for the optimal delegation (B.2), we obtain the optimal delegation parameters for our targeting regime as in Proposition 3.

B.4 Nominal Income Growth Targeting

The policy regime is described by the following loss function:

$$L_t^N = \frac{1}{2}[\pi_t^2 + \lambda_x x_t^2 + \lambda_N(\pi_t^c + \Delta x_t)^2 + 2c_\pi \pi_t x_{t-1}] \tag{B.10}$$

Substituting (3) and (5) into the loss function, we obtain

$$\begin{aligned} L_t^N &= \frac{1}{2}[\pi_t^2 + \lambda x_t^2 + (\lambda_x - \lambda)x_t^2 + \lambda_N((1+\nu\sigma_\nu)\Delta x_t + \pi_t)^2 + 2c_\pi \pi_t x_{t-1}] \\ \Rightarrow L_t^N &= \frac{1}{2}[\pi_t^2 + \lambda x_t^2 + \lambda_N \pi_t^2 + (\lambda_x - \lambda + (1+\nu\sigma_\nu)^2\lambda_N)x_t^2 \\ &\quad + (1+\nu\sigma_\nu)^2\lambda_N x_{t-1}^2 + 2(1+\nu\sigma_\nu)\lambda_N \pi_t x_t \\ &\quad + 2(-(1+\nu\sigma_\nu)\lambda_N + c_\pi)\pi_t x_{t-1} + 2(-(1+\nu\sigma_\nu)^2\lambda_N)x_t x_{t-1}]. \end{aligned} \tag{B.11}$$

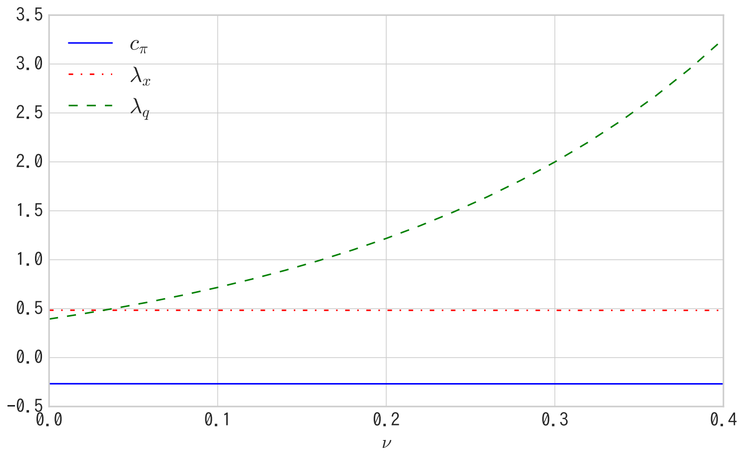
For the loss function (B.11), we have the following mapping between its delegating parameters in the general case [the Θ matrix in (B.1)]:

$$\theta_{\pi\pi} = \lambda_N \qquad \theta_{xx} = \lambda_x - \lambda + (1+\nu\sigma_\nu)^2\lambda_N \qquad \theta_{ss} = (1+\nu\sigma_\nu)^2\lambda_N$$
$$\theta_{\pi x} = (1+\nu\sigma_\nu)\lambda_N \qquad \theta_{\pi s} = -(1+\nu\sigma_\nu)\lambda_N + c_\pi \qquad \theta_{xs} = -(1+\nu\sigma_\nu)^2\lambda_N$$

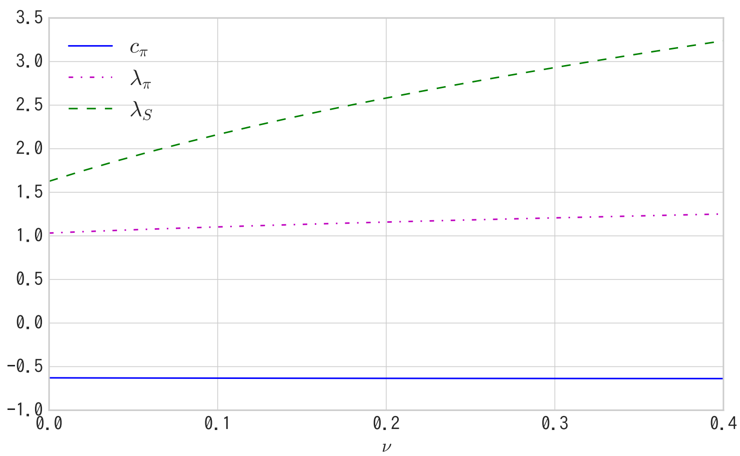
Substituting this in the parameter restriction for the optimal delegation (B.2), we obtain the optimal delegation parameters for our targeting regime as in Proposition 4.

Table 1: Deep parameters

	Parameters	Values
β	Discount rate	0.99
σ	Relative risk aversion coefficient for consumption	2.0
φ	Inverse of the elasticity of labor supply	5.0
α	Degree of decreasing return on labor	0.25
ϵ	Elasticity of substitution for individual intermediate goods	9.0
θ	Calvo lottery	0.75
η	Elasticity of substitution between domestic and foreign goods	1.0
ν	Degree of openness	0.4
λ	Weight on the output gap in the true loss function	0.5

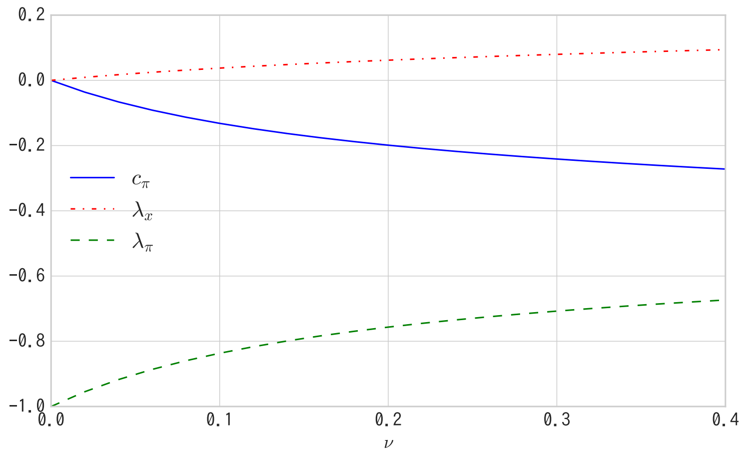


(a) Real Exchange Rate Target

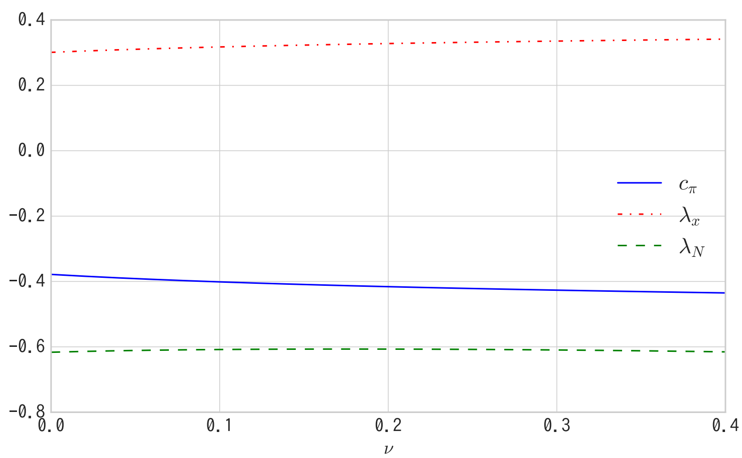


(b) Speed Limit Policy

Figure 1: Real Exchange Rate Target vs. Speed Limit Policy

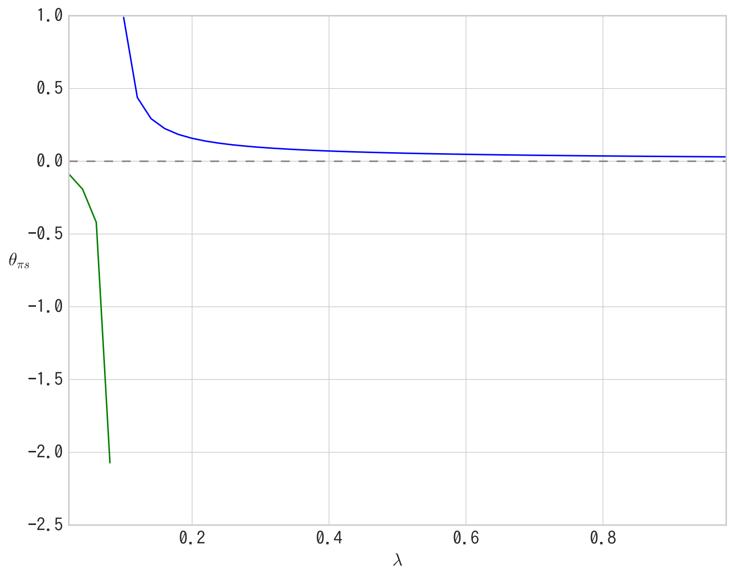


(a) CPI Inflation Target

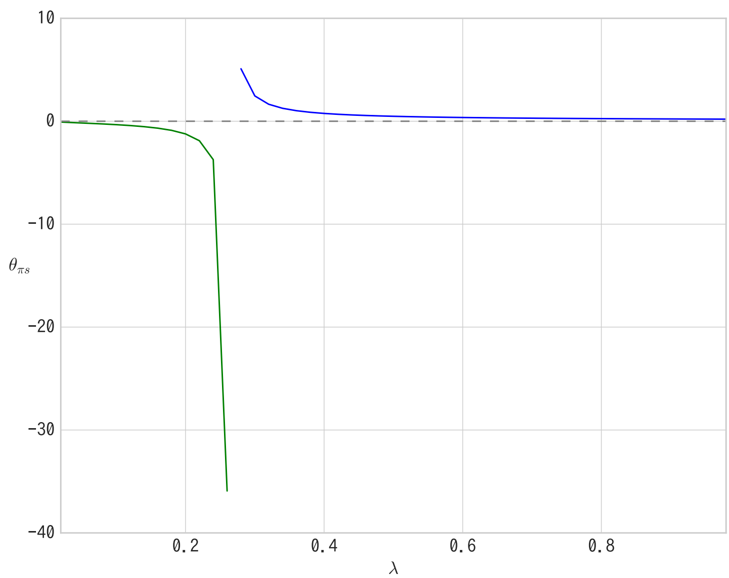


(b) Nominal Income Growth Target

Figure 2: Other Regimes



(a) CPI Inflation Target



(b) Nominal Income Growth Target

Figure 3: Whether the Second-Order Condition $\theta_{\pi s} < 0$ Holds With Respect to λ