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Delegating nominal income growth targeting in a small-open economy*

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Abstract

The present study examines the role of nominal income growth targeting (NIGT) in a small-open economy and shows that the central bank's selection of a price index crucially affects the performance of NIGT in a small open economy. NIGT based on a producer price index can always achieve the same outcome as the commitment policy. However, NIGT based on a consumer price index fails to create the same outcome as the commitment policy unless the stabilisation weight on the output gap is considerably small in the true loss function.

Keywords: Optimal monetary policy; Delegation; Nominal income growth targeting; Price index;

JEL classification: E52; E58; F41

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1 Introduction

It has been argued that the outcomes of a commitment policy may be preferable to those of a discretionary policy in the standard new Keynesian model. A central bank that commits its future policy stances to the public sector can operate monetary policy by managing the expectations of the private sector, which can induce policy inertia into the economy. In contrast, a discretionary policy, which treats the future expectations as given, cannot induce policy inertia into the economy. This difference is referred to as stabilisation bias.

Discretionary policy suffers from stabilization bias, which can be overcome by a government's delegation to the central bank an objective that differs from the true objective function. Bilbiie (2014) analytically derived a condition by which a delegated policy regime corresponds with a commitment policy. The analysis shows that each coefficient for stabilisation terms in the delegated loss function is characterised by deep parameters.

Following Bilbiie (2014), Ida and Okano (2017) analytically examined how the effect of an open economy changes the characteristics of delegated policy regimes, using a framework of a small-open economy based on Gali (2015). They also analytically showed the condition under which delegated monetary policy regimes correspond with a commitment solution in a small-open new Keynesian model.

Considering the monetary policy analysis in a closed economy framework, there is no need for differentiation between the consumer price index (CPI) and producer price index (PPI) inflation. However, extending the analysis to the small-open economy yields an important distinction between CPI and PPI inflation. Indeed, as Linnemann and Schabert (2006) noted, when the central bank responds to changes in CPI inflation, the Taylor principle may not apply, leading to the indeterminacy problem. Moreover, Gali (2015) emphasized the distinction between CPI and PPI inflation from a welfare analysis perspective, indicating that in a canonical representation of the small open new Keynesian model, the central bank should stabilise the PPI inflation for maximising household's welfare.

We show that the difference between PPI and CPI inflation changes the properties of delegating the optimal monetary policy by especially focusing on nominal income growth targeting (NIGT), which may use either PPI or CPI as the price index. First, a nominal income growth target based on PPI inflation coincides with the commitment solution. This policy regime can achieve the same outcome as the commitment policy. In contrast, a CPI-based nominal income growth target fails to achieve the same outcome as the commitment policy unless the stabilisation weight on the output gap is considerably small in the true loss function.

The paper is organised as follows. Section 2 describes the characteristics of a small-open economy new Keynesian model. Section 3 considers PPI and CPI-based NIGT regimes. Analytical derivations of the conditions show that these optimal delegation schemes coincide with a commitment policy, and show the properties of each policy regimes. Section 4 briefly concludes.

2 Model

The model is based on Gali (2015). The home country is infinitesimally small relative to the rest of the world. Representative households in the home country purchase both home and foreign goods. They can have access to a complete set of state-contingent securities that are traded both domestically and internationally. Firms face a monopolistically competitive environment and nominal price rigidities specified by Calvo (1983). As shown in Gali (2015), such situations leads to the following log-linearized system:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_\nu x_t + u_t, \tag{1}$$

$$x_t = E_t x_{t+1} - \sigma_\nu^{-1}(i_t - E_t \pi_{t+1} - \bar{rr}_t), \tag{2}$$

$$\pi_t^c = \pi_t + \nu(s_t - s_{t-1}), \tag{3}$$

$$q_t = (1-\nu)s_t \tag{4}$$

$$s_t = \sigma_\nu x_t. \tag{5}$$

where π_t is producer currency inflation, x_t is the output gap, i_t is the nominal interest rate, π_t^c is CPI inflation, s_t is the terms of trade gap, and q_t is the real exchange rate gap. Gap variables are expressed by the log-deviation of endogenous variables from the efficient level of their variables. In addition, $\bar{rr}_t$ denotes the natural rate of interest, which holds the real interest rate under flexible price equilibrium. u_t is the exogenous cost-push shock, which follows an AR(1) process. Finally, coefficients for each structural equation are defined as follows:

$$
\begin{aligned}
\sigma_\nu &= \frac{\sigma}{1+\nu[\sigma\eta+(1-\nu)(\sigma\eta-1)-1]}, \\
\kappa_\nu &= \delta\left(\sigma_\nu + \frac{1+\psi}{1-\alpha}\right), \\
\delta &= \frac{(1-\omega)(1-\omega\beta)}{\omega}\frac{1-\alpha}{1-\alpha+\alpha\epsilon}.
\end{aligned}
$$

The parameters β, σ, η, and ψ represent the discount factor, the relative risk aversion coefficient, the substitutability between domestic and foreign goods, and the inverse labour supply elasticity, respectively. ν is the degree of openness, and α is the degree of diminishing return to scale for labour supply. ω characterises the degree of nominal price rigidities (i.e. Calvo's lottery), and ϵ denotes the elasticity of substitution for individual goods.

Equation (1) represents the small-open economy new Keynesian Phillips curve (NKPC) which is derived from the firm's profit maximisation problem subject to Calvo pricing. Equation (2) is a dynamic IS equation resulting from a household's intertemporal optimisation problem. Equation (3) is the relationship between CPI and PPI inflation. Equation (4) states that the real exchange rate proportionally changes in response to changes in the terms of trade. Finally, Equation (5) represents the relationship between the terms of trade and the output gap.

The effect of the open economy is characterised by changes in both ν and $\sigma\eta$, where the parameter ν is the degree of openness. For a given $\sigma\eta > 1$, through the risk-sharing condition, the home output gap changes proportionally to a change in the terms of trade, which implies real exchange rate fluctuations.

In addition, the change in the terms of trade affects the sensitivity of inflation to the real marginal cost in the NKPC through two channels. First, home inflation reacts positively to improvement in the terms of trade by a change in the real exchange rate through international consumption risk sharing. Second, the terms of trade produces a change in home inflation because it induces a change in the real marginal cost. Whether home inflation increases depends on the movement of the value of $\sigma\eta$. In the case of $\sigma\eta = 1$, both κ_ν and σ_ν reduce to $\delta(1 + \frac{1+\psi}{1-\alpha}) \equiv \kappa$ and 1, respectively. Thus, the open economy effect disappears in this case.

The central bank conducts its monetary policy following a targeting rule derived from the minimization problem of the central bank's loss function. More specifically, the central bank minimises the following social loss function subject to structural equations:

$$L_t = E_t \sum_{j=0}^{\infty} \beta^j (\pi_{t+j}^2 + \lambda x_{t+j}^2). \tag{6}$$

where the parameter λ denotes the stabilisation weight on the output gap relative to inflation stabilisation.

In this paper, the central bank minimises the loss function (6) subject to NKPC under cost-push shock. A commitment solution produces the following targeting rule:

$$\pi_t = -\frac{\lambda}{\kappa_\nu}(x_t - x_{t-1}). \tag{7}$$

Except for structural equations coefficients, a targeting rule with commitment is the same shape as the one obtained in a closed economy. The central bank that conducts a commitment policy can manipulate the private sector expectations by gradually changing its policy variable. Thus, the presence of the lagged output gap in the targeting rule allows the central bank to employ an inertial behaviour of policy stance; therefore, a commitment policy is superior to a purely discretionary policy (e.g., Woodford, 2003; McCallum and Nelson, 2004). The term of 'commitment policy' in this paper implies a timeless perspective proposed by Woodford (2003).

The optimisation problem of the central bank leads to the reduced form of endogenous variables, which is solved by the standard factorisation method. Specifically, the reduced form of both inflation and the output gap are solved as follows:

$$\pi_t = \psi_\pi^{co} x_{t-1} + \psi_{u\pi}^{co} u_t, \tag{8}$$

$$x_t = \psi_x^{co} x_{t-1} + \psi_{ux}^{co} u_t, \tag{9}$$

where

$$\psi_\pi^{co} = \frac{\lambda}{\kappa_\nu}(1-\mu_1); \psi_{u\pi}^{co} = -\frac{\lambda}{\kappa_\nu}\psi_{xu}^{c},$$
$$\psi_x^{co} = \mu_1; \psi_{ux}^{co} = -\frac{\kappa_\nu}{\beta\lambda}\frac{1}{\mu_2-\rho_u}.$$

Here, μ_1 and μ_2 are the eigenvalues obtained from the solution of that factorization method. The parameter ρ_u denotes the degree of persistency of the cost-push shock. The superscript *co* denotes the commitment solution.

Equations (8) and (9) present the reduced forms of endogenous variables depending on the lagged variable x_{t-1}. Thus, if the central bank can make a credible commitment to the private sector, such a commitment policy enables endogenous variables to gradually change in response to a cost-push shock. In other words, the central bank can alleviate a policy trade-off generated by a cost-push shock by managing the private sector's expectations.

3 Nominal income growth targeting in a small-open economy

3.1 Two specifications of nominal income growth targeting

If the central bank cannot commit to its monetary policy, it naturally conducts a discretionary policy. However, a pure discretionary policy may produce a worse outcome than the commitment policy. Several studies argue that the discretionary policy can achieve a preferable outcomes if the government delegates the objective function with policy inertia, which is different from the social welfare criteria, to the central bank (Jensen, 2002; Walsh, 2003; Vestin, 2006). Jensen (2002) showed that performance under NIGT is similar to that under the commitment policy in a closed economy model.

However, in an open economy framework, PPI inflation does not correspond with CPI inflation (e.g, Equation (3)). How do these differences change the condition that the delegation problem under NIGT coincides with the optimal policy?

First, consider NIGT based on PPI. Specifically, the objective function under PPI-based NIGT is defined as follows:

$$L_{P,t} = \pi_t^2 + \lambda_x x_t^2 + \lambda_N(\pi_t + \Delta x_t)^2 + 2c_\pi x_{t-1}\pi_t. \tag{10}$$

This policy regime imparts inertia through a change in the output gap. The subscript P denotes PPI-based NIGT.

In contrast, as shown in Ida and Okano (2017), the targeting regime under CPI-based NIGT leads to policy inertia through a change in not only the output gap but also the terms of trade. In particular, CPI-based NIGT is defined as follows:

$$L_{C,t} = \pi_t^2 + \lambda_x^C x_t^2 + \lambda_N^C(\pi_t^c + \Delta x_t)^2 + 2c_\pi^C x_{t-1}\pi_t, \tag{11}$$

where the subscript C is the delegated parameter under NIGT based on CPI.

Finally, for both regimes, as explained in Bilbiie (2014), a fourth term is technically added to the objective function for an analytical solution corresponding to the solution under a commitment policy. In other words, this term reflects the linear inflation contract term proposed by Walsh (1995) and Svensson (1997).

3.2 The properties of nominal income growth targeting

We show that PPI-based NIGT can always achieve the same outcome as the commitment policy, whereas CPI-based NIGT can do so if and only if the parameter λ takes a considerably small value. The delegation problem under PPI-based NIGT leads to the following result:

Proposition 1. *(PPI-based NIGT) The Markov-perfect equilibrium values of domestic inflation and output gap, which occur if the central bank minimises the delegated loss function (10), are identical to the timeless-optimal commitment solutions (8) and (9) if* *and only if the delegation parameters are given as follows:*

$$\lambda_N = \frac{\gamma\lambda^2}{(\kappa_\nu - \lambda)(\gamma\kappa_\nu + \gamma\lambda + \lambda)}, \tag{12}$$

$$c_\pi = -\frac{\gamma\lambda}{\gamma\kappa_\nu + \gamma\lambda + \lambda}, \tag{13}$$

$$\lambda_x = \frac{\gamma^2\lambda}{\gamma\kappa_\nu + \gamma\lambda + \lambda}. \tag{14}$$

The proof of this proposition is given in the Appendix. This regime satisfies the properties of NIGT suggested by Jensen (2008) and Billbie (2014). In particular, it is possible that λ_N takes a negative value when $\kappa_\nu > \lambda$. In contrast to Billbie (2014), λ_N is now affected by ν and $\sigma\eta$ which specify the open economy effect. In other words, unsurprisingly, except for the effect of the open economy, this regime can completely replicate the commitment solution because c_π never takes any positive values. As shown in Billbie (2014), a negative value of c_π is required to satisfy the second-order condition of the optimal delegation problem under NIGT.

This condition only holds in the case of PPI. If this second-order condition is violated, the solution for NIGT does not lead to the same outcome as the commitment policy. This is possible if c_π takes a positive value. Under CPI-based NIGT, the second-order condition is invalid when the parameter λ does not take a small value. This is because c_π may take a positive value when λ takes a larger value.

Indeed, as shown in Ida and Okano (2017), the optimal delegated parameters under CPI-based NIGT are summarised as follows:

Proposition 2. *(CPI-based NIGT) The Markov-perfect equilibrium values of domestic inflation and output gap, which occur if the central bank minimises the delegated loss function (10), are identical to the timeless-optimal commitment solutions (8) and (9) if and only if the delegation parameters are given as follows:*

$$\lambda_N^C = \frac{\gamma}{\chi_N}, \tag{15}$$

$$c_\pi^C = -\Psi_N \left(\Psi_N \frac{\kappa_\nu}{\lambda} - 1\right)\left(\frac{\gamma}{\chi_N}\right), \tag{16}$$

$$\lambda_x^C = \frac{\gamma^2}{\chi_N}\Psi_N \left(\Psi_N \frac{\kappa_\nu}{\lambda} - 1\right), \tag{17}$$

where $\Psi_N = 1+\nu\sigma_\nu$ *and* $\chi_N = \frac{\kappa_\nu}{\lambda}\Psi_N^2\left(\frac{\kappa_\nu}{\lambda}\gamma+1\right)-\gamma-\Psi_N$*. In particular, these delegation parameters under nominal income targeting lead to the same outcome as commitment if and only if* λ *takes a small value.*

Ida and Okano (2017) provide the proof of this proposition. In contrast to the case of PPI-based NIGT, the optimal delegation parameters are more complicated. As mentioned in Ida and Okano (2017), CPI-based NIGT does not lead to the same outcome as commitment unless the parameter λ takes a considerably small value. As noted earlier, the parameter c_π^C takes a negative value to satisfy the second-order condition of the optimal delegation problem. It follows from Equation (16) that this only holds when $\nu\sigma_\nu\kappa_\nu > \lambda$. Indeed, Ida and Okano (2017) showed that this condition is violated when the parameter λ takes a value above 0.2.

Why does this difference occur in open economies? In contrast to the closed economy, a change in the terms of trade adjusts the difference between PPI and CPI inflation. In the case of CPI-based NIGT, the objective function under NIGT can be rewritten as follows:

$$L_{C,t} = \frac{1}{2}\{H_t + 2[c_\pi^C - (1+\nu\sigma_\nu)\lambda_N^C]\pi_t x_{t-1}\}, \tag{18}$$

where

$$\begin{aligned} H_t = {} & (1+\lambda_N^C)\pi_t^2 + [\lambda_x^C - \lambda + (1+\nu\sigma_\nu)^2\lambda_N^C]x_t^2 + 2(1+\nu\sigma_\nu)\lambda_N^C\pi_t x_t \\ & + (1+\nu\sigma_\nu)^2\lambda_N^C x_{t-1}^2 - 2(1+\nu\sigma_\nu)^2\lambda_N^C) x_t x_{t-1}. \end{aligned}$$

H_t is the correction term excluding the inflation linear contract term $\pi_t x_{t-1}$. Therefore, the second-term of the right-hand side can be regarded as the linear inflation contract term in an open economy. The term $(1+\nu\sigma_\nu)\lambda_N^C$ crucially affects the sign of this term. This term indicates that policy inertia derived from the terms of trade might weaken the effectiveness of the linear contract term, which guarantees that the parameter c_π^C takes a negative value.

4 Concluding remarks

This study has examined the role of NIGT in a small-open economy. We analytically showed that the central bank's selection of the price index crucially affects the performance of NIGT in a small open economy. NIGT based on a PPI can always achieve the same outcome as the commitment policy. However, NIGT based on a CPI fails to create the same outcome as the commitment policy unless the stabilisation weight on the output gap is considerably small in the true loss function.

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A Proof of Proposition 1 (Not for publication)

To solve a discretionary policy, we define the Bellman equation as follows:

$$V(x_{t-1}; u_t) = \min \frac{1}{2}\left[L_{P,t} + \beta E_t V(x_t; u_{t+1})\right].$$

The first-order condition with respect to x_t is as follows:

$$\begin{aligned}&2(1+\lambda_N)\gamma\pi_t + 2\lambda_N\pi_t + 2\lambda_N(1+\gamma)(x_t - x_{t-1}) + 2\lambda_x x_t + 2c_\pi\gamma x_{t-1}\\&+\frac{1}{2}\beta E_t \frac{\partial V(x_t; u_{t+1})}{\partial x_t} = 0\end{aligned} \tag{19}$$

In addition, from the envelop theorem, we obtain

$$\frac{\partial V(x_{t-1;}u_t)}{\partial x_{t-1}} = -2\lambda_N\pi_t - 2\lambda_N\Delta x_t + 2c_\pi\pi_t. \tag{20}$$

Substituting a one-period ahead Equation (20) and Equations (8) and (9) into Equation (19) and then considering the optimal targeting rule, we obtain the following second-order difference equation:

$$\begin{aligned}&\beta\left[(\lambda_N - c_\pi)\frac{\lambda}{\kappa_\nu} - \lambda_N\right]E_t x_{t+1} + \left[(1+\gamma)\lambda_N + \beta\lambda_N - [\gamma + \lambda_N(1+\gamma)]\frac{\lambda}{\kappa_\nu} + \lambda_x\right.\\&\left.-\beta(\lambda_N - c_\pi)\frac{\lambda}{\kappa_\nu}\right]x_t + \left[c_\pi\gamma - (1+\gamma)\lambda_N + (\gamma + \lambda_N(1+\gamma))\frac{\lambda}{\kappa_\nu}\right]x_{t-1} = 0.\end{aligned} \tag{21}$$

As explained in Bilbiie (2014), in this equation, the optimal delegation parameter is found by noticing that Equation (21) evaluated at the conjectured solution (x_t^{co}, π_t^{co}) should be identity. Therefore, to identify the optimal delegation parameter, all coefficients should be zero. This leads to the following relationship:

$$(\lambda_N - c_\pi)\frac{\lambda}{\kappa_\nu} - \lambda_N = 0, \tag{22}$$

$$(1+\gamma)\lambda_N + \beta\lambda_N - [\gamma + \lambda_N(1+\gamma)]\frac{\lambda}{\kappa_\nu} + \lambda_N - \beta(\lambda_N - c_\pi)\frac{\lambda}{\kappa_\nu} = 0, \tag{23}$$

$$c_\pi\gamma - (1+\gamma)\lambda_N + (\gamma + \lambda_N(1+\gamma))\frac{\lambda}{\kappa_\nu} = 0. \tag{24}$$

Solving Equations (22)–(24), we obtain the optimal delegation parameters in Proposition 1.