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The role of money and optimal monetary policy in a two-country economy

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Abstract

The importance of the role of money is evident from the monetary policies of advanced countries. This paper examines the effect of a non-separable utility between consumption and real money balance on optimal monetary policy in a two-country framework. It shows that relative real money balances between two countries affect the international risk-sharing condition directly when considering the non-separable utility. Departure from the separable utility function changes the structural equations in a two-country economy with nominal rigidities. In addition, the shape of the loss function is different from that of a standard loss function. Therefore, the assumption of a non-separable utility affects the properties of optimal monetary policy significantly. The presence of a non-separable utility function between consumption and money crucially changes the termination date of a zero interest rate policy for both countries.

Keywords: Optimal monetary policy; Money demand function; Two-country model; Zero lower bound on nominal interest rates

JEL classification: E52; E58; F41

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1 Introduction

The new Keynesian model (NKM) approach to recent monetary policy analysis is prevalent (Clarida, Gali and Gertler, 1999; Woodford, 2003; Walsh, 2017; Gali, 2015). The standard NKM assumes that the central bank implements a monetary policy by manipulating its policy rate. As argued in a study by Woodford (2003), this theory is constructed in a cashless economy in which money does not play a significant role because the money demand function indicates that a change in the nominal interest rate automatically leads to determination of money aggregate. This implies the redundancy of the money demand function when the model presumes separability between consumption and real money balances in a household's utility function.[1]

However, monetary policies in advanced countries reveal the important role of money. Central banks in advanced country reduce their policy rates to mitigate the risk of deflation arising from the Lehman crisis in the United States. Consequently, central banks have faced with non-negativity constraints on nominal interest rates. Therefore, the central banks increase the monetary base to combat deflation and recession when faced with zero lower bound (ZLB). Such policy stances in advanced countries are referred to as quantitative easing.

This paper focuses on the fact that a change in money supply in one country affects the monetary policy strategy of other countries. For instance, the quantitative easing policy that the Federal Reserve Board (FRB) implemented in 2008 might have significantly affected the monetary policy stance implemented by the Bank of Japan (BOJ). In fact, a large-scale asset purchases (LSAPs) by FRB depreciated the exchange rate in terms of dollar, whereas the BOJ faced a severe appreciation of the exchange rate. However, the standard NKM that studies the effect of ZLB on the economy does not consider the role of money.

This paper addresses the role of money in a two-country economy. As mentioned

[1]Woodford (2003) examines the role of the non-separable utility between consumption and money in a closed economy. Also, Kurozumi (2006) examines the determinacy condition in a model with the non-separable utility.

earlier, the role of money is limited, unless it is assumed that consumption and real money balances are not separable in a household's utility function. A two-country model with sticky prices has been developed by several previous studies (Betss and Devereux, 2001; Clarida, Gali, and Gertler, 2002; Corsetti and Pesenti, 2001; Corsetti and Pesenti, 2005; Devereux and Engel 2003; Engel, 2009; Engel, 2011). These studies abstract the role of non-separable utility in the model. The present study considers the non-separable utility between consumption and real money balance in a two-country framework. More specifically, this paper examines the role of money in a two-country model with home bias constructed by Engel (2009, 2011).[2]

Clarida, Gali, and Gertler (2002) shows that, under the assumption that both producer-currency pricing and home bias do not hold, consumption in the home and foreign countries is identical provided that a complete market exists in which home and foreign households can trade state-contingent securities domestically and internationally. This is referred to as the consumption risk-sharing condition for open economies. Even if we relax the assumption of no home bias, relative consumption between countries is adjusted by a movement of the real exchange rate (Chari, Kehoe, and McGrattan, 2002; Pappa, 2004).[3] However, this paper shows that relative real money balances between two countries affect the international risk-sharing condition directly when we consider a case where consumption and real money balances are not separable in the household's utility function.

How does such a departure from the separable utility function change the structural equations in a two-country economy with nominal rigidities? Woodford (2003) considers the role of the non-separable utility function in a closed economy model. It allows the new Keynesian Phillips curve (NKPC) to be affected by real money balances. Interestingly, in a two-country model, in which consumption and real money balances are not separable,

[2]Engel (2009, 2011) examines how local currency pricing changes the properties of optimal monetary policy in a two-country economy model with home bias. This paper introduces the presence of home bias in the model, whereas it abstracts that of pricing to market associated with local currency pricing.

[3]See Chari, Kehoe, and McGrattan (2002) and Pappa (2004) for a detailed discussion about this issue.

relative real money balances directly affect not only the dynamic IS equation and the money demand function, but also the NKPC. In addition, the non-separable utility changes the shape of the central bank's loss function. Clarida, Gali, and Gertler (2002) and Engel (2009, 2011) show, under policy coordination,[4] the loss function contains the squared output gaps and the producer price index (PPI) inflation rates of both countries in a two-country economy with producer-currency pricing. The shape of the loss function is different from such a standard loss function when consumption and real money balances are not separable in the utility function. First, the central banks need to stabilize differences between home and foreign real money balances. Second, the loss function includes the cross term for relative output gap and relative real money balances.

Hence, the assumption of the non-separable utility between consumption and real money balances significantly affects the properties of optimal monetary policy. This paper regards optimal monetary policy in a timeless perspective as suggested by Woodford (2003). It focuses on commitment under policy coordination.[5] Under optimal monetary policy, a foreign natural interest rate shock induces higher inflation and output gap in the foreign country. The foreign central bank increases its nominal interest rate gradually under optimal monetary policy. In particular, for higher substitutability, the foreign policy rate is smaller initially, whereas the foreign central bank raises its policy rate aggressively in subsequent periods. The home central bank maintains a positive policy rate to stabilize the inflation rate. An increase in the policy rate leads to a negative response of the output gap. For higher substitutability, the home central bank reinforces increase in the policy rate under the optimal monetary policy. Therefore, a foreign natural interest rate shock generates a policy trade-off between inflation and the output gap in the home country.

Optimal policy under a foreign money demand shock is different from that under a

[4] As mentioned, policy coordination implies that both domestic and foreign central banks jointly minimize a weighted sum of the loss function of domestic and foreign central banks.

[5] This paper does not cover gain from policy coordination because when optimal monetary policy is considered under no policy coordination, one faces the complicated derivation of a well-defined loss function of the central bank. For instance, see Engel (2009, 2011) for a detailed discussion of this issue.

foreign natural interest rate shock. For a foreign country, higher substitutability leads to more aggressive increase in the policy rate after the shock. Thus, the foreign central bank accommodates a huge decline in the output gap in the initial period instead of increasing the inflation rate for subsequent periods. On the other hand, the home central bank aggressively cuts its policy rate in response to the shock. Thus, under optimal monetary policy, the home central bank cuts its policy rates aggressively. In contrast, a foreign money demand shock leads to a boom in the output gap and an increase in inflation in the home country.

Furthermore, this paper examines how the assumption of a non-separable utility between consumption and money affects the optimal monetary policy when the ZLB simultaneously hits both countries. A new Keynesian model emphasizes the role of ZLB (Eggertsson and Woodford, 2003; Jung, et al., 2005; Adam and Billi, 2006; Adam and Billi, 2007, Guerrieri and Iacoviello, 2015; McKay, Nakamura and Steinsson, 2016; Nakata, 2017). Consider the case where a natural interest rate takes a negative value. It cannot stimulate the real economy once it hits ZLB. As argued in Eggertsson and Woodford (2003) and Jung, Teranishi and Watanabe (2005), the central bank can insulate aggregate demand through history-dependent commitment policy even when the nominal interest rate reaches the zero bound.

Nakajima (2008) investigates optimal monetary policy in a two-country economy, but the study considered only the domestic interest rate as zero. Ida (2013) derives optimal monetary policy rules in a two-country economy in which both home and foreign countries simultaneously face the ZLB. Cook and Devereux (2011) examine also the role of fiscal policy under the ZLB in a two-country framework with Calvo pricing.[6] Fujiwara et al. (2010, 2014) examine also optimal monetary policy responses to non-negativity constraints on nominal interest rate existing in both home and foreign countries.

The previous studies abstract the role of money in the NKM. However, the present study addresses the recent evidence that money aggregate plays a significant role in

[6]Their model is close to our model, however, this paper is different from the study by Cook and Devereux (2011), in that the present paper focuses on the role of money in the model.

non-conventional monetary policy. In unconventional monetary policy, central banks anticipate the effect of quantity of money on real economy in addition to a commitment to a zero interest rate policy (ZIRP). Central banks, such as the FRB, the Bank of England, and the BOJ, expect that quantitative easing would have an effect of money on the real economy.[7] It is well known that these central banks follow the quantitative easing policy and we show that the ZLB plays a significant role in a two-country model with the non-separable utility function between money and consumption. The previous studies do not account for the effect of money on the real economy in a two-country model in which central banks simultaneously face the ZLB. Although this paper cannot explicitly derive the policy rule associated with money aggregate, we can conjecture that the central bank can implicitly follow such a rule in terms of the micro-founded loss function of the central banks.[8]

This paper explores the case of a foreign negative natural interest rate shock under optimal policy when ZLB hits both countries. The foreign central bank maintains a ZIRP for a considerably long period even when the natural rate of interest takes a positive value. Accordingly, foreign inflation overshoots in response to such a monetary policy stance, which introduces policy inertia into the foreign output gap. On the other hand, a foreign negative interest rate decreases home inflation, whereas it increases the home output gap. Therefore, the home central bank employs a ZIRP also. Since the home country experiences a positive output gap, the termination date of a home ZIRP is considerably shorter than that in the foreign country. After the termination of a ZIRP, the home central bank sharply increases its policy rate. In particular, in contrast to the foreign nominal interest rate following the decision to terminate a ZIRP, the home central bank allows policy rate increase from a ZIRP termination date.

Also, I consider the case of a foreign money demand shock. Interestingly, it considerably changes the property of optimal monetary policy in contrast to a natural interest

[7]See English et al. (2013) and Williamason (2015) for a detailed discussion of the effect of quantitative easing policies.

[8]This paper focuses on the commitment policy under policy coordination. Therefore, the foreign central bank can also implement the optimal rule based on money aggregate.

rate shock. The money demand shock leads to a negative response of both inflation and the output gap in the foreign country, whereas the home country experiences surge in both inflation and the output gap. Nevertheless, the home central bank employs a ZIRP for a few periods. After the termination of a ZIRP, the home central bank sharply increases its policy rate and maintains for some time, whereas the ZIRP termination is shorter than that for the foreign country. In contrast to a movement of nominal interest rate in the home country, the foreign central bank raises its policy rate gradually after the termination of a ZIRP. Such a policy stance leads to increase in inflation and the output gap in the home country.

The present paper is structured as follows. Section 2 derives a two-country new Keynesian model with a non-separable utility function between consumption and real money balances. Section 3 describes equilibrium conditions in the model. Section 4 provides the log-linearized equations of the model around the efficient steady state. Section 5 derives the central bank's loss function by calculating the second-order approximation of the household's utility function in both countries. Then I examine optimal monetary policy in a two-country model with the non-separable utility function. It demonstrates the properties of optimal monetary policy. Section 6 considers the properties of optimal monetary policy when the ZLB hits both countries simultaneously. Section 7 briefly concludes. Appendix A provides the detailed derivation of the second-order approximation of the household's utility function. Appendix B derives the optimal monetary policy with commitment under policy coordination.

2 The model

A standard two-country model, home and foreign, with nominal price rigidities developed by Clarida, Gali and Gertler (2002) and Engel (2009, 2011) is adopted. Based on a study by Engel (2011), the model employs different preferences in the two countries. This paper allows for home bias. In each country, households obtain utility from consumption and real money balances and disutility from labor supply. This model assumes that consumption and real money balances are not separable in a household's utility function.

Households in each country have access to a complete set of state-contingent securities that are traded both domestically and internationally.

Each country has firms that face a monopolistically competitive environment. According to Calvo (1983), such firms set staggered nominal prices. Since firms set their prices in their own currency, this implies that the present paper assumes the producer-currency pricing (PCP) model. Finally, unless otherwise noted, analogous equations hold for a foreign country.

2.1 Households

The intertemporal utility of an infinitely lived representative household subscripted by h is

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t \Big[u(C_t(h), z_t(h)) - v(N_t(h)) \Big], \tag{1}$$

where C_t denotes the consumption, Z_t denotes the real money balances, and N_t denotes the household's labor supply. In addition, the parameter β is discount factor. The utility function $u(\cdot)$ is strictly concave and continuously differentiable with respect to both C_t and Z_t. The disutility of labor supply $v(\cdot)$ is strictly convex and continuously differentiable. Moreover, unlike Clarida, Gali and Gertler (2002) and Engel (2011), the utility function is not separable between consumption and real money balances in this paper. Thus, I assume $u_{cm}(\cdot) \geq 0$[9]. This paper assumes the following Cobb-Douglas preferences for consumption:

$$C_t = (C_{H,t}(h))^{a/2} (C_{F,t}(h))^{(1-a/2)}, \tag{2}$$

where $C_{H,t}(h)$ represents consumption of home goods and $C_{F,t}(h)$ represents the consumption of foreign goods. The parameter a denotes the degree of home bias. When the parameter a takes unity, home and foreign preferences are the same as those in the study

[9] As in Benhabib, Schmitt-Gohe and Uribe (2001), here the case of $u_{cm}(\cdot) \leq 0$ is not considered.

by Clarida, Gali, and Gertler (2002). Here, $C_{H,t}(h)$ and $C_{F,t}(h)$ are CES aggregates over a continuum of goods produced in each country:

$$C_{H,t} = \left[\int_0^1 C_{H,t}(h,j)^{(\theta-1)/\theta} dj\right]^{\theta/(\theta-1)}, \tag{3}$$

$$C_{F,t} = \left[\int_0^1 C_{F,t}(h,j)^{(\theta-1)/\theta} dj\right]^{\theta/(\theta-1)}, \tag{4}$$

where the parameter θ is the elasticity of substitution for individual goods.[10]

The labor supply $N_t(h)$ is an aggregate of the labor supply that the household sells to each of a continuum of firms located in the home country:

$$N_t(h) = \int_0^1 N_t(h,j)dj, \tag{5}$$

The representative household maximizes Equation (1) subject to the following budget constraint:

$$P_tC_t(h) + M_t(h) + E_t[Q_{t,t+1}B_{t+1}(h)] = B_t(h) + M_{t-1}(h) + W_t(h)N_t(h) + \Pi_t + T_t, \tag{6}$$

where $B_t(h)$ is the nominal bond, M_t is the nominal money supply, and T_t denotes the nominal lump-sum tax. Also, $W_t(h)$ and Π_t are the nominal wage and the dividend from intermediate goods firms. The variable $Q_{t,t+1}$ denotes the stochastic discount factor, which is defined as follows:

$$E_t(Q_{t,t+1}) = \frac{1}{1+i_t}, \tag{7}$$

where i_t is the nominal interest rate. In addition, P_t is the aggregate price index for consumption, which is given as follows:

$$P_t = \kappa^{-1}(P_{H,t})^{a/2}(P_{F,t})^{(1-a/2)}, \tag{8}$$

where $\kappa = (1-a/2)^{(1-a/2)}a/2^{a/2}$. $P_{H,t}$ is the price of domestic goods, and $P_{F,t}$ is the price of foreign goods. Equation (8) are derived from the household's cost minimization

[10] In this model, this parameter is time-invariant. As in Steinsson (2003), we can introduce a time-varying elasticity of substitution if we examine the case of a price mark-up shock.

problem. Both $P_{H,t}$ and $P_{F,t}$ follow the CES aggregate over prices of individual varieties j:

$$P_{H,t} = \left[\int_0^1 P_{H,t}(h,j)^{1-\theta} dj\right]^{1-\theta}, \tag{9}$$

$$P_{F,t} = \left[\int_0^1 P_{F,t}(h,j)^{1-\theta} dj\right]^{1-\theta}. \tag{10}$$

Note that the foreign households also have analogous preferences and face an analogous budget constraint.

Similar to Engel (2011), we can drop subscript h for the household and use the fact that the aggregation per capita consumption of each good is equal to consumption of each good by each household. From the first order conditions of the utility maximization problem, the following equations are obtained:

$$C_{H,t} = \frac{a}{2}\left(\frac{P_{H,t}}{P_t}\right)^{-1} C_t, \tag{11}$$

$$C_{F,t} = \left(1-\frac{a}{2}\right)\left(\frac{P_{F,t}}{P_t}\right)^{-1} C_t, \tag{12}$$

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\theta} C_{H,t}, \tag{13}$$

$$C_{F,t}(j) = \left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{-\theta} C_{H,t}, \tag{14}$$

$$\frac{u_z(C_t, Z_t)}{u_c(C_t, Z_t)} = \frac{i_t}{1+i_t}, \tag{15}$$

$$-\frac{v_n(N_t)}{u_c(C_t, Z_t)} = \frac{W_t}{P_t}, \tag{16}$$

$$Q_{t,t+1} = \beta \frac{u_c(C_{t+1}, Z_{t+1})}{u_c(C_t, Z_t)} \frac{P_t}{P_{t+1}}. \tag{17}$$

Equations (11) and (12) are derived from the household's cost minimization problem under the assumption of the Cobb-Douglas preferences for consumption. Equations (13) and (14) represent demand equations for both $C_{H,t}$ and $C_{F,t}$. Equation (15) is the money demand function. This equation states that the marginal rate of substitution between consumption and real money balances is equal to the opportunity cost of holding money. Equation (16) states the intratemporal optimality condition that the marginal rate of substitution between consumption and household labor supply is equal to the real wage.

As noted earlier, there are complete markets in each country. Therefore, by taking the expectations of each side of Equation (17) and using the definition of the stochastic discount factor, the following familiar Euler equation is derived:

$$\beta E_t(1+i_t)\frac{u_c(C_{t+1}, Z_{t+1})}{u_c(C_t, Z_t)}\frac{P_t}{P_{t+1}} = 1. \tag{18}$$

Equation (18) requires that in equilibrium, the marginal utility of consumption intertemporally equalizes through the adjustment of the real interest rate. Note that the analogous Euler equation is applied for the foreign country. In particular, by the assumption of the international tradability of state-contingent securities, the intertemporal efficient condition for the foreign country is given by

$$\beta E_t(1+i_t^*)\frac{u_c(C_{t+1}^*, Z_{t+1}^*)}{u_c(C_t^*, Z_t^*)}\frac{\varepsilon_t P_t}{\varepsilon_{t+1} P_{t+1}} = 1. \tag{19}$$

where ε_t is the nominal exchange rate in terms of the home currency price. In addition, an asterisk is used to denote variables associated with foreign countries.

Finally, as in Engel (2011), this paper presumes at this stage that labor supply of households is the same, that is, $N_t = N_t(h)$.

2.2 Firms

The firms in each country are characterized by monopolistic competition and each firm produces differentiated goods. Each home good $Y_t(j)$ is produced by the following production function:

$$Y_t(j) = A_t N_t(j), \tag{20}$$

where A_t is an aggregate productivity disturbance.

The firm's profit function is given by

$$\Pi_t(j) = P_{H,t}(j)C_{H,t}(j) + \varepsilon_t P_{H,t}^*(j)C_{H,t}^*(j) - (1-\tau)W_t N_t(j), \tag{21}$$

where $P_{H,t}(j)$ the home-currency price of the goods in the case where it is sold in the home country. On the other hand, $P_{H,t}^*(j)$ denotes the foreign-currency price of the goods when it is sold in the foreign country. The parameter τ denotes optimal subsidy. Finally, $C_{H,t}(j)$ is the aggregate sales of the goods in the home country:

$$C_{H,t}(j) = \int_0^1 C_{H,t}(h,j)dj. \tag{22}$$

Analogous equations hold good in the foreign country. Note the following:

$$Y_t(j) = C_{H,t}(j) + C^*_{H_t}(j), \tag{23}$$

$$Y^*_t(j) = C_{F,t}(j) + C^*_{F_t}(j). \tag{24}$$

Following Calvo (1983), this paper assumes that firms set their nominal prices on the basis of a staggered manner. Thus, a fraction $1-\omega$ of all firms adjusts their price, whereas the remaining fraction of firms ω do not. Therefore, a fraction ω of prices remains unchanged from the previous period. These firms consider uncertainty during price revision until the next price adjustment.

We now consider the firms that can adjust their prices. As in Engel (2011), when the firm can reset its price, it will be able to change its sales price in both the countries. In the PCP model, firms set their nominal prices in its own currency. The home firm that can reset its price sets $P_{H,t}(j)$ and $P^{**}_{H,t}(j) = \varepsilon_t P^*_{H,t}(j)$ in terms of home currency. In this case, the firm's optimization problem is given by

$$E_t \sum_{k=0}^{\infty} (\omega\beta)^k \frac{u_c(C_{t+k}, Z_{t+k})}{u_c(C_t, Z_t)} \left[P^o_{H,t}(j) C_{H,t+k}(j) + \varepsilon_t P^{o**}_{H,t}(j) C^*_{H,t+k}(j) - (1-\tau) W_{t+k} N_{t+k}(j) \right], \tag{25}$$

where $P^o_{H,t}(j)$ is the optimal home price index and $P^{o**}_{H,t}(j)$ is the optimal price for foreign sales. The firms that reset their prices select its reset prices $P^o_{H,t}(j)$ and $P^{o**}_{H,t}(j)$ to maximize the firm's objective function (25), subject to the sequence of demand curves given by Equation (13) and the corresponding foreign demand function for home goods.

The first order conditions of this profit maximization problem are as follows:[11]

$$P^o_{H,t} = \frac{E_t \sum_{k=0}^{\infty}(\omega\beta)^k u_c(C_{t+k}, Z_{t+k})(1-\tau)W_{t+k}P^{\theta}_{H,t+k}C_{H,t+k}/A_{t+k}}{E_t \sum_{k=0}^{\infty}(\omega\beta)^k u_c(C_{t+k}, Z_{t+k})P^{\theta}_{H,t+k}C_{H,t+k}}, \tag{26}$$

$$P^{o**}_{H,t} = \frac{E_t \sum_{k=0}^{\infty}(\omega\beta)^k u_c(C_{t+k}, Z_{t+k})(1-\tau)W_{t+k}(\varepsilon_{t+k}P_{H,t+k})^{\theta}C^*_{H,t+k}/A_{t+k}}{E_t \sum_{k=0}^{\infty}(\omega\beta)^k u_c(C_{t+k}, Z_{t+k})(\varepsilon_{t+k}P_{H,t+k})^{\theta}C^*_{H,t+k}}. \tag{27}$$

Under Calvo pricing, price levels are defined as follows:

$$P_{H,t} = \left[(1-\omega)(P^o_{H,t})^{1-\theta} + \omega(P_{H,t-1})^{1-\theta}\right], \tag{28}$$

$$P^*_{H,t} = \left[(1-\omega)(P^{o*}_{H,t})^{1-\theta} + \omega(P^*_{H,t-1})^{1-\theta}\right]. \tag{29}$$

3 Equilibrium

This section describes the equilibrium conditions in a two-country model. The equilibrium conditions for the goods market in each country are given as follows:

$$Y_t = C_{H,t} + C^*_{H_t}, \tag{30}$$

$$Y^*_t = C_{F,t} + C^*_{F_t}. \tag{31}$$

Substituting Equation (11) and the corresponding equation for the foreign country, Equation (30) can be rewritten as follows:

$$Y_t = \kappa\left[\frac{a}{2}S_t^{1-a/2}C_t + \left(1-\frac{a}{2}\right)(S^*_t)^{-a/2}C^*_t\right], \tag{32}$$

$$Y^*_t = \kappa\left[\frac{a}{2}(S^*_t)^{-a/2}C^*_t + \left(1-\frac{a}{2}\right)(S_t)^{-a/2}C_t\right], \tag{33}$$

where S_t and S^*_t are the home terms of trade and the foreign terms of trade, respectively:

$$S_t = \frac{P_{F,t}}{P_{H,t}}, \tag{34}$$

$$S^*_t = \frac{P^*_{H,t}}{P^*_{F,t}}. \tag{35}$$

[11]Under the PCP model, this equation is redundant because the law of one price holds. However, under the local-currency pricing model, this equation is required since the law of one price no longer holds. See Corsetti, Dedola and Leduc, (2010) and Engel (2011) for a detailed discussion of this issue.

Note that since the law of one prices hold under the PCP model, we have $S_t = (S_t^*)^{-1}$.

Here, in this model purchasing power parity (PPP) does not hold because of the presence of home bias. Even when PPP does not hold, since households in each country can trade a state-contingent security internationally, combining Equation (18) with Equation (19) and imposing a suitable normalization of initial condition, we obtain the following equation:

$$\frac{u_c(C_t^*, Z_t^*)}{u_c(C_t, Z_t)} = \frac{\varepsilon_t P_t}{P_t^*} = S_t^{(a-1)}. \tag{36}$$

When the PPP condition holds in the absence of home bias, the relationship $u_c(C_t, Z_t) = u_c(C_t^*, Z_t^*)$ holds. The standard two-country NKM assumes a cashless-limit case or the case that consumption and real money balances are separable in the household's utility function. As shown in Clarida, Gali and Gertler (2002), $u_c(C_t) = u_c(C_t^*)$ in such a case. This implies that consumption in the home country completely co-moves one in the foreign country. When home bias is present, as shown in Chari, Kehoe, and McGrattan (2002), the relative consumption is affected by the real exchange rate. In contrast to their model, our model allows that both home and foreign real money balances change the relative consumption under the assumption of a non-separable utility function between consumption and real money balances.

Under flexible prices, all firms set their prices equal to a constant markup over marginal cost. In addition, this paper assumes that the government can set a constant optimal subsidy rate for monopolistic firms. Such an optimal subsidy allows the government to achieve an efficient allocation in the deterministic steady state.

$$P_{H,t}^n = \varepsilon_t^n P_{H,t}^{n*} = (1-\tau)\mu \frac{W_t^n}{A_t}, \tag{37}$$

where a variable with (n) denotes one in the flexible price equilibrium. In addition, the parameter μ is the firm's price markup, which is given by $\theta/(\theta-1)$. When optimal subsidies are imposed in place, the following equation holds

$$\bar{P}_{H,t} = \bar{\varepsilon}_t \bar{P}_{H,t}^* = (1-\tau)\mu \frac{\bar{W}_t}{A_t}, \tag{38}$$

where a variable with $(-)$ denotes one in the efficient level equilibrium. Thus, it follows from Equations (37) and (38) that an optimal subsidy rate is given by

$$(1-\tau)\mu = 1.$$

4 Log-linearization

The system derived in the previous section is now log-linearized around the steady state. Here lower case variables are used to denote a log-deviation from the steady state. Specifically, a log-linearized variable around the steady state is expressed by $h_t = \log(H_t/\bar{H})$, where $\bar{H}$ represents a steady-state value.

First, log-linearization of Equation (32) is given by

$$y_t = \frac{a(2-a)}{2}s_t + \frac{a}{2}c_t + \frac{(2-a)}{2}c_t^*. \tag{39}$$

Similarly, log-linearization of Equation (33) is given by

$$y_t^* = -\frac{a(2-a)}{2}s_t + \frac{a}{2}c_t^* + \frac{(2-a)}{2}c_t. \tag{40}$$

Next, log-linearizing Equation (36) gives the following

$$\sigma(c_t - c_t^*) = \chi(z_t^* - z_t) + (a-1)s_t, \tag{41}$$

where $\sigma = -u_{cc}\bar{C}/u_c$ and $\chi = u_{cz}\bar{Z}/u_z$. In the case of the non-separable utility function, as noted earlier, relative real money balances affect relative consumption, inducing a change in the terms of trade. If the utility function is separable, the parameter χ is zero. Hence, the first-term of the right-hand side is zero. Therefore, relative consumption between the two countries is only adjusted by the change in the terms of trade as providing there is home bias. In particular, if we make an assumption of both the separable utility and no home bias, a movement of home consumption completely coincides with that of foreign consumption due to the presence of an international complete market (e.g, Clarida, Gali and Gertler, 2002).

Subtracting Equation (40) from Equation (39) results in the following equation:

$$y_t - y_t^* = a(2-a)s_t + (a-1)(c_t - c_t^*). \tag{42}$$

Substituting Equation (42) into Equation (41), the following relationship is derived

$$s_t = \frac{1}{\Theta}[\sigma(y_t - y_t^*) + (a-1)\chi(z_t - z_t^*)], \tag{43}$$

where $\Theta = \sigma a(2-a) + (a-1)^2$.

At this point, inserting Equation (41) into Equation (43), the following equation is obtained:

$$c_t = \psi y_t + (1-\psi)y_t^* + \zeta(z_t^* - z_t), \tag{44}$$

where

$$\psi = \frac{\Theta + (a-1)}{2\Theta},$$
$$\zeta = \frac{\chi a(2-a)}{2\Theta}.$$

It is assumed that an increase in real money balances leads to an increase in consumption in the closed economy model. However, this inference might not be correct in an open economy model. Equation (36) indicates that this is because the difference between home and foreign real money balances directly changes relative consumption. Equation (43) shows that it turns out that home consumption is negatively affected by home real money balances only if the household's utility function in each country is not separable between consumption and money. However, home consumption co-moves with a change in foreign real money balances.[12] Similarly, an analogous equation holds in the foreign country as follows:

$$c_t^* = \psi y_t^* + (1-\psi)y_t - \zeta(z_t^* - z_t). \tag{45}$$

Next, log-linearization of the firm's optimization condition leads to the following NKPC:

$$\pi_t = \beta\pi_{t+1} + \delta(w_t - p_{H,t} - a_t), \tag{46}$$

[12]If the parameter χ takes a negative value, a polar result is obtained.

where

$$\delta = \frac{(1-\omega)(1-\omega\beta)}{\omega}.$$

Also, $\pi_t = \log(P_{H,t}/P_{H,t-1})$ is the PPI inflation rate. An analogous equation holds for the foreign country.

Log-linearization of Equation (16) leads to

$$w_t - p_{H,t} = \eta n_t + \sigma c_t - \chi z_t + \frac{(2-a)}{2} s_t, \tag{47}$$

where $\eta = v_{NN}\bar{N}/v_n$. Substituting the log-linearized production function, Equations (44) and (47) leads to

$$w_t - p_{H,t} = \left[\frac{\sigma(\Theta+1)}{2\Theta} + \eta\right] y_t + \frac{\sigma(\Theta-1)}{2\Theta} y_t^* - \chi\frac{\Theta+1}{2\Theta} z_t + \chi\frac{\Theta-1}{2\Theta} z_t + \eta a_t. \tag{48}$$

Under efficient price equilibrium, Equation (48) becomes

$$w_t^n - p_{H,t}^n = \left[\frac{\sigma(\Theta+1)}{2\Theta} + \eta\right] y_t^n + \frac{\sigma(\Theta-1)}{2\Theta} y_t^{n*} - \chi\frac{\Theta+1}{2\Theta} z_t^n + \chi\frac{\Theta-1}{2\Theta} z_t^{n*} + \eta a_t. \tag{49}$$

where a variable with subscript n denotes one under efficient price equilibrium. Subtracting Equation (49) from Equation (48), we obtain

$$\tilde{w}_t - \tilde{p}_{H,t} = \left[\frac{\sigma(\Theta+1)}{2\Theta} + \eta\right] \tilde{y}_t + \frac{\sigma(\Theta-1)}{2\Theta} \tilde{y}_t^* - \chi\frac{\Theta+1}{2\Theta} \tilde{z}_t + \chi\frac{\Theta-1}{2\Theta} \tilde{z}_t. \tag{50}$$

where a variable with a tilde denotes one in terms of deviation of a variable from one of its flexible price equilibrium. Finally, substituting Equation (50) into Equation (46), we obtain the NKPC expressed in terms of the output gap.

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_1 \tilde{y}_t + \kappa_2 \tilde{y}_t^* - \kappa_3 \tilde{z}_t + \kappa_4 \tilde{z}_t^* \tag{51}$$

where

$$\kappa_1 = \delta\left[\frac{\sigma(\Theta+1)}{2\Theta} + \eta\right], \kappa_2 = \delta\frac{\sigma(\Theta-1)}{2\Theta},$$
$$\kappa_3 = \chi\frac{\Theta+1}{2\Theta}, \kappa_4 = \chi\frac{\Theta-1}{2\Theta}.$$

In contrast to the previous studies that examine monetary policy in a two-country economy, this paper shows that foreign real money balances directly affect home inflation because of the presence of a non-separable utility. Thus, the money demand function plays a significant role in this model. For instance, given the level of real money balances in the home country, a decline in the policy rate in the foreign country induces an increase in foreign real money balances, thus increasing home inflation.

An analogous equation holds for the foreign country:

$$\pi_t^* = \beta E_t \pi_{t+1}^* + \kappa_1 \tilde{y}_t^* + \kappa_2 \tilde{y}_t - \kappa_3 \tilde{z}_t^* + \kappa_4 \tilde{z}_t, \tag{52}$$

Here, the home money demand function expressed by the gap term is given by

$$\tilde{z}_t = \mu_1 \tilde{y}_t + \mu_2 \tilde{y}_t^* + \mu_3 \tilde{z}_t^* - \mu_4 i_t + \varsigma_t, \tag{53}$$

where ς_t denotes the money demand shock. In addition, coefficients in Equation (53) are defined as

$$\mu_0 = \frac{2\Theta + \chi a(2-a)}{2\Theta}, \mu_1 = \frac{\eta_c \chi[\Theta + (a-1)]}{2\Theta\mu_0}, \mu_2 = \frac{\eta_c \chi[\Theta - (a-1)]}{2\Theta\mu_0},$$
$$\mu_3 = \frac{\eta_c \chi a(2-a)}{2\Theta\mu_0}, \mu_4 = \frac{\eta_i}{\mu_0},$$

where η_c is the income elasticity of money demand and η_i is the interest elasticity of money demand. In contrast to the standard money demand function, Equation (53) includes foreign real money balances, which are positively related to home real money balances. Thus, other things being equal, a decline in the policy rate in the foreign country increases foreign money balances, which leads to a decrease in the home inflation rate. Similarly, for the foreign country the money demand function is given by

$$\tilde{z}_t^* = \mu_1 \tilde{y}_t^* + \mu_2 \tilde{y}_t + \mu_3 \tilde{z}_t - \mu_4 i_t^* + \varsigma_t, \tag{54}$$

Finally, a dynamic IS equation is given as follows:

$$\tilde{y}_t = E_t \tilde{y}_{t+1} + \vartheta_1 E_t \Delta \tilde{y}_{t+1}^* - \sigma_0^{-1}(i_t - E_t \pi_{t+1} - \bar{rr}_t) + \vartheta_2 E_t \Delta \tilde{z}_{t+1} - \vartheta_3 \Delta \tilde{z}_{t+1}^*, \tag{55}$$

where $\bar{rr}_t$ denotes the natural rate of interest rate that holds the real interest rate in the efficient price equilibrium, which is given by

$$\bar{rr}_t = \sigma_0 \left[\frac{\Theta + 1}{2\Theta} E_t \Delta y_{t+1}^n + \frac{(\Theta - 1)}{2\Theta} E_t \Delta y_{t+1}^{n*} \right]. \tag{56}$$

In addition, the coefficients for the dynamic IS curve are defined as follows:

$$\sigma_0 = \sigma\frac{\Theta+1}{2\Theta}, \vartheta_1 = \sigma^{-1}\frac{\Theta-1}{\Theta+1}, \vartheta_2 = \sigma^{-1}\chi, \vartheta_3 = \sigma^{-1}\chi\frac{\Theta-1}{\Theta+1}.$$

Similarly, the corresponding IS equation for the foreign country is obtained, given by

$$\tilde{y}_t^* = E_t\tilde{y}_{t+1}^* + \vartheta_1 E_t\Delta\tilde{y}_{t+1} - \sigma_0^{-1}(i_t^* - E_t\pi_{t+1}^* - \bar{rr}_t^*) + \vartheta_2 E_t\Delta\tilde{z}_{t+1}^* - \vartheta_3\Delta\tilde{z}_{t+1}, \tag{57}$$

where $\bar{rr}_t^*$ denotes the natural rate of interest for the foreign country.

5 Optimal Monetary Policy

This section examines how a non-separable utility function between consumption and real money balances changes the properties of optimal monetary policy in a two-country economy model. Section 5.1 derives the loss function of the central bank in a two-country model in which consumption and money are not separable in the household's utility function. In contrast to the previous studies, the present paper addresses the role of a non-separable utility in a two-country model. Section 5.2 presents an argument on optimal monetary policy. Section 5.2.1 derives optimal monetary policy with commitment, and Section 5.2.2 shows the effect of a non-separable utility between consumption and real money balance in a two-country economy by illustrating the results of impulse response functions.

5.1 The derivation of the central bank's loss function

To investigate optimal monetary policy in an NKM, we must derive a well-defined loss function with a micro-foundation. Rotemberg and Woodford (1997) and Woodford (2003) show that the second-order approximation of the household's utility function corresponds to the central bank's loss function. In a two-country framework, Clarida, Gali and Gertler (2002) and Engel (2011) derive the central bank's loss function by using the second-order approximation of the utility function. In a two-country framework, we can consider an optimal policy under both policy and no policy coordination.[13] As mentioned earlier, this paper focuses on the case of policy coordination.

The utility function of the planner is given by

$$W_t = u(C_t, Z_t) + u(C_t^*, Z_t^*) - v(N_t) - v(N_t^*), \tag{58}$$

To obtain the well-defined loss function, we need to eliminate distortions created by monopolistic competition and real money balances. The first distortion is eliminated by an optimal subsidy rate that eliminates price markup created by monopolistic competition. Fiscal authorities choose an optimal subsidy rate that restores the natural level of output to the efficient level of one in a zero-inflation steady state. As noted earlier, such an optimal subsidy is given as follows:

$$(1-\tau)\mu = 1.$$

The second distortion is a result of an opportunity cost of holding money. As shown in Woodford (2003), this opportunity cost should be considerably small in steady state to obtain a well-defined loss function of the central bank. In particular, Woodford (2003) argues that real money balances are sufficiently close to being satiated in the optimal steady state. To do so, we can eliminate the distortion produced by the opportunity cost of money.[14]

Calculating the second-order approximation of this planner's objective function (58) around the nondistorted steady state, we can obtain the following central bank's loss function:

$$\sum_{t=0}^{\infty} W_t \approx -\Omega \sum_{t=0}^{\infty} \beta^t L_t + t.i.p. + O(||\xi||^3), \tag{59}$$

where $t.i.p$ includes the terms that are independent of monetary policy and $O(||\xi||^3)$ indicates the terms of third or higher orders. Here, the periodic loss function L_t in

[13]As discussed in Clarida, Gali and Gertler (2002), an optimal subsidy rate is different between policy coordination and no policy coordination. Engel (2011) argues that we may have difficulty in having the derivation of the central bank's loss function under no policy coordination in the LCP model.

[14]See Chapter 6 in Woodford (2003) for a detailed discussion of this issue.

Equation (59) is given by

$$L_t = \pi_t^2 + (\pi_t^*)^2 + \lambda_1[\tilde{y}_t^2 + (\tilde{y}_t^*)^2] + \lambda_2\tilde{y}_t\tilde{y}_t^* + \lambda_3[i_t^2 + (i_t^*)^2] + \lambda_4(\tilde{z}_t^* - \tilde{z}_t)^2$$
$$+ \lambda_5(\tilde{z}_t - \tilde{z}_t^*)(\tilde{y}_t - \tilde{y}_t^*), \tag{60}$$

where

$$\Omega = \frac{1}{2}\frac{\omega\theta}{(1-\omega)(1-\omega\beta)}u_c\bar{C},$$
$$\varrho = \sigma - \chi\eta_c,$$
$$\lambda_1 = \frac{(1-\omega)(1-\omega\beta)}{\omega}\frac{\varrho[2\psi(\psi-1)(1-\varrho^{-1})+1]+\eta}{\theta},$$
$$\lambda_2 = \frac{4(1-\omega)(1-\omega\beta)}{\omega}\frac{(\varrho-1)\psi(\psi-1)}{\theta},$$
$$\lambda_3 = \frac{(1-\omega)(1-\omega\beta)\eta_i}{\omega\theta\bar{v}},$$
$$\lambda_4 = \frac{2(1-\omega)(1-\omega\beta)}{\omega}\frac{(\varrho-1)\zeta^2}{\theta},$$
$$\lambda_5 = \frac{(1-\omega)(1-\omega\beta)}{\omega}\frac{(\varrho-1)(\psi-1)\zeta}{\theta},$$

where $\bar{v}$ denotes the velocity of money demand. Appendix A provides a detailed derivation of the loss function. As in the case of the standard two-country NKM, the loss function (60) includes the squared output gaps and the PPI inflation rates of both countries. Stabilization of the PPI inflation rates in each country captures the welfare costs associated with nominal price rigidities.

Importantly, this loss function indicates several comments in contrast to the standard loss function derived in a two-country model with nominal price rigidities. This paper emphasizes that the shape of the loss function is different from the standard loss function derived in Clarida, Gali and Gertler (2002) and Engel (2009, 2011) when a non-separable utility between consumption and real money balances are presumed in the model.[15]

First, in contrast to the loss function derived in Clarida, Gali and Gertler (2002) and Engel (2011), the central banks need to stabilize the variation between home and foreign

[15] Woodford (2003) derives the central bank's loss function in a closed economy with a non-separable utility function between consumption and real money balances.

real money balances, because such a variability of relative real money balances directly affects relative consumption through the condition (41). As in Clarida, Gali and Gertler (2002), complete international risk sharing leads to $C_t = C_t^*$ under the assumption of both the separable utility and no home bias. In contrast, in our model, the relative real money balances due to a non-separable utility between money and consumption does not create $C_t = C_t^*$. Therefore, a fluctuation in relative real money balances worsens social welfare in both countries.

Second, the loss function includes the cross term for relative output gap and relative real money balances. To understand this term intuitively, this term is expanded as follows:

$$(\tilde{z}_t - \tilde{z}_t^*)(\tilde{y}_t - \tilde{y}_t^*) = \tilde{z}_t\tilde{y}_t - \tilde{z}_t^*\tilde{y}_t - \tilde{z}_t\tilde{y}_t^* + \tilde{z}_t^*\tilde{y}_t^*, \tag{61}$$

When we assume a non-separable utility, an increase in real money balance leads to an increase in consumption in both countries. This is reflected by the first and the fourth term in Equation (60). However, according to the home (foreign) money demand function, an increase in foreign (home) real money balance increases home (foreign) real balances, resulting in increases in home (foreign) consumption. Thus, a rise in real money balances in the foreign (home) country indirectly creates a positive effect of home (foreign) consumption. Both the second and the third terms in Equation (61) reflect this.

Note that when money and consumption are separable in the utility function, the loss function set coefficients for both the fourth and the fifth terms as zero. Therefore, Equation (60) reduces to[16]

$$L_t = \pi_t^2 + (\pi_t^*)^2 + \lambda_1[\tilde{y}_t^2 + (\tilde{y}_t^*)^2] + \lambda_2\tilde{y}_t\tilde{y}_t^* + \lambda_3[i_t^2 + (i_t^*)^2]. \tag{62}$$

In particular, if real money balances are absent in the model, the central bank's loss function corresponds to the case of the result in study by Engel (2011).

[16]Ida (2013) derives optimal monetary policy rules in a two-country model with the assumption of the separable utility assumption. The loss function in his model corresponds to Equation (62) in this paper.

5.2 The optimization problem of central banks

In this paper, both home and foreign central banks implement their monetary policies under policy coordination. Under a coordination regime, the central banks minimize their loss function, Equation (60), subject to Equations (51), (53) and (55) and the corresponding constraints in the foreign country. Policy coordination implies that both domestic and foreign central banks jointly minimize a weighted sum of the loss function of domestic and foreign central banks.

We note that both home and foreign money demand functions are additional constraints on the optimal monetary policy of both home and foreign central banks if consumption and real money balances are not separable in the utility function. If we postulate a separable utility function, the central banks no longer regard the money demand function as a constraint on optimal monetary policy because real money balances are automatically determined if the central bank sets its interest rate in accordance with its policy function.

This paper focuses on a commitment policy, in which central banks can commit to future monetary policy stances at current period. This paper regards the commitment policy as optimal from a timeless perspective as suggested by Woodford (2003).[17][18] When the central banks commit to a policy, they can introduce policy inertia into the economy. This policy inertia enables the central banks to influence the private-sector expectations. A discretionary policy is often subject to commitment; that is, the discretionary policy cannot use private-sector expectations. The welfare loss under a commitment policy is smaller than that under a discretionary policy (Woodford, 2003; McCallum and Nelson, 2004; Walsh, 2017).[19] Moreover, as pointed out by Pappa (2004), a commitment policy can eliminate a deflationary bias induced by a discretionary policy, which is the source

[17]See Chapter 7 in Woodford (2003) for a detailed discussion of optimal monetary policy from a timeless perspective.

[18]Fujiwara, Kam, and Sunakawa (2016) consider the sustainable plan for optimal monetary policy under the case of both policy coordination and no policy coordination in a two-country model.

[19]If we introduce endogenous persistence into the forward-looking model, it is possible that the gain from commitment decreases (e.g., Steinsson, 2003).

of an additional gain from commitment associated with an open economy.

Suppose ϕ_{1t} and ϕ_{2t} are Lagrange multipliers associated with both home and foreign NKPCs. Let ϕ_{3t} and ϕ_{4t} be the Lagrange multiplier associated with both home and foreign IS equations. Finally, I denote ϕ_{5t} and ϕ_{6t} as the Lagrange multiplier with respect to both home and foreign money demand equations. In this instance, the economic system can be written as follows:

$$A_0 X_t = A_1 X_{t-1} + B_1 R_t + \Gamma_t, \tag{63}$$

where

$$\begin{aligned} X_t &= [X_{1t} X_{2t}]' \\ X_{1t} &= [\bar{rr}_t \ \bar{rr}_t^* \ \varsigma_t \ \varsigma_t^* \ \phi_{1t} \ \phi_{2t} \ \phi_{3t} \ \phi_{4t} \ \phi_{5t} \ \phi_{6t}]', \\ X_{2t} &= [\tilde{z}_t \ \tilde{z}_t^* \ \pi_t \ \pi_t^* \ \tilde{y}_t \ \tilde{y}_t^*]', \\ R_t &= [i_t \ i_t^*]'. \end{aligned}$$

In addition, A_0, A_1, and B_1 are matrices constructed by deep parameters, and Γ_t denotes a shock vector. I simulate the property of optimal monetary policy by using the Dynare software package.[20]

5.3 Calibration

This section describes deep parameters used in this paper. The discount factor is set to 0.99. The relative risk aversion coefficient for consumption σ is 5.0 based on the value of Steinsson (2003). In addition, following Steinsson (2003), this paper assumes that the parameter η equals 2.0. Following Rotemberg and Woodford (1997) and Pappa (2004), the elasticity of substitution between goods θ is set as 7.88. The income elasticity of money demand η_i is set to 1.0, following Woodford (2003). As in Woodford (2003), I assume that the interest elasticity of money demand η_c takes a value 28. With regard to the parameter a, that denotes the degree of home bias, this parameter is set to 1.2.

[20]The dynare is available on the web page: http://www.dynare.org/.

Thus, we allow the presence of home bias for home goods. The velocity of money $\bar{v}$ is set to 1.0.

Next, set the parameter χ that denotes the degree of substitutability between consumption and real money balances in the household's utility function. In Woodford (2003), this parameter takes a value of 0.02. Kurozumi (2008) shows that a value of the parameter χ crucially affects the determinacy condition, which uniquely determines the rational expectations equilibrium. In this paper, the parameter χ takes a range from 0 to 0.08 as a benchmark case.[21] This paper set the parameter χ to 0.01 as a benchmark calibration.

Finally, the standard deviations of the natural interest rate σ_r and the money demand shock σ_ς is set to 1.0 and 1.0, respectively. With regard to autoregressive coefficients for structural shocks, we choose 0.7, 0.5 for ρ_r and ρ_ς , respectively. These calibrated values for economic shocks are presumed to be the same in the foreign country. Table 1 summarizes the baseline parameters used in this paper.

[Table 1 around here]

5.4 Results

This section provides the simulation results from impulse response analyses and reports the impulse response analysis when a foreign natural interest rate shock occurs. It examines the case where a money demand shock occurs in the foreign country.

5.4.1 A foreign natural interest rate shock

Figure 1 shows the impulse response to a foreign natural interest rate shock under optimal monetary policy. Both inflation and the output gap increase in the foreign country. The foreign central bank raises its nominal interest rate gradually under optimal monetary policy. In particular, for the case in which the parameter χ takes a higher value, an

[21]This paper confirms that the Blanchard-Kahn condition, which is required to achieve a unique rational expectations equilibrium, is satisfied under these ranges of the parameter χ.

initial increase in the foreign policy rate is smaller, whereas the foreign central bank more aggressively raises its policy rate subsequently. Since a change in real money balances induces a co-movement in consumption, the central bank employs a smaller response of the interest rate to the shock in order to restrain an undesirable fluctuation of inflation and the output gap. A higher value of the parameter χ weakens the ability of the foreign central bank to impart policy inertia into the inflation rate. Accordingly, the central bank cannot introduce policy inertia into the output gap. Real money balances are only affected by a change in the foreign policy rate. Thus, in the case of the natural interest rate shock, the parameter χ may not play a significant role for the foreign country.

[Figure 1 around here]

In contrast to a change in the foreign nominal interest rate, the home central bank sets its policy rate to a positive value. In contrast to the case of the foreign country, a change in the parameter χ significantly affects endogenous variables in the home country. For a higher value of the parameter χ the home central bank reinforces such a rise in the policy rate under optimal monetary policy. In particular, a foreign natural interest rate shock generates a policy trade-off between inflation and the output gap in the home country. Such a trade-off is more severe as the parameter χ takes a higher value because a higher value of the parameter χ leads to a larger decline in the output gap in the initial period. On the other hand, the home central bank implements monetary tightening aggressively to combat a situation where a higher value of the parameter χ generates an increase in inflation. In addition, as in the case of the foreign country, a higher value of the parameter χ counteracts policy inertia into both home inflation and the home output gap. A change in the home policy rate affects home real money balances. Finally, the real exchange rate remains unchanged even if the substitutability between consumption and money increases.

5.4.2 A foreign money demand shock

Next, this section reports the impulse response analysis for a foreign money demand shock. The impulse response of a foreign money demand shock is different from that of a foreign natural rate shock. This impulse response is shown in Figure 2. For a money demand shock, the effect on the real economy is negligible as long as the parameter χ takes a much smaller value, which is close to the case of the separable utility. This implies that the money demand shock has little effect on the real economy. However, a larger value of the parameter χ significantly affects the real economy. In fact, in the case of the money demand shock, endogenous variables of both countries are clearly affected by any change in the parameter χ.

For the foreign country, a money demand shock induces a decline in both inflation and the output gap. On the other hand, the foreign interest rate remains unchanged in response to the shock in the case of the parameter $\chi = 0.01$. However, when the parameter χ takes higher values, the foreign central bank accommodates a huge decline in both inflation and the output gap, whereas it increases the interest rate. This is because the foreign central bank reacts aggressively to the endogenous variables since the output gap co-moves an increase in the money demand in the presence of the non-separable utility function between money and consumption.

[Figure 2 around here]

In contrast to the case of a natural interest rate shock, a change in the parameter χ affects the impulse responses for the home country. In addition, compared to the case of the foreign country, an increase in the foreign money demand leads to a boom in the home country. This boom is created by a reduction of home central bank's monetary easing. In particular, for a larger value of the parameter χ, the optimal monetary policy requires the home central bank to cut its policy rates more aggressively. Such a reduction of the home policy rate induces a further increase in both inflation and the output gap in the home country.

Finally, in contrast to the natural interest rate shock, the real exchange rate depreciates in the initial period. Then, the money demand shock induces an appreciation of the real exchange rate in the subsequent periods. As shown in Figure 2, a larger value of the parameter χ produces a larger depreciation of the real exchange rate initially and a persistent appreciation subsequently. It follows from Equation (41) that an increase in foreign money produces a depreciation of the real exchange rate in terms of home currency.

Intuitively, a foreign money demand shock increases foreign real money balances. Such an increase in the real money balances produces a decline in foreign inflation through the foreign NKPC. The foreign output gap declines also through the foreign IS equation. A decline in both foreign inflation and the foreign output gap induces a depreciation of the real exchange rate, which generates an increase in home inflation through the home NKPC. In addition, as an increase in foreign real money balances leads to an increase in the home money demand. This induces an increase in the output gap in the home country. Therefore, a foreign money demand shock produces a recession in the foreign country, whereas the boom is created by that shock in the home country.

6 The role of money and the zero lower bound on nominal interest rates in a two-country economy

This section examines the effect of the ZLB on the real economy when the non-separable utility function between consumption and real money balances in a two-country framework is assumed. Section 6.1 reviews briefly the role of the ZLB in a standard NKM. Section 6.2 reports the impulse response of a foreign negative natural interest rate shock under the ZLB. Section 6.3 considers how a foreign negative money demand shock affects the real economy of each country under the ZLB.

6.1 The zero lower bound on nominal interest rates

The importance of the presence of the ZLB in a NKM has been argued in literature (Eggertsson and Woodford, 2003; Jung, et al., 2005; Adam and Billi, 2006; McKay, Nakamura and Steinsson, 2016; Nakata, 2017). Consider the case where a natural interest rate temporarily takes a negative value. Since the central bank cannot reduce its policy rate to a negative value, it cannot stimulate the real economy once the ZLB hits the economy. As argued in Eggertsson and Woodford (2003) and Jung et al. (2005), the central bank can insulate the aggregate demand through the history dependence of a commitment policy even when the nominal interest rate reaches the zero bound.

However, the aforementioned studies examine optimal monetary policy with the zero bound in closed economies and have limited bearing on the current global situation. Nakajima (2008) investigates optimal monetary policy under the zero bound in a two-country economy, but in his work only the domestic interest rate is zero. Ida (2013) derives optimal monetary policy rules in a two-country economy in which both home and foreign country simultaneously face the ZLB. He shows that optimal policy rules that excludes the information from past policy rates only retains the optimality when two countries simultaneously face the ZLB. Fujiwara et al. (2010, 2014) examine also optimal monetary policy in the case where nonnegativity constraints on nominal interest rate exist in both home and foreign countries. They point out that the global liquidity trap induces its international dependence, and show that price level targeting can achieve higher welfare than inflation targeting in such a case. According to their studies, when the ZLB is binding in both the countries, there are gains from a commitment policy when both domestic and foreign central banks coordinate monetary policies.

As noted earlier, the previous studies do not stress the role of money in the NKM. However, recent evidence indicates that money aggregate plays a significant role in unconventional monetary policy, in which central banks would expect the effect of the quantity of money on the real economy in addition to a commitment to a ZIRP. Central banks, such as the FRB, the Bank of England, and the BOJ, address the effect of money on the real economy as quantitative easing. Since these central banks have employed quantitative easing policies, the presence of the ZLB should play a significant role in a two-country model that the role of money is included. Importantly, previous studies do not include the effect of money on the real economy in a two-country model even in which central banks simultaneously face the ZLB.

As in Fujiwara et al. (2014), I consider a situation where both home and foreign country simultaneously face the ZLB. Thus, both home and foreign central banks face the following nonnegativity constraints on nominal interest rates:[22]

$$i_t \geq 0,\ i_t^* \geq 0. \tag{64}$$

Fujiwara et al. (2014) argue that the termination date of a ZIRP for the home country is different from that for the foreign country when both home and foreign countries simultaneously face the ZLB.[23] Their results still hold in this model regardless of a deterministic shock case. Importantly, the substitutability between consumption and money in the household's utility function generates the difference between the termination date of a ZIRP for the home country and that for the foreign country.

6.2 A foreign natural interest rate shock

Figure 3 portrays the responses of endogenous variables to a foreign negative natural rate of interest shock. The negative shock for the foreign natural interest rate induces a recession in the foreign country. The foreign central bank employs a ZIRP and maintains that policy for long term. Indeed, as shown in Figure 3, the ZIRP terminates at period 33. After the termination of the ZIRP, the foreign central bank gradually increases its policy rate. This optimal response of the foreign central bank is consistent with the assertion of Eggertsson and Woodford (2003).

[Figure 3 around here]

On the other hand, a foreign negative interest rate shock generates a trade-off between inflation and the output gap in the home country. The home central bank conducts a ZIRP also in response to a decline in the rate of inflation. Since the output gap is a positive response to the shock, the home central bank terminates the ZIRP at period 15.

[22]Unlike Fujiwara et al. (2010, 2014), this paper examines the role of money in a two-country model in which both home and foreign country face the ZLB under the assumption that a deterministic shock occurs in both country. The simulations in this paper are implemented by the Dynare. See also Adam and Billi (2006, 2007) and Nakov (2008) a detailed discussion about a stochastic shock case.

[23]Their simulation is based on two-state Markov switching that the natural interest rates of both countries follow the Makov process.

This termination date is shorter than that by roughly 15 periods compared to the case of the foreign country.

Surprisingly, unlike the argument in Eggertsson and Woodford (2003), the home central bank raises its policy rate aggressively when it decides to depart from the ZIRP. In particular, it accommodates considerable overshooting of the nominal interest rate. Although deflation in the home country is caused by a decline in the foreign natural interest rate, the natural interest rate in the home country still takes a positive value. The home central bank needs to prevent an undesirable boom created by introducing a ZIRP induced by a foreign natural interest rate shock. Therefore, the home central bank raises its policy rate immediately and aggressively when it overcomes the deflationary pressure.

[Figure 4 around here]

How does the parameter χ affect the effect of the ZLB on both countries? Figure 4 shows how a change of the parameter χ affects the response of nominal interest rates of both the countries. It implies that the termination date of the foreign ZIRP in the case of $\chi = 0.04$ is slightly longer than that in the case of $\chi = 0.01$. Moreover, for the home country, the central bank implements a more overshooting reaction of the policy rate than one in the case of $\chi = 0.01$.

6.3 A foreign money demand shock

Next, consider the effect of the ZLB in the case of a foreign money demand shock. Interestingly, the money demand shock considerably changes the property of optimal monetary policy in contrast to the case of a natural interest rate shock. A money demand shock, which increases money demand in the foreign country, creates a decline in both inflation and the output gap. While the foreign central bank initially decides on an increase in the policy rate in response to the shock, it attempts to employ the ZIRP also because the shock worsens the real economy. Indeed, the foreign central bank continues to maintain its zero interest rate for a long time. It terminates the ZIRP at period 17, and then employs the gradualism for the policy rate stance.

On the contrary, the home country experiences an increase in both inflation and the output gap. However, under policy coordination, the home central bank, which considers the foreign ZIRP effect on the home country, maintains the nominal interest rate at zero for shorter periods. Indeed, the home central bank decides to terminate a ZIRP at period 3. As in the case of the foreign natural rate shock, the home central bank that implements optimal monetary policy allows the policy rate to overshoot immediately once the central bank determines the termination of the ZIRP.

[Figure 5 around here]

How does a change in the parameter χ affect the path of the nominal interest rates in both the countries in the case of the money demand shock? Figure 6 illustrates the path of the nominal interest rates of both the countries when the money demand shock occurs in the foreign country.[24] According to this figure, a larger value of the parameter χ requires the foreign central bank to increase its policy rate more aggressively after the shock. However, after such an aggressive monetary tightening, as in the case of the $\chi = 0.02$, the central bank decides to set its nominal interest rate zero. The termination date of the ZIRP in the case of $\chi = 0.04$ is longer than that of the case of $\chi = 0.02$. In fact, as shown in Figure 6, the foreign central bank keeps the nominal interest rate at zero by period 22 in the case of $\chi = 0.04$. On the contrary, the termination date of the ZIRP of the home country in the case of $\chi = 0.04$ is also longer than that in the case of $\chi = 0.02$. In addition, an overshooting response of the home nominal interest rate is amplified by a larger value of the parameter χ.

[Figure 6 around here]

[Figure 7 around here]

Finally, the paper checks how the degree of persistence of the foreign money demand shock affects the interest rate path of both of the countries. Figure 7 shows the interest

[24]Here, the interest rate path for the case of $\chi = 0.01$ is not reported. This is because it is confirmed that the home nominal interest rate increases in response to the money demand shock. Thus, in the case of $\chi = 0.01$, the home nominal interest rate does not face the ZLB.

rate path of both of the countries when persistence of the foreign money demand shock (ρ_ζ^*) changes. The foreign central bank raises its policy rate more aggressively as the parameter ρ_ζ^* takes a larger value. According to this figure, a larger value of the parameter ρ_ζ^* forces the foreign central bank to retain the ZIRP for a longer period. Indeed, the termination date of the ZIRP of the foreign central bank for the case of $\rho_\zeta^* = 0.8$ is about twice the duration of the case of $\rho_\zeta^* = 0.3$. A larger value of the parameter ρ_ζ^* also prolongs the termination date of the ZIRP in the home country. The nominal interest rate of the home country overshoots more predominately as the effect of foreign money demand shock is persistent.

These results imply that the money demand function significantly affects the path of endogenous variables when both the countries face the ZLB simultaneously. Therefore, from a global perspective, this paper addresses that the model should consider the case that money aggregate significantly influences the real economy in the presence of the nonnegativity constraints on the nominal interest rates.

7 Conclusions

The NKM has been used in recent monetary policy analysis. In the case of the standard NKM, the money demand function may be redundant because the model presumes the separation between consumption and real money balances in the household's utility function. However, recent evidences from monetary policy in advanced countries reveal the importance of role of money since central banks in advanced country reduce their policy rates to combat the risk of deflation derived from the Lehman crisis that occurred in the United States. As a result, they have faced nonnegativity constraints on nominal interest rates. Therefore, the central banks decided to increase a quantity of monetary base to combat deflation and recession once they face the ZLB. The standard NKM that studies the ZLB effect on the economy does not consider the role of money.

This paper addresses the role of money in a two-country economy. The present study considered the non-separable utility between consumption and real money balance in a two-country framework. This paper examined the role of money in a two-country model constructed by Engel (2009, 2011), who constructed a two-country model with home bias. Unlike Clarida, Gali, and Gertler (2002) and Engel (2009, 2011), this paper showed that relative real money balances between two countries directly affect the risk-sharing condition for consumption when one allows the case where consumption and real money balances are not separable in the household's utility function.

Such a departure of the separable utility function changes the structural equations in a two-country economy with nominal rigidities. Interestingly, in a two-country model in which consumption and real money balances are not separable, relative real money balances between two countries directly affect not only the dynamic IS equation and the money demand function, but also the NKPC. In addition, the non-separable utility changes the shape of the central bank's loss function. In contrast to Clarida, Gali, and Gertler (2002) and Engel (2009, 2011), the shape of the loss function is different from such a standard loss function when consumption and real money balances are not separable in the utility function. First, the central banks need to stabilize a variability of difference between home real money balances and foreign real money balances. Second, the loss function includes the cross term for relative output gap and relative real money balances.

The assumption of the non-separable utility between consumption and real money balances significantly affects the properties of optimal monetary policy. Furthermore, this paper attempts to examine how the assumption of a non-separable utility between consumption and money affects the optimal monetary policy when the ZLB simultaneously hits both countries. Several studies that explore the properties of optimal monetary policy in a two-country model do not account for the effect of money on the real economy in a two-country model in which central banks simultaneously face the ZLB. This paper showed that the presence of a non-separable utility function between consumption and money crucially changes the termination date of a ZIRP for both countries.

Finally, there are possible further extensions of the work in this paper. First, as in Deverex and Engel (2003), Monacelli (2005), and Engel (2009, 2011), it is interesting how the relaxation of the assumption of producer-currency pricing changes the properties of optimal monetary policy in the case of a non-separable utility function. Second, this paper provided the log-linearized structural equations around the zero-inflation steady state. However, as argued by Ascari and Ropele (2007) and Ascari and Sbordone (2014), it might be important whether the presence of a non-separable utility function changes optimal monetary policy in the case where a positive trend inflation exists in both countries. Third, this paper simply assumed a deterministic economic shock when examining the effect of the ZLB in a two-country economy. This is because the two-country model may make the numerical algorithm for solving the problem of the ZLB more complicated in the case of a stochastic shock. Indeed, as argued in the existing literature, we may often have difficulty in numerically solving optimal policy due to the presence of state variables (e.g., Adam and Billi, 2006; Adam and Billi, 2007; Nakov, 2008; Guerrieri and Iacoviello, 2015). Accordingly, the numerical solution might be more complicated as state variables associated with commitment increase in a two-country model. As in the studies by Adam and Billi (2006, 2007) and Nakov (2008), however, the properties of optimal monetary policy obtained in this paper may be changed by the introduction of a stochastic shock. For instance, as the extension of this study, it may be interesting how the non-linear effects of the ZLB are introduced in this paper.

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A Appendix A: Derivation of the central bank's loss function

This appendix provides a detailed derivation of the central bank's loss function in a two-country model with a non-separable utility between consumption and real money balances. As shown in Woodford (2001, 2003), the loss function corresponds to the second-order approximation of the household's utility function.

A.1 Preliminaries

I derive the loss function approximated around a steady state. Before doing so, I define relevant notation. First, $\bar{H}$ denotes the value of the steady state, and H_t^n is the value of the efficient level. Second, as in the main text, I define $h_t = \log(H_t/\bar{H})$ as the deviation of H_t from the steady state. To implement the second-order approximation of the planner's objective function, I introduce the following equation:

$$H_t - \bar{H} = \bar{H}\left(\frac{H_t}{\bar{H}} - 1\right) \approx h_t + \frac{1}{2}h_t^2. \tag{A.1}$$

To derive the second-order approximation of the planner's objective function, I introduce several useful equations. First, expressing Equation (41) in terms of log-deviation from their efficient price equilibrium counterparts, we obtain the following equations:

$$\tilde{c}_t = \psi\tilde{y}_t + (1-\psi)\tilde{y}_t^* + \zeta(\tilde{z}_t^* - \tilde{z}_t), \tag{A.2}$$

$$\tilde{c}_t^* = \psi\tilde{y}_t^* + (1-\psi)\tilde{y}_t - \zeta(\tilde{z}_t^* - \tilde{z}_t). \tag{A.3}$$

Also, expressing production functions for both home and foreign countries in terms of log-deviation from their efficient price equilibrium counterparts, we obtain

$$\tilde{n}_t = \tilde{y}_t + \frac{\theta}{2}var(p_{H,t}), \tag{A.4}$$

$$\tilde{n}_t^* = \tilde{y}_t^* + \frac{\theta}{2}var(p_{F,t}^*). \tag{A.5}$$

Furthermore, home and foreign money demand functions, which are expressed by log-deviation from their efficient price equilibrium counterparts are as follows:

$$\tilde{z}_t = \eta_c \tilde{c}_t - \eta_i i_t, \tag{A.6}$$

$$\tilde{z}^*_t = \eta_c \tilde{c}^*_t - \eta_i i^*_t. \tag{A.7}$$

A.2 Second-order approximation of the planner's objective function

I now provide the second-order approximation of the following household utility function. The derivation of the loss function in the main text is based on the idea of Clarida, Gali, and Gertler (2002). I implement the second-order approximation of the following planner's objective function around their efficient price equilibrium. The following step of the derivation of the loss function is mainly based on that of Clarida, Gali and Gertler (2002) and Engel (2009, 2011).

The planner's objective function under policy coordination is given by

$$W_t = u(C_t, Z_t) + u(C^*_t, Z^*_t) - v(N_t) - v(N^*_t), \tag{A.8}$$

The second-order approximation of first and second terms of the right-hand side of the planner's objective function are follows:

$$u(C_t, Z_t) \approx u_c c^n_t \left(\tilde{c}_t + \frac{1}{2}\tilde{c}^2_t + s_z \tilde{z}_t + s_z(1-\sigma_z)\tilde{z}^2_t + \chi \tilde{c}_t \tilde{z}_t \right) + t.i.p. + +O(||\xi||^3), \tag{A.9}$$

$$u(C^*_t, Z^*_t) \approx u_c c^{*n}_t \left(\tilde{c}^*_t + \frac{1}{2}(\tilde{c}^*_t)^2 + s_z \tilde{z}^*_t + s_z(1-\sigma_z)(\tilde{z}^*_t)^2 + \chi \tilde{c}^*_t \tilde{z}^*_t \right) + t.i.p. + +O(||\xi||^3), \tag{A.10}$$

where

$$\sigma_z = \frac{u_{zz}\bar{Z}}{u_z}, s_z = -\frac{u_z \bar{Z}}{u_c \bar{C}}.$$

The term $u_c c^n_t$ and $u_c C^{n*}_t$ of the second-order approximation of Equations (A.9) - (A.10) are given by

$$u_c c^n_t = u_c \bar{C} + u_c \bar{C}(1-\sigma)c^n_t + O(||\xi||^3), \tag{A.11}$$

$$u_c c^{n*}_t = u_c \bar{C} + u_c \bar{C}(1-\sigma)c^{n*}_t + O(||\xi||^3). \tag{A.12}$$

As assumed in Woodford (2003), this paper presumes that the real money balances satiate at some finite level, and that the variation in interest rates in efficient price equilibrium is quite small. Since we can make use of Woodford (2003, chapter 6) assumption 6.1, therefore, we obtain the following useful relationships:[25]

$$s_z \sigma_z = -(\bar{v}\eta_i)^{-1} \tag{A.13}$$

$$\eta_c = \bar{v}\chi\eta_i. \tag{A.14}$$

Using Equations (A.7), (A.8), (A.11), (A.12), (A.13), and (A.14), we can rewrite Equations (A.11) and (A.12) as follows:

$$u(C_t, Z_t) \approx u_c\bar{C}\left[(1+s_z)\tilde{c}_t + \frac{1}{2}(1-\sigma+\chi\eta_c)\tilde{c}_t^2 - \frac{1}{2}(\bar{v})^{-1}\eta_i(i_t - i_t^T)^2 + (1-\sigma)c_t^n\tilde{c}_t\right]$$
$$+ t.i.p. + O(||\xi||^3), \tag{A.15}$$

$$u(C_t^*, Z_t^*) \approx u_c\bar{C}\left[(1+s_z)\tilde{c}_t^* + \frac{1}{2}(1-\sigma+\chi\eta_c)(\tilde{c}_t^*)^2 - \frac{1}{2}(\bar{v})^{-1}\eta_i(i_t^* - i_t^{T*})^2 + (1-\sigma)c_t^{*n}\tilde{c}_t^*\right]$$
$$+ t.i.p. + O(||\xi||^3), \tag{A.16}$$

where I used the relationship $s_z = \Delta\bar{v}$. $\Delta = \bar{i}/(1+\bar{i})$ denotes opportunity costs of money holdings in the steady state. Also, note that $i^T = i^{T*} = \eta_i\bar{v}\Delta$.

Next, the second-order approximation of both third and fourth terms of the right-hand side are given by

$$V(N_t) = V'(\bar{N})\bar(N)\left[\tilde{n}_t + \frac{1}{2}\tilde{n}_t^2 + (1+\eta)n_t^n\tilde{n}_t\right] + t.i.p. + O(||\xi||^3), \tag{A.17}$$

$$V(N_t^*) = V'(\bar{N})\bar(N)\left[\tilde{n}_t^* + \frac{1}{2}(\tilde{n}_t^*)^2 + (1+\eta)n_t^{*n}\tilde{n}_t^*\right] + t.i.p. + O(||\xi||^3), \tag{A.18}$$

I used the relationship $V'(\bar{N}) = V'(\bar{N}) + V'(\bar{N})(1+\eta)n_t^n$ and the corresponding equation for the foreign country in the derivation of the above equations. Substituting Equation (A.5) and using the relationship $n_t^n\tilde{n}_t = n_t^n\tilde{y}_t + O(||\xi||^2)$, we obtain

$$V(N_t) = V'(\bar{N})\bar(N)\left[\tilde{y}_t + \frac{1}{2}\tilde{y}_t^2 + (1+\eta)n_t^n\tilde{y}_t + \frac{\theta}{2}var(p_{H,t})\right] + t.i.p. + O(||\xi||^3). \tag{A.19}$$

[25]See Woodford (2003) for a detailed discussion of this relationship.

Similarly,

$$V(N_t^*) = V^{'}(\bar{N})\bar(N)\left[\tilde{y}_t^* + \frac{1}{2}(\tilde{y}_t^*)^2 + (1+\eta)n_t^{n*}\tilde{y}_t^* + \frac{\theta}{2}var(p_{F,t}^*)\right] + t.i.p. + O(||\xi||^3). \tag{A.20}$$

Also, substituting Equation (A.2) into Equation (A.15) and rearranging, we obtain the following expression

$$u(C_t, Z_t) = u_c\bar{C}\left[\tilde{y}_t + \frac{1}{2}(1-\sigma+\chi\eta_c)(\psi\tilde{y}_t + (1-\psi)\tilde{y}_t^* + \zeta(\tilde{z}_t^* - \tilde{z}_t))^2 - \frac{1}{2}(\bar{v})^{-1}\eta_i(i_t - i_t^T)^2 \right.$$
$$\left.(1+\eta)n_t^n(\psi\tilde{y}_t + (1-\psi)\tilde{y}_t^* + \zeta(\tilde{z}_t - \tilde{z}_t^*))\right] + t.i.p. + O(||\xi||^3). \tag{A.21}$$

Similarly, we can obtain the corresponding equation for the foreign country:

$$u(C_t^*, Z_t^*) = u_c\bar{C}\left[\tilde{y}_t^* + \frac{1}{2}(1-\sigma+\chi\eta_c)(\psi\tilde{y}_t^* + (1-\psi)\tilde{y}_t - \zeta(\tilde{z}_t^* - \tilde{z}_t))^2 - \frac{1}{2}(\bar{v})^{-1}\eta_i(i_t^* - i_t^{T*})^2 \right.$$
$$\left.(1+\eta)n_t^{n*}(\psi\tilde{y}_t^* + (1-\psi)\tilde{y}_t - \zeta(\tilde{z}_t^* - \tilde{z}_t))\right] + t.i.p. + O(||\xi||^3), \tag{A.22}$$

Combining (A.21) with (A.22), we obtain

$$u(C_t, Z_t) + u(C_t^*, Z_t^*) = u_c\bar{C}\left[\tilde{y}_t + \tilde{y}_t^* + \frac{1}{2}(1-\sigma+\chi\eta_c)[(2\psi^2 - 2\psi + 1)(\tilde{y}_t^2 - (\tilde{y}_t^*)^2)\right.$$
$$+ 4\psi(1-\psi)\tilde{y}_t\tilde{y}_t^* + \zeta^2(\tilde{z}_t^* - \tilde{z}_t)^2] - 2\zeta(2\psi-1)(\tilde{y}_t - \tilde{y}_t^*)(\tilde{z}_t - \tilde{z}_t^*)$$
$$\left. - \frac{1}{2}(\bar{v})^{-1}\eta_i(i_t + i_t^*)^2 + (1+\eta)n_t^n\tilde{y}_t + (1+\eta)n_t^{n*}\tilde{y}_t^*\right] + t.i.p. + O(||\xi||^3). \tag{A.23}$$

I assumed that $i_T = i_{T*} = 0$ in this derivation.

Since the relationship $u_c\bar{C} = V^{'}(\bar{N})\bar{N}$ holds in the efficient level of the steady state, taking into the above relationship consideration and combining Equation (23) with Equations (A.19) and (A.20), we obtain

$$W_t = -u_c\bar{C}\left[\frac{\sigma+\chi\eta_c}{2}(1 + 2\psi(\psi-1)(1-\varrho^{-1})(\tilde{y}_t^2 + (\tilde{y}_t^*)^2) + 2(\sigma+\chi\eta_c - 1)\zeta^2(\tilde{z}_t^* - \tilde{z}_t)^2\right.$$
$$+ 4\psi(1-\psi)\tilde{y}_t\tilde{y}_t^* + 2\zeta(2\psi-1)(\tilde{y}_t - \tilde{y}_t^*)(\tilde{z}_t - \tilde{z}_t^*) + \frac{1}{2}(\bar{v})^{-1}\eta_i(i_t + i_t^*)^2 + \eta\tilde{y}_t^2 + \eta(\tilde{y}_t^*)^2$$
$$\left.+ \frac{\theta}{2}(var(p_{H,t}) + var(p_{F,t}^*))\right] + t.i.p. + O(||\xi||^3). \tag{A.24}$$

Furthermore, from the firm j's demand function, we obtain the following equations:[26]

$$var_j \log y_t(j) = \theta^2 var_j \log P_{H,t}(j),$$
$$var_j \log y_t^*(j) = \theta^2 var_j \log(P_{F,t^*})(j),$$

Now define

$$\Delta_{H,t} = var_j \log P_{H,t},$$
$$\Delta_{F,t} = var_j \log P_{F,t}^*.$$

Then, we obtain the following equations:[27]

$$\sum_{t=0}^{\infty} \Delta_{H,t} = \frac{\omega}{(1-\omega)(1-\omega\beta)} \sum_{t=0}^{\infty} \beta^t \pi_t^2 + t.i.p. + O(||\xi||^3), \quad \text{(A.25)}$$
$$\sum_{t=0}^{\infty} \Delta_{F,t} = \frac{\omega}{(1-\omega)(1-\omega\beta)} \sum_{t=0}^{\infty} \beta^t (\pi_t^*)^2 + t.i.p. + O(||\xi||^3). \quad \text{(A.26)}$$

Substituting Equations (A.25) and (A.26) into the discounted sum of Equation (A.24), we obtain the loss function (60) in the main text.

B Appendix B: Optimization problem of optimal monetary policy under policy coordination

In this appendix, I describe the detailed derivation of the central bank's optimization problem[28]. As noted in the main text, under a coordination regime, the central banks minimize their loss function, Equation (60), subject to Equations (51), (53) and (55) and the corresponding constraints in the foreign country. Policy coordination implies that both domestic and foreign central banks jointly minimize a weighted sum of the loss function of domestic and foreign central banks.

To implement optimal monetary policy under policy coordination, we define the Lagrangian as follows:

[26] See Walsh (2017) for a detailed derivation of these equations.

[27] See Woodford (2003) for a detailed derivation of these equations.

[28] This appendix provides the optimal monetary policy when the ZLB is present in both countries.

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \big\{ \pi_t^2 + (\pi_t^*)^2 + \lambda_1[\tilde{y}_t^2 + (\tilde{y}_t^*)^2] + \lambda_2 \tilde{y}_t \tilde{y}_t^* + \lambda_3[i_t^2 + (i_t^*)^2] + \lambda_4(\tilde{z}_t^* - \tilde{z}_t)^2$$
$$+ \lambda_5(\tilde{z}_t - \tilde{z}_t^*)(\tilde{y}_t - \tilde{y}_t^*) - 2\phi_{1t}\big(\beta\pi_{t+1} + \kappa_1\tilde{y}_t + \kappa_2\tilde{y}_t^* - \kappa_3\tilde{z}_t\kappa_4\tilde{z}_t^* - \pi_t\big)$$
$$- 2\phi_{2t}\big(\beta\pi_{t+1}^* + \kappa_1\tilde{y}_t^* + \kappa_2\tilde{y}_t - \kappa_3\tilde{z}_t^*\kappa_4\tilde{z}_t - \pi_t^*\big)$$
$$- 2\phi_{3t}\big[\tilde{y}_{t+1} + \vartheta_1\Delta\tilde{y}_{t+1}^* - \sigma_0^{-1}(i_t - \pi_{t+1} - \bar{r}r_t) + \vartheta_2\Delta\tilde{z}_{t+1} - \vartheta_3\Delta\tilde{z}_{t+1}^* - \tilde{y}_t\big]$$
$$- 2\phi_{4t}\big[\tilde{y}_{t+1}^* + \vartheta_1\Delta\tilde{y}_{t+1} - \sigma_0^{-1}(i_t^* - \pi_{t+1}^* - \bar{r}r_t^*) + \vartheta_2\Delta\tilde{z}_{t+1}^* - \vartheta_3\Delta\tilde{z}_{t+1} - \tilde{y}_t^*\big]$$
$$- 2\phi_{5t}\big(\mu_1\tilde{y}_t + \mu_2\tilde{y}_t^* + \mu_3\tilde{z}_t^* - \mu_4 i_t + \varsigma_t - \tilde{z}_t\big)$$
$$- 2\phi_{6t}\big(\mu_1\tilde{y}_t^* + \mu_2\tilde{y}_t + \mu_3\tilde{z}_t - \mu_4 i_t^* + \varsigma_t^* - \tilde{z}_t^*\big)\big\} \tag{B.1}$$

First order conditions of this optimization problem are as follows:

$$\pi_t - \phi_{1t-1} + \phi_{1t} - (\sigma_0\beta)^{-1}\phi_{3t-1} = 0, \tag{B.2}$$
$$\pi_t^* - \phi_{2t-1} + \phi_{2t} - (\sigma_0\beta)^{-1}\phi_{4t-1} = 0, \tag{B.3}$$
$$\lambda_1\tilde{y}_t + \lambda_2\tilde{y}_t^* - \lambda_5(\tilde{z}_t^* - \tilde{z}_t) - \kappa_1\phi_{1t} - \kappa_2\phi_{2t} - \beta^{-1}\phi_{3t-1} + \phi_{3t} + \vartheta_1(\phi_{4t} - \beta^{-1}\phi_{4t-1})$$
$$- \mu_1\phi_{5t} - \mu_2\phi_{6t} = 0, \tag{B.4}$$
$$\lambda_1\tilde{y}_t^* + \lambda_2\tilde{y}_t + \lambda_5(\tilde{z}_t^* - \tilde{z}_t) - \kappa_2\phi_{1t} - \kappa_1\phi_{2t} - \beta^{-1}\phi_{4t-1} + \phi_{4t} + \vartheta_1(\phi_{3t} - \beta^{-1}\phi_{3t-1})$$
$$- \mu_2\phi_{5t} - \mu_1\phi_{5t} = 0, \tag{B.5}$$
$$\lambda_3 i_t + \sigma_0^{-1}\phi_{3t} + \mu_4\phi_{5t} = 0, \tag{B.6}$$
$$\lambda_3 i_t^* + \sigma_0^{-1}\phi_{4t} + \mu_4\phi_{6t} = 0, \tag{B.7}$$

Combining these optimal conditions with structural equations of both countries, we obtain the economic system (63).

Table 1: Parameter Values

Parameters	Values	Explanation
β	0.99	Discount factor
σ	5.0	Risk aversion coefficient for consumption
η	2.0	Inverse of elasticity of labor supplyt
ω	0.75	Price stickiness
θ	7.88	Elasticity of substitution for individual goods
η_y	1.0	Income elasticity of money demand
η_r	28	Interest elasticity of money demand
χ	0.01	Substitutability between consumption and real money balances
a	1.2	Degree of home bias
ρ_r	0.7	Auto-regressive coefficient for the home natural interest rate shock
ρ_ς	0.5	Auto-regressive coefficient for the home money demand shock
ρ_r^*	0.7	Auto-regressive coefficient for the foreign natural interest rate shock
ρ_ς^*	0.5	Auto-regressive coefficient for the foreign money demand shock
σ_r	1.0	Standard deviation of natural rate shock for home country
σ_ς	1.0	Standard deviation of money demand shock for home country
σ_r^*	1.0	Standard deviation of natural rate shock for foreign country
σ_ς^*	1.0	Standard deviation of money demand shock for foreign country

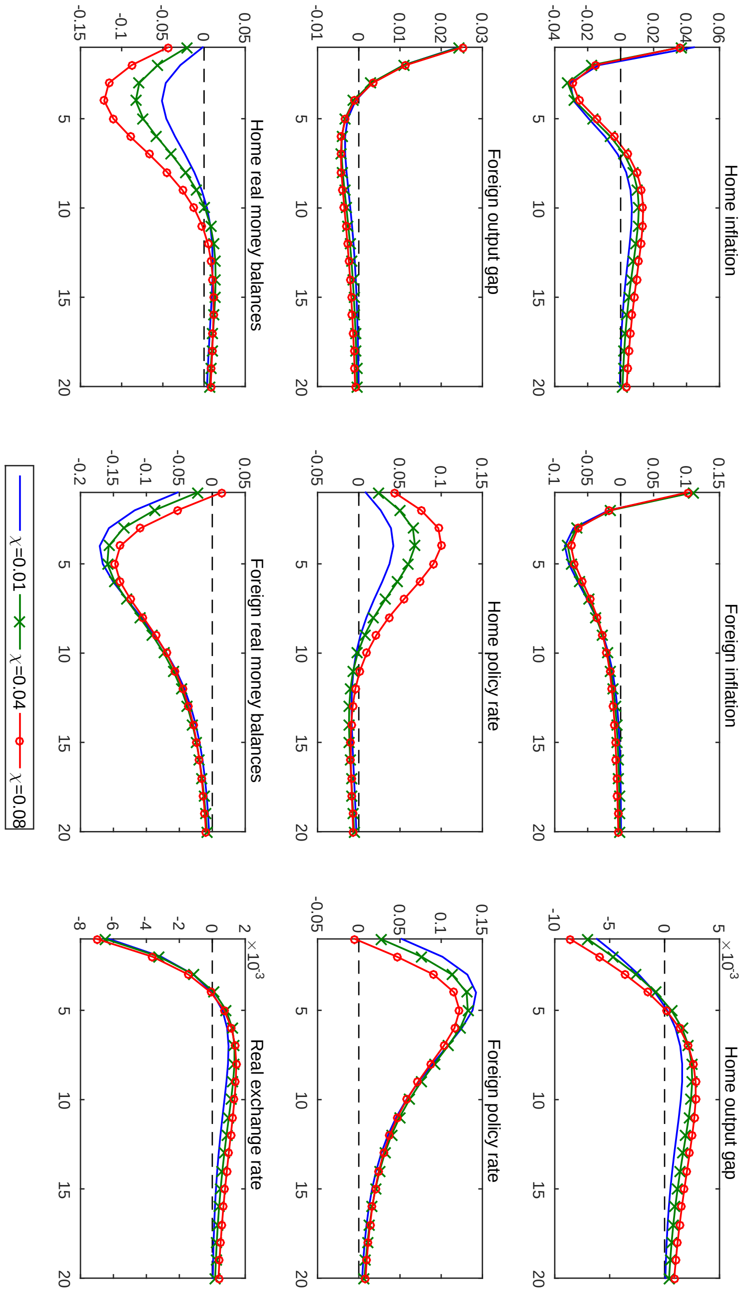

Figure 1: Optimal monetary policy under a foreign natural interest rate.

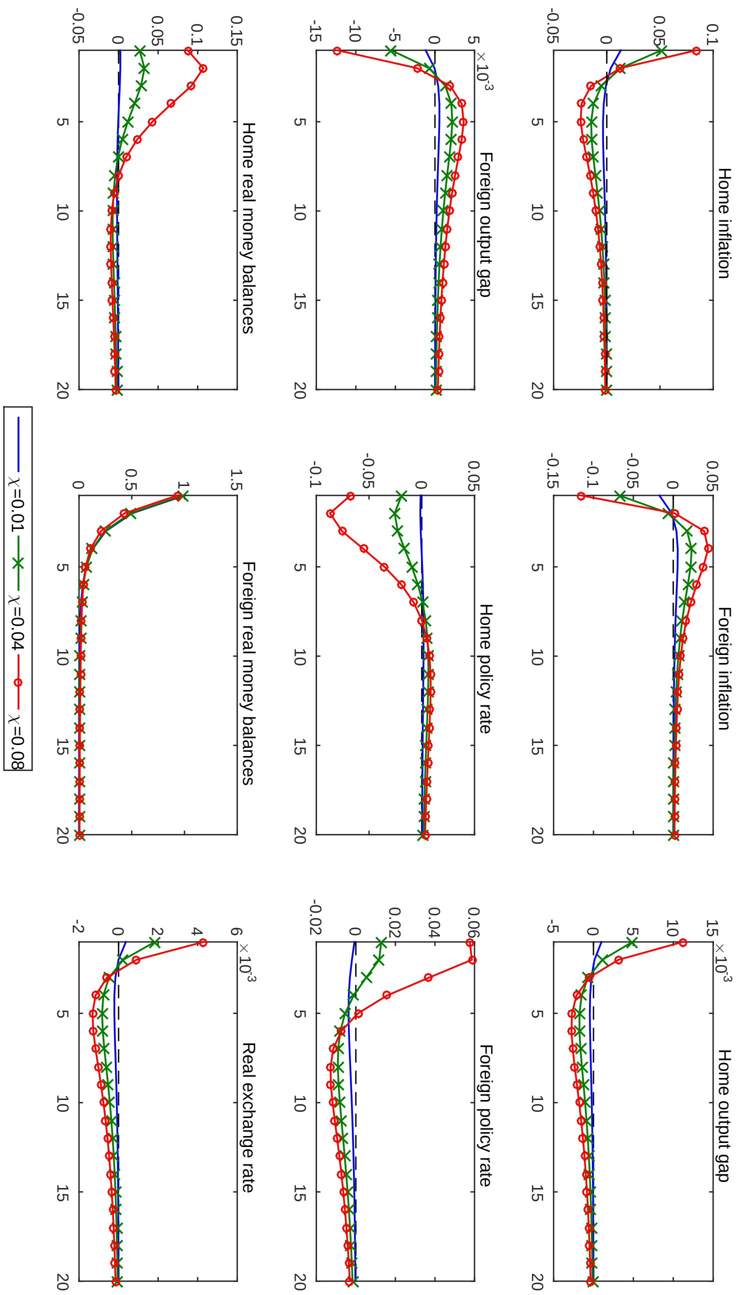

Figure 2: Optimal monetary policy under a foreign money demand shock.

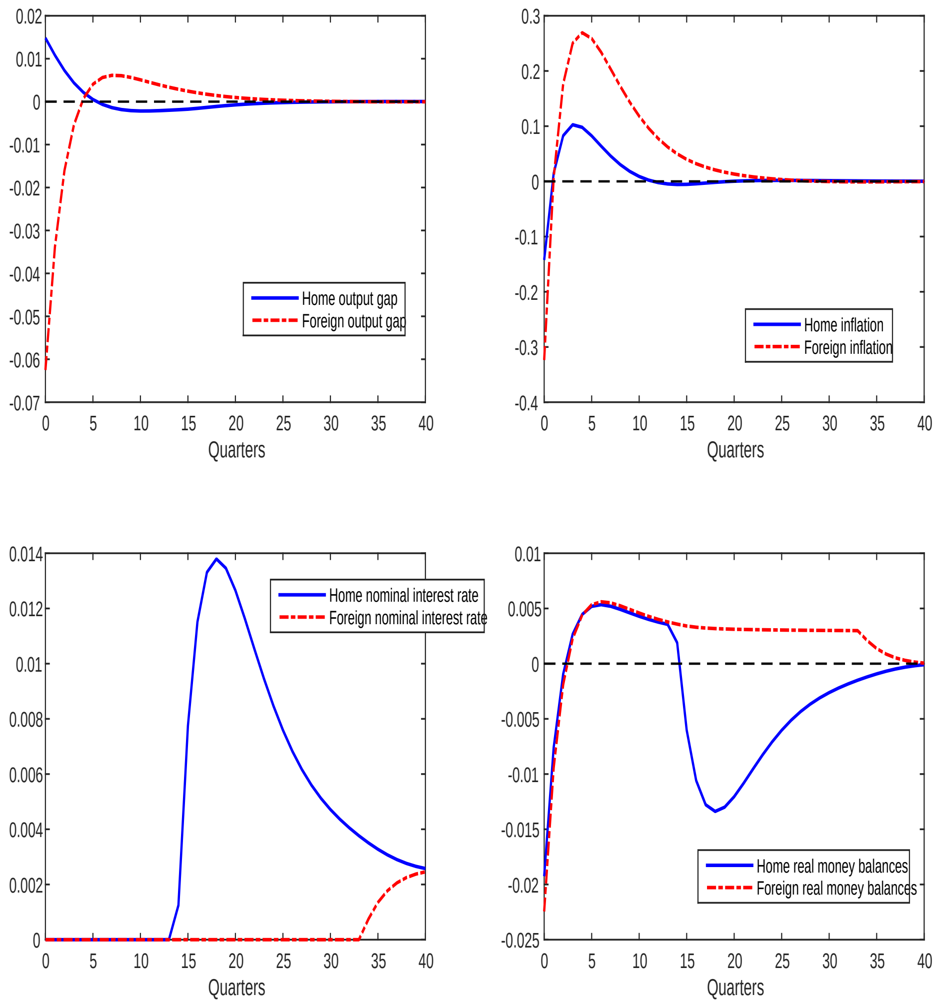

Figure 3: Optimal monetary policy under the ZLB when a foreign natural interest rate occurs.

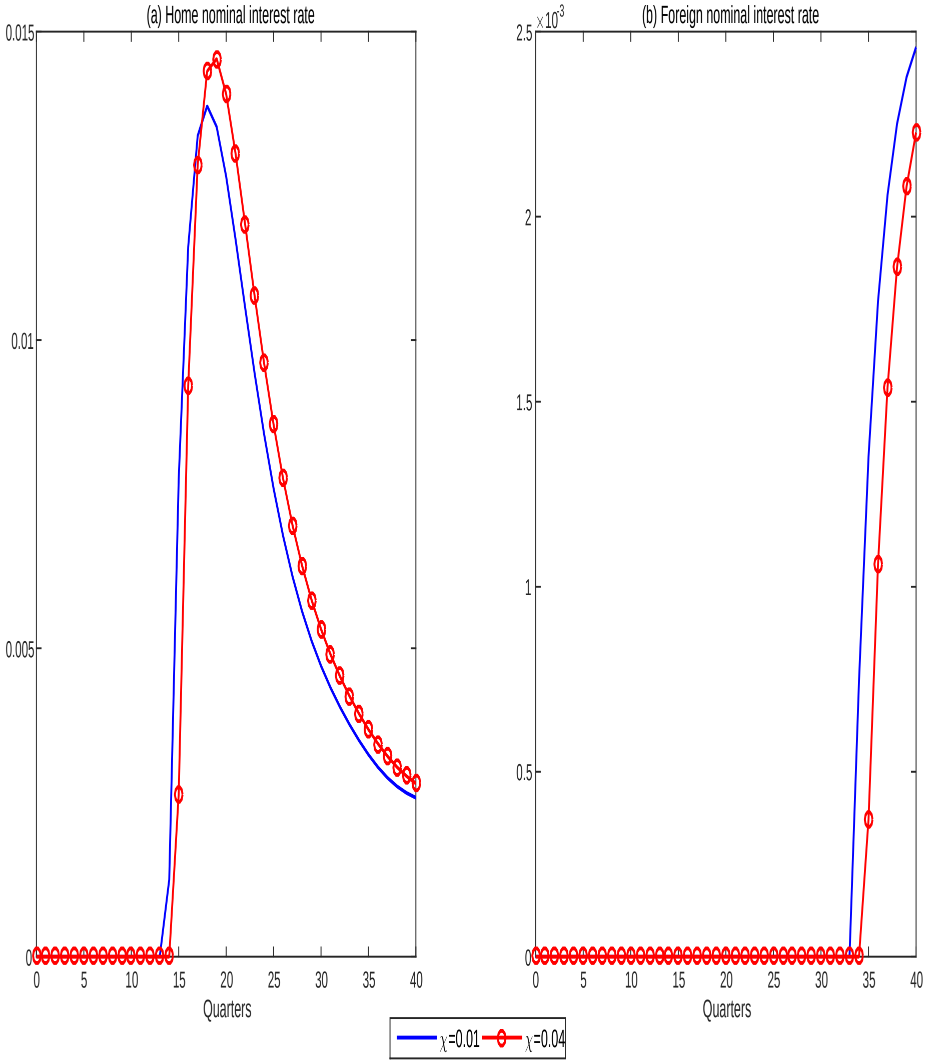

Figure 4: Optimal monetary policy under the ZLB when a foreign natural interest rate occurs.

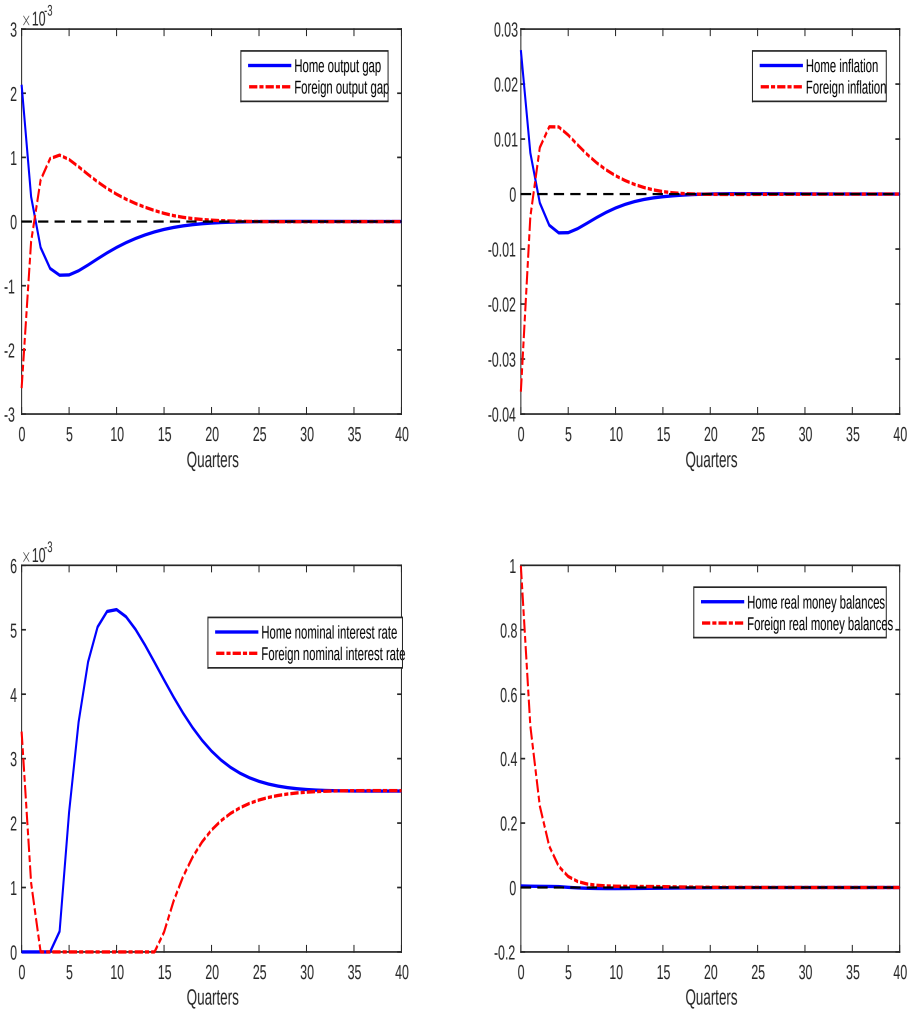

Figure 5: Optimal monetary policy under the ZLB when a foreign money demand shock occurs in the case of $\chi = 0.02$.

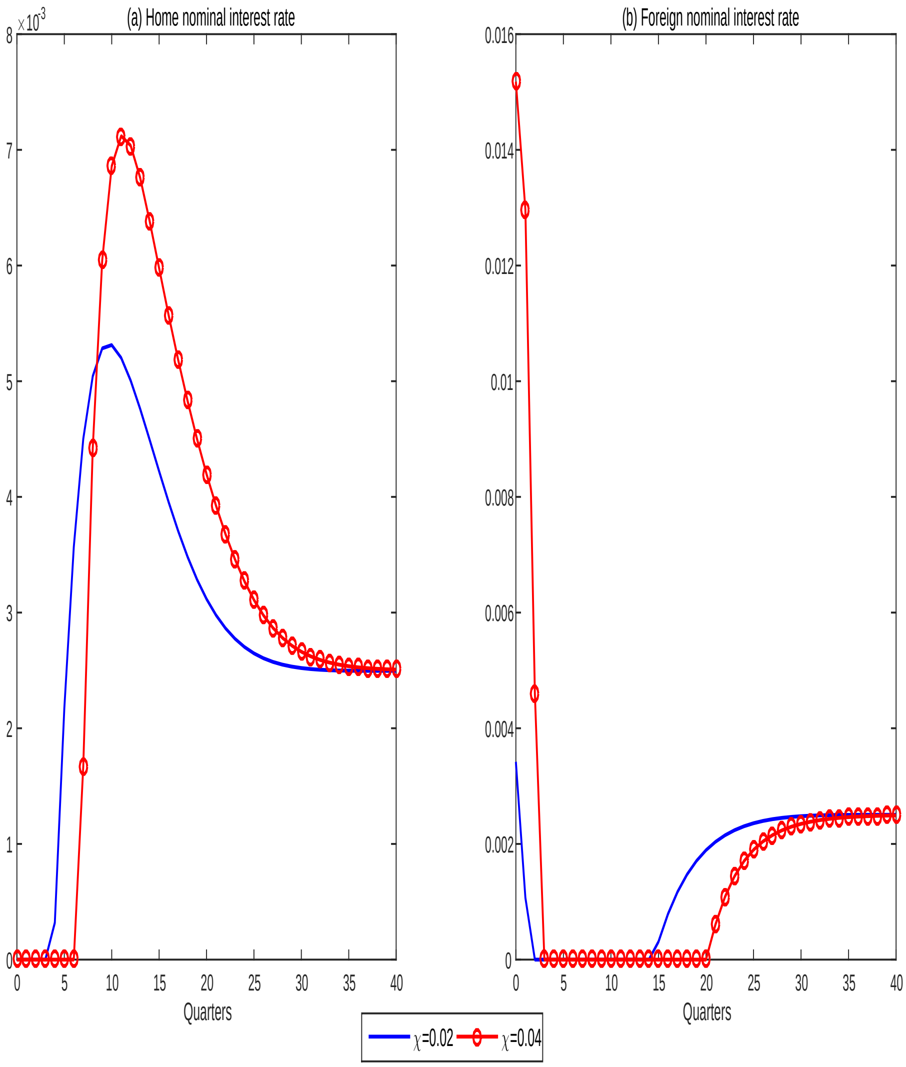

Figure 6: Optimal monetary policy under the ZLB when a foreign money demand shock occurs.

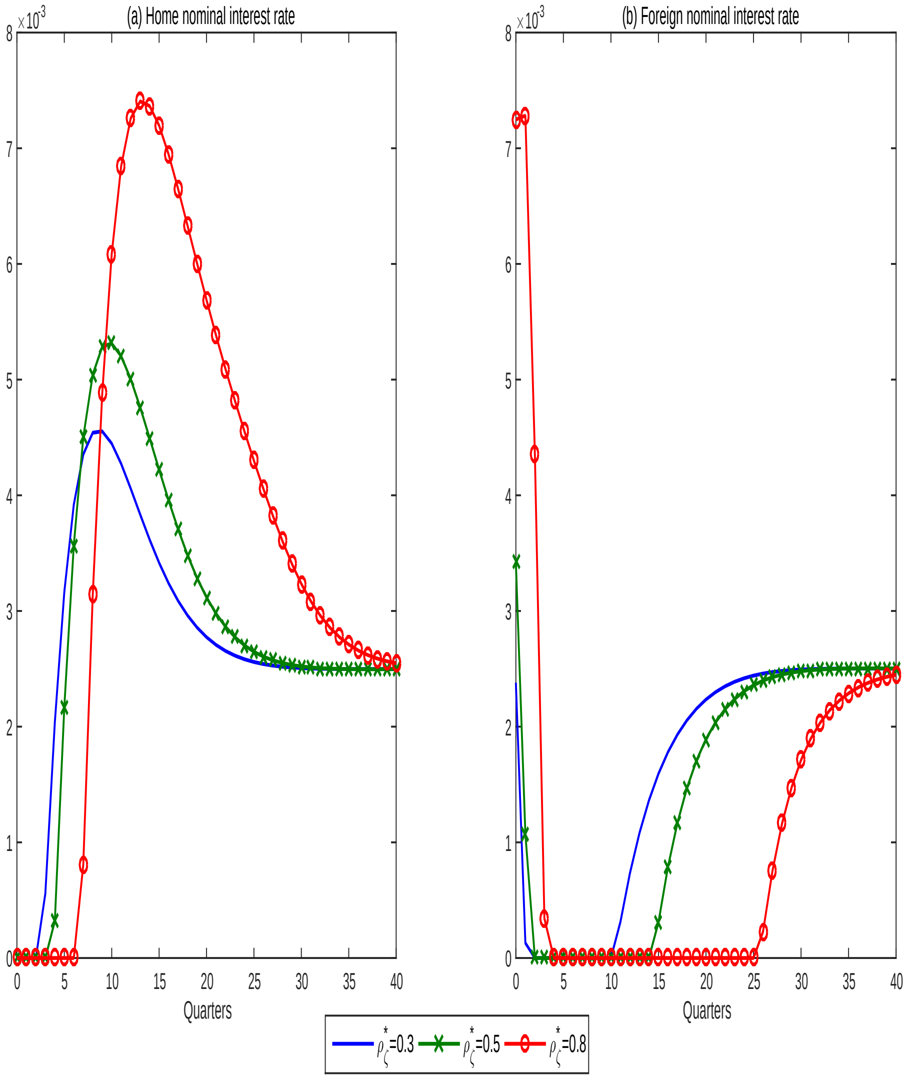

Figure 7: Optimal monetary policy and persistence of a foreign money demand shock under the ZLB in the case of $\chi = 0.02$.