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Cross-checking monetary policy rule and Equilibrium determinacy

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Does a cross-checking monetary policy rule uniquely lead to equilibrium determinacy?*

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Abstract

This paper examines equilibrium determinacy under cross-checking optimal monetary policy. The result shows that an optimal rule derived from cross-checking optimal policy easily expands the regions of the unique rational expectations equilibrium (REE). Interestingly, this result is turned over in the case of the cost channel. This paper shows that introducing a cost channel is likely to make the REE indeterminate under cross-checking optimal monetary policy.

Keywords: Cross-checking monetary policy; Taylor principle; Determinacy; Cost channel;

JEL classification: E52; E58

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1 Introduction

How should the central bank implement optimal monetary policy considering the robustness of the model? According to Tillmann (2012), the central bank refers to monetary policy rules that are robust to the misspecification of the model because it faces the model uncertainty—the true model is not known with certainty. In such a situation, Tillmann (2012) shows that it is desirable for the central bank to attach some weights to the deviation of the policy rate from the prescription suggested by the Taylor rule when the central bank follows a discretionary policy in a forward-looking economy. This gain is associated with the fact that such a policy can reduce stabilization bias generated by a discretionary policy in terms of the traditional loss function that contains inflation and output stabilization.[1]

Ilbas, Roisland and Sveen (2012) also argue the importance of the deviation term of the policy rate from the prescription suggested by the Taylor rule under optimal monetary policy. Walsh (2017b) considers the role of rule-based policy (RBP) regimes when the central bank follows a discretionary policy. The cross-checking optimal monetary policy can be regarded as optimal with the information from the policy rule. Moreover, Taylor (2009) also argues that the deviation of the policy rate from the Taylor rule induced the housing boom from 2000 to 2006 in the United States. It appears, therefore, that these assertions justify the significance of cross-checking optimal monetary policy. They may also address the effectiveness of cross-checking optimal monetary policy, whereas it is unclear whether such a rule can lead to a unique rational expectations equilibrium (REE).

It has been argued that a non-fundamental shock can create multiple equilibria in a forward-looking sticky price model. Hence, it is very important how policymakers make REE determinate. In the standard new Keynesian model, Bullard and Mitra (2002) argue that the REE is uniquely determinate when the traditional Taylor principle is satisfied, which requires that the central bank should respond with a more than one-for-one increase in the interest rate when inflation rate increases. If a rise in the nominal

[1] See Walsh (2017a) for a detailed explanation of the stabilization bias.

interest rate cannot produce an increase in the real interest rate, both expected inflation and output increase in response to a sun-spot shock regardless of monetary tightening. As a result, the economy reaches a non-fundamental equilibrium.[2]

The objective of this paper is to examine equilibrium determinacy under cross-checking optimal monetary policy with discretion. To the best of my knowledge, no studies show how a unique REE is achieved in the case where the central bank implements a discretionary policy by using the information from the Taylor rule. This is the first study to show the condition under which the cross-checking rule achieves a unique REE.

This paper contributes to the literature in the following ways. First, this paper examines how the cross-checking optimal policy affects the determinacy condition under which the REE is uniquely determinate. It shows the condition for a unique REE to be achieved under a cross-checking optimal rule in the standard new Keynesian model. Interestingly, a decrease in a positive weight on a cross-checking rule expands the determinacy regions. Indeed, when the central bank puts a much smaller weight on the information from the Taylor rule, the REE is always determinate when it employs some weight in response to the output gap in a cross-checking optimal rule. Notably, there are cases that the REE is always determinate even if the central bank sets the coefficient for inflation stabilization to zero in the Taylor rule. This is the natural extension of Tillmann (2012) and Walsh (2017b).

The second contribution of this paper is to show that this result is easily turned over in the case of the cost channel. A cost channel implies that a change in the nominal interest rate directly affects the inflation rate through an increase in the firm's working capital. Previous studies have addressed the importance of a cost channel of monetary policy.[3] This paper shows that the central bank, which employs cross-checking monetary

[2]This is the case where the standard Taylor rule is employed in the model. When the forward-looking Taylor rule is assumed, the upper bounds for the response to a change in inflation exist in addition to the Taylor principle. See Bullard and Mitra (2002) for a detailed discussion of this issue.

[3]See Barth and Ramey (2001), Ravenna and Walsh (2006), Chowdhury et al. (2006), and Tillmann (2008) for a detailed discussion of the cost channel. See also Section 4.

policy, can achieve the unique REE if the degree of a cost channel is small. On the contrary, when a strong cost channel is introduced into the economy, a cross-checking optimal policy rule fails to attain the unique REE. Here, this paper presumes that the strength of the cost channel is measured by the degree of loan rate pass-through.

Interestingly, in the case of the cost channel, the REE is likely to be indeterminate as the central bank puts more weight on the stabilization of the output gap. The indeterminacy problem is more severe when the central bank puts less weight on the cross-checking term. The result implies that in the case of a strong cost channel, a smaller weight on the RBP policy term, which leads to an attenuated response to the inflation rate, results in equilibrium indeterminacy even if the instrument rule satisfies the Taylor principle. This result still holds in the forward-looking Taylor rule. Under that rule, the indeterminacy problem is also more severe when the weight on the output gap stabilization takes a larger value.

The case of the cost channel in this paper is the natural extension of Llosa and Tuesta (2009) for the case of cross-checking optimal monetary policy.[4] First, this paper differs from Llosa and Tuesta (2009) in that it assumes that the central bank follows a discretionary policy with the information from the Taylor rule. The paper shows that the determinacy condition derived by Llosa and Tuesta (2009) is strongly affected by the degree of the cross-checking parameter. Second, they assume that the cost-channel parameter takes a range from 0 to 1, whereas this paper employs a wider range of parameters pointed out by the existing literature. This is important because a higher value of the cost-channel parameter implies the amplification of financial instability. Indeed, larger values for a cost channel parameter are reported by the existing literature (Ravenna and Walsh, 2006; Castelnuovo, 2007). Therefore, in contrast to Llosa and Tuesta (2009), this paper both analytically and numerically studies how the degree of the cost channel changes the region where the REE is determinate.

The rest of this paper is organized as follows. Section 2 briefly explains the standard

[4]Note that in contrast to Llosa and Tuesta (2009), this paper abstracts the E-stability condition under adoptive learning when the central bank cross-checks its monetary policy.

new Keynesian model, deriving the cross-checking optimal monetary policy. Section 3 shows the condition under which a cross-checking optimal policy rule can uniquely attain the REE in the standard new Keynesian model. Section 4 explores how the condition is altered in the new Keynesian model with a cost-channel. Section 5 briefly concludes.

2 Model

The model description in this paper is based on Tillmann (2012) and Walsh (2017b). The model is a standard new Keynesian model, which comprises three equations: a dynamic IS equation, a new Keynesian Phillips curve (NKPC), and monetary policy.

The first equation is a dynamic IS curve derived from the intertemporal optimal conditions from households. The dynamic IS curve is given by

$$x_t = E_t x_{t+1} - \sigma^{-1}(r_t - E_t \pi_{t+1} - r_t^n), \tag{1}$$

where x_t is the output gap, π_t is the inflation rate, and r_t is the nominal interest rate. Variable r_t^n denotes the natural rate of interest that stands for the real interest rate in the flexible price equilibrium. This shock can be regarded as a demand shock in this paper. E_t is the expectations operator conditional on the information about period t. The parameter σ is the relative risk aversion coefficient for consumption.

The second equation is a NKPC, which is derived from the optimal condition of the firms that are subject to monopolistic competition and Calvo (1983) type nominal price rigidities. The NKPC is given by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t, \tag{2}$$

where the parameter κ is the sensitivity of inflation to a fluctuation of the output gap and β denotes the discount factor.[5]

[5]This paper abstracts the cost-push shock in the NKPC. The reason is that the presence of the cost-push shock generates the stabilization bias associated with a discretionary policymaking of the central bank. Tillmann (2012) argues that cross-checking optimal policy help alleviate the stabilization bias. In contrast to Tillmann (2012), this paper focuses on an alternative gain from cross-checking optimal

Finally, this paper describes monetary policy of the central bank. The central bank aims at stabilizing quadratic in inflation and the output gap and thus minimizes the loss function, which is given as follows:

$$L_t^{stab} = \pi_t^2 + \lambda x_t^2, \tag{3}$$

where the parameter λ represents the weight on the stabilization of the output gap relative to inflation stabilization. As in Ilbas, Roisland and Sveen (2012), Tillmann (2012), and Walsh (2017b), this paper assumes that the central bank cross-checks its monetary policy by referring to the information of the instrument rule suggested by Taylor (1993). In other words, the central bank cares about the robustness of its interest rate policy. Like Tillmann (2012), this paper also assume that squared deviation of the policy rate from the Taylor rule describes the concern about the robustness of the interest rate policy. This assumption can be regarded as the RBP term in the cross-checking monetary policy labeled by Walsh (2017b). More specifically, such a central bank's concern is captured by

$$L_t^{rbp} = (r_t - r_t^T)^2, \tag{4}$$

where r_t^T is the policy rate indicated by the Taylor rule, which is given as follows:

$$r_t^T = \phi_\pi \pi_t + \phi_x x_t, \tag{5}$$

where ϕ_π is the coefficient of inflation stabilization and ϕ_x is the coefficient of output gap stabilization. The central bank, which is concerned about the robustness of its interest rate policy, attempts to minimize equation (4).

In the case of cross-checking optimal monetary policy, the central bank minimizes the loss function that is a convex combination between L_t^{stab} and L_t^{rob} weighted by γ. Concretely, such a loss function is given as follows:

$$L_t = E_t \sum_{t=0}^{\infty} \beta^t \{(1-\gamma) L_t^{stab} + \gamma L_t^{rbp}\}, \tag{6}$$

policy in terms of the determinacy problem. Therefore, this paper discards the presence of the cost-push shock in the NKPC.

As Tillmann (2012) explains, the parameter γ can be regarded as measuring the share of members of the monetary policy committee referring to the Taylor rule. It turns out that Equation (6) reduces to (3) when $\gamma = 0$.

The central bank that implements a discretionary policy minimizes equation (6) subject to equations (1) and (2). As shown in Tillmann (2012), from the first order conditions of the central bank's optimization problem, the following optimal monetary policy rule is derived:

$$r_t = r_t^T + \frac{1-\gamma}{\gamma A}(\kappa \pi_t + \lambda x_t), \tag{7}$$

where $A = \kappa\phi_\pi + \phi_x + \sigma$.[6] When $\gamma = 1$, the policy rule completely coincides with the implied rate suggested by the Taylor rule, whereas the central bank implements the targeting rule by referring to the implied policy rate from the Taylor rule as long as $\gamma = 1$. It follows that the smaller value of the γ, the stronger is the response to inflation and the output gap. Actually, since a positive demand shock causes the positive response of both π_t and x_t, we can easily confirm $\partial r_t / \partial \gamma = -(\kappa\pi_t + \lambda x_t)/\gamma^2 A < 0$.

3 Benchmark results

This section examines how cross-checking optimal monetary policy changes the condition to attain the REE. While previous studies argue that the Taylor principle must be satisfied to guarantee the unique REE, it remains unclear how the cross-checking optimal rule achieves the unique REE. For instance, is the REE uniquely pinned down when the monetary policy committee decides to refer to the information from the Taylor rule under a discretionary policy? How does the degree of the committee's preferences to the cross-checking policy γ affect the uniqueness of the REE?

To this end, the paper examines the case of the optimal monetary policy with the information from the standard Taylor rule (5). Substituting the optimal policy rule (7)

[6]See Tillmann (2012) for a detailed derivation of this equation.

into equation (1), the system of two endogenous variables x_t and π_t are written as follows:

$$X_t = ME_t X_{t+1} + \mu r_t^n, \tag{8}$$

where $X_t = [x_t \pi_t]'$ and

$$M = \frac{1}{\sigma + \kappa\phi_\pi + \phi_x + (1-\gamma)\gamma^{-1}A^{-1}(\kappa^2 + \lambda)} \begin{bmatrix} \sigma & 1 - \beta(\phi_\pi + (1-\gamma)\gamma^{-1}A^{-1}\kappa) \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_x + (1-\gamma)\gamma^{-1}A^{-1}\lambda) \end{bmatrix}.$$

Note that the form of μ is omitted because it does not affect the following discussion. Since x_t and π_t are endogenous jump variables, both eigenvalues of M should be inside the unit circle for determinacy. Thus, we have the following result:

Proposition 1 *Under the optimal monetary policy rule (7) the necessary and sufficient condition for a unique rational expectations equilibrium is given as follows:*

$$\kappa(\phi_\pi - 1) + (1-\beta)\phi_x + \frac{1-\gamma}{\gamma A}[\kappa^2 + (1-\beta)\lambda] > 0, \tag{9}$$

Proof. The proof of this proposition is provided in Appendix A. ■

This proposition provides the modified Taylor principle. The first and second terms of the left-hand side corresponds to the Taylor rule. This is associated with the information from the Taylor rule. On the other hand, the third term is associated with the condition derived from a discretionary policy. Third term never takes a value less than zero. In other words, condition (9) reinforces the condition of the standard Taylor principle, which is given as follows:

$$\kappa(\phi_\pi - 1) + (1-\beta)\phi_x > 0. \tag{10}$$

This is the case of $\gamma = 1.0$, which corresponds to that of Bullard and Mitra (2002). The Taylor principle requires that the central bank should aggressively raise its interest rate in response to a change in inflation unless it does not consider the stabilization of the output gap (i.e., $\phi_x = 0$).

Interestingly, the analytical result shows that in the case of $\phi_x = 0$, if the central bank puts some weights on the RBP term, the condition (9) is satisfied even when the coefficient ϕ_π is less than unity. As an extreme case, the condition is satisfied even if the central bank does not employ inflation stabilization in the Taylor rule (i.e., $\phi_\pi = 0$). Therefore, the result implies that a combination of targeting rule and an RBP leads to a preferable outcome.

How does the strengthened Taylor rule affect the determinacy regions? The findings of this paper are illustrated using numerical methods, mainly by employing the deep parameters calibrated in Tillmann (2012). The discount factor (β) is set to 0.99, and the inverse of the intertemporal elasticity of consumption (σ) is set to 1.0. The slope coefficient of the NKPC (κ) is set to 0.024 and set the weight on output gap stabilization relative to inflation stabilization in the loss function (λ) to 0.048.

Next, four values of the parameter γ are chosen. First, the case of $\gamma = 1.0$ is set, which corresponds to the determinacy condition under the standard Taylor rule shown in Bullard and Mitra (2002). Second, the parameter γ is set to 0.25.[7] As shown in Tillmann (2012), this value can produce the smallest welfare loss in the model with cross-checking optimal policy. Third, in the case of $\gamma = 0.5$, half of committee members refer to the information from the Taylor rule. Fourth, the parameter γ is set to 0.1.

[Insert Figure 1 around here]

Figure 1 portrays the determinacy condition under a cross-checking optimal monetary policy rule. In an extreme case, regardless of values of both ϕ_π and ϕ_x, the REE is always determinate when $\gamma = 0.01$. This is not a surprising result. When $\gamma = 0.01$, the central bank can respond aggressively to inflation and the output gap because this value suffices to satisfy the condition (9). In other words, this might indicate that when $\gamma = 0.01$, the central bank can almost generate a unique REE through targeting rule without *ex post facto* referring to the information from the Taylor rule.

When the central bank puts a smaller weight on the information from the Taylor rule (i.e., the case of $\gamma = 0.1$), the REE is always determinate in the case where it employs a

[7]Note that in Tillmann (2012), $\gamma = 0.25$ minimizes the welfare loss when both demand and supply shocks coexist.

stronger response to the output gap. When the value of ϕ_x is above approximately 1.8, the REE is always determinate even if the central bank sets ϕ_π to zero.

Unfortunately, even when $\gamma = 0.1$, the central bank needs to put some weight on inflation stabilization in the case of a smaller value of ϕ_x. In contrast to $\gamma = 0.01$, the result shows that even if the central bank employs the cross-checking optimal policy rule, it is possible that the REE is indeterminate if the central bank does not target both inflation and output stabilization.

It follows from Figure 1 that the indeterminacy regions expands as γ approaches unity. The case of the parameter $\gamma = 1$ corresponds to the determinacy regions generated by the standard Taylor rule. This line is satisfied as long as the standard Taylor principle (10) holds. To sum up, a decrease in γ expands the determinacy regions. In other words, a some weight on γ can reinforce the modified Taylor principle.

4 Determinacy and cross-checking monetary policy in an economy with a cost channel

This section extends the benchmark model to the case of a cost channel. The benchmark result in Section 3 is likely to overturn in an economy with a cost channel.[8] Section 4.1 briefly explains the introduction of a cost channel into the standard new Keynesian model. Section 4.2 explores the condition that the REE is uniquely determinate in the case of the cost channel.

4.1 An introduction of the cost channel

Several studies incorporate the role of a cost channel into the standard new Keynesian model. Ravenna and Walsh (2006) show that the presence of a cost channel generates a trade-off between inflation and output stabilization when the central bank implements optimal monetary policy. Chowdhury, Hoffmann and Schabert (2006) argue that the

[8]See Barth and Ramey (2000) and Rabanal (2007) for a detailed discussion of empirical studies of the cost channel.

inflation rate increases in response to a rise in the policy rate after monetary tightening in a model in which cost channels matter.

In a model with a cost channel, the firms borrow the working capital from financial intermediaries to finance the wage bill. In an economy with a cost channel, the term for the nominal interest rate is augmented in the NKPC (e.g., Ravenna and Walsh, 2006; Chowdhury, et al., 2006; Tillmann, 2008). The NKPC in an economy with a cost channel is given by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(x_t + \delta r_t^l), \tag{11}$$

where r_t^l is the lending rate and δ is the presence of the cost channel. When $\delta = 0$, equation (11) reduces to the standard one. As in previous studies for the cost channel, we assume that $\delta = 1$.[9] As shown in Chowdhury et al. (2006) and Tillmann (2009), the lending rate is related to the nominal interest rate:

$$r_t^l = \psi_r r_t, \tag{12}$$

where ψ_r is the degree of incomplete pass-through of the lending rate. In other words, the parameter ψ_r represents the strength of the cost channel (e.g., Chowdhury, et al. 2006; Castelnuovo, 2007; Tillmann, 2009).

The cost channel increases inflation through the supply side effect of monetary policy. If the cost channel dominates the demand channel that monetary tightening induces a decline in inflation rate, an increase in the policy rate generated by monetary tightening results in an increase in inflation. In this case, should the central bank more aggressively raise its policy rate to stabilize the inflation rate? The question is whether a cross-checking monetary policy helps achieve the unique REE in an economy with a cost channel.

In the recent literature, several studies show the determinacy condition in a forward-looking sticky price model with a cost channel. Bruckner and Schabert (2003) find that the cost channel produces an additional upper bound to the coefficient for inflation

[9] See Ravenna and Walsh (2006) and Tillmann (2009) for a detailed discussion of the parameter δ.

stabilization in the Taylor rule. Surico (2008) also finds that a positive weight on the stabilization of the output gap in a monetary policy rule is likely to cause indeterminacy in a model with a cost channel. Furthermore, Llosa and Tuesta (2009) show that standard policies can induce equilibrium indeterminacy and expectational instability under adaptive learning when a cost channel is present.

Finally, in this study the standard loss function is used in the case of the cost channel. In fact, Ravenna and Walsh (2006) show that the central bank's loss function contains the standard policy objectives of both inflation and output gap stabilization.[10] Tillmann (2009) also uses the standard loss function to examine optimal monetary policy with the model uncertainty in a model with a cost-channel. Therefore, the standard loss function is applicable to the case of cross-checking optimal monetary policy in an economy with a cost-channel.

Note that as shown in Woodford (2003), the term of the interest rate stabilization term is include in the central bank's loss function as an additional target variable when the household's utility function depends on real money balances.[11] Therefore, it may be interesting how alternative specifications of the central bank's loss function affect the determinacy condition under cross-checking optimal policy.

4.2 Determinacy and cross-checking optimal rule in the case of a cost channel

In the case of the cost channel, the cross-checking optimal policy rule is derived as follows:

$$r_t = r_t^T + \frac{1-\gamma}{\gamma \tilde{A}}[\kappa(1-\sigma\psi_r)\pi_t + \lambda x_t], \tag{13}$$

where $\tilde{A} = \kappa(1-\psi_r\sigma)\phi_\pi + \phi_x + \sigma$.

In the benchmark new Keynesian model, a smaller value of γ strengthens the response

[10]See Ravenna and Walsh (2006) and Demirel (2013) for a detailed derivation of the central bank's loss function in an economy with a cost channel.

[11]Ida (2018) derives the cross-checking optimal policy rule under interest rate smoothing and examines the determinacy condition under that rule

of the nominal interest rate to fluctuations in endogenous variables. How does the presence of the cost channel affects a change in the nominal interest rate in response to fluctuations in macro-variables? Following equation (13), when both π_t and x_t are positive responses to the positive demand shock, we obtain the following results:

Proposition 2 *Under a positive demand shock, a decline in γ leads to the attenuated response of the nominal interest rate if $1/\sigma < \psi_r$.*

Proof. Differentiating (13) with respect to γ leads to the following result:

$$\frac{\partial r_t}{\partial \gamma} = -\frac{\kappa(1-\sigma\psi_r)\pi_t + \lambda x_t}{\gamma^2 \tilde{A}}. \tag{14}$$

Under a positive demand shock, since a numerator takes a positive value as long as $1/\sigma > \psi_r$, this condition should take a negative value. Therefore, it is possible that a decline in γ induces an attenuated response of the policy rate to endogenous variables for larger values of ψ_r such that $1/\sigma < \psi_r$. ■

This proposition states that if a strong cost-channel is introduced into the economy, a decline in γ counteracts the response of the nominal interest rate under cross-checking policy rule. This is the contrast result to the benchmark case where $\psi_r = 0$. Additionally, this never occurs in Llosa and Tuesta (2009). How does this modification of the policy rule affect equilibrium determinacy?

To answer this question, substituting the optimal policy rule (7) into equation (1) and using equation (11), the system of two endogenous variables x_t and π_t are written as follows:

$$X_t = \tilde{M} E_t X_{t+1} + \mu r_t^n, \tag{15}$$

where

$$\tilde{M} = \frac{1}{\Gamma}\begin{bmatrix} \sigma(1-\kappa\psi_r)(\phi_\pi + (1-\gamma)\gamma^{-1}\tilde{A}^{-1}\kappa)) & 1-(\beta+\kappa\psi_r)(\phi_\pi + (1-\gamma)\gamma^{-1}\tilde{A}^{-1}\kappa) \\ \sigma\kappa(1+\psi_r)(\phi_x + (1-\gamma)\gamma^{-1}\tilde{A}^{-1}\lambda) & \kappa + \sigma\beta + (1+\beta)(\phi_x + (1-\gamma)\gamma^{-1}A^{-1}\lambda) \end{bmatrix},$$

with

$$\Gamma = \sigma + \kappa(1-\psi_r\sigma)\phi_\pi + \phi_x + (1-\gamma)\gamma^{-1}A^{-1}[\kappa^2(1-\psi_r\sigma) + \lambda].$$

Since x_t and π_t are endogenous jump variables, both eigenvalues of $\tilde{M}$ should be inside the unit circle for determinacy. Thus, we have the following result:

Proposition 3 *Let $1/\sigma > 2\psi_r$. In the case of the cost channel, under the optimal monetary policy rule (13) the necessary and sufficient condition for a unique rational expectations equilibrium is given as follows:*

$$\phi_x + \kappa(1-\psi_r\sigma)\phi_\pi + (1-\beta)\sigma + \frac{1-\gamma}{\gamma\tilde{A}}\left[(1-\psi_r\sigma)\kappa^2 + \lambda\right] > 0, \tag{16}$$

$$(1-\beta-\kappa\psi_r)\phi_x + \kappa\phi_\pi + \frac{1-\gamma}{\gamma\tilde{A}}[\kappa^2 + (1-\beta-\kappa\psi_r)\lambda] > 0, \tag{17}$$

$$2(1+\beta)\sigma + \kappa + (1+\beta+\kappa\psi_r)\left(\phi_x + \frac{(1-\gamma)\lambda}{\gamma\tilde{A}}\right) + \kappa(1-2\psi_r\sigma)\left(\phi_\pi + \frac{(1-\gamma)\kappa}{\gamma\tilde{A}}\right) > 0. \tag{18}$$

Proof. The proof of this proposition is provided in Appendix B. ■

Equation (17) is the modified Taylor principle derived Llosa and Tuesta (2009) who show the determinacy condition in the case of the cost channel. According to this condition, if the RBP term is absent, equilibrium indeterminacy might occur when ϕ_x takes a larger value in accordance with an increase in the value of ψ_r. Thus, when $\gamma = 1.0$, in the case where incomplete pass-through of the lending rate to the policy rate is very severe, the REE is indeterminate when the central bank takes a larger weight on output stabilization in the Taylor rule, given the value of ϕ_π. This is consistent with the result of Llosa and Tuesta (2009).[12] Can the third-term derived from the targeting rule ease this problem? In particular, as the benchmark result suggests, does the cross-checking rule help alleviate the indeterminacy problem? Does a larger weight on output stabilization lead to indeterminacy as in the case of Llosa and Tuesta (2009)?

Equation (18) is related to the condition derived by Surico (2008) and Llosa and Tuesta (2009). If the cross-checking optimal monetary policy is absent, as shown by Surico (2008) and Llosa and Tuesta (2009), the condition (18) imposes the upper bound for the response of inflation in the Taylor rule. Under the cost channel, responding too

[12] In their study, the cost channel parameter ψ_r is set to 1.0.

strongly to inflation leads to real indeterminacy. Finally, Equation (16) is also satisfied as long as $1/\sigma > \psi_r$.

It remains unclear how the presence of γ affects the conditions (16) - (18) under several parameterizations of ψ_r and ϕ_π given the value of γ. Numerically showing the regions where the REE is uniquely determinate, since the conditions (16) - (18) are complicated, will answer these questions.

Figure 2 shows the determinacy/indeterminacy regions when the central bank cross-checks its monetary policy in the presence of the cost channel. Surprisingly, in contrast to the benchmark case, when $\gamma = 0.01$, unless the degree of the cost channel is considerably weak, the REE is always indeterminate even if the central bank reacts more aggressively to inflation given the stabilization of the output gap. Thus, as Proposition 2 states, the smaller γ is, the more attenuated is the response of the policy rate in the cross-checking optimal policy rule. Therefore, an extremely small value of γ is likely to violate the conditions (16) - (18) compared to the case where the cross-checking policy is absent.

If the central bank sets γ to 0.1, an aggressive response of the policy rate to inflation leads to the unique REE even when ψ_r takes a larger value. Therefore, compared to the case of $\gamma = 0.01$, $\gamma = 0.1$ expands the determinacy regions. However, this prescription is overturned when $\gamma = 0.25$. Thus, as shown in Figure 2, the upper bound for the response of inflation in the Taylor rule is required to guarantee the unique REE as ψ_r takes above 3.0. This upper bound is more severer as the parameter γ takes a larger value, whereas a larger value of γ forces the central bank to employ the Taylor rule that ϕ_π is set to above unity. When $\gamma = 1.0$, the REE is always indeterminate when ψ_r takes a value around 5.0.

[Insert Figure 2 around here]

This result is intuitive. As shown in Proposition 2, a change in γ weakens the response of the interest rate to inflation and the output gap once a strong cost channel is introduced into the economy. Therefore, if the central bank cross-checks its monetary policy, given the output gap stabilization a higher response to inflation rate may be required to attain the unique REE. In other words, a larger value of ϕ_π compensates the attenuated response of the interest rate induced from the cross-checking monetary policy in the case of the cost channel. Tillmann (2012) argues that a smaller weight on γ can improve social welfare, whereas our result shows that such a parameterization can easily make the REE indeterminate. In particular, unless the degree of the cost channel is weak, the central bank that employs $\gamma = 0.01$ can be prone to cause multiple equilibria even though it employs any value of ϕ_π.

[Insert Figure 3 around here]

As shown by Surico (2008) and Llosa and Tuesta (2009), in a model with the cost channel, a larger value of the output gap stabilization in the Taylor rule is likely to induce equilibrium indeterminacy. Does their result hold in the case of a cross-checking optimal monetary policy? Figure 3 portrays how determinacy/indeterminacy regions change as the parameter ϕ_x increases in the case of $\gamma = 0.1$.[13] ϕ_x is allowed to to take negative values. This is because the condition (17) may be easily satisfied if the central bank sets ϕ_x to negative values for any large values of ψ_r. In this case, the REE is determinate if ϕ_π is above 1.0 in the case of $\psi_r < 2.0$. The REE is indeterminate, however, even if ϕ_π takes larger values in the case of $\psi_r > 3.0$.

Compared to the case where the central bank employs the negative response to the output gap (i.e., $\phi_x = -0.5$), the central bank that sets ϕ_x to zero can expand the regions where the REE is uniquely determinate. However, the upper bound for inflation response is required as ψ_r takes a larger value. When $\phi_x = 0.5$, for any larger value of ψ_r, the upper bound for inflation stabilization disappears, whereas the lower bound for it increases. Interestingly, the indeterminacy regions expands again when the central bank puts a larger weight on the output gap stabilization. In particular, when $\phi_x = 2.0$, the REE is always indeterminate even if the central bank tries to respond more aggressively to inflation in the case where ψ_r is above around 2.0.

[Insert Figure 4 around here]

[13] I abstract the result in the case of $\gamma = 0.01$. This is because we can easily reach the inference that under this value the REE is almost all indeterminate.

Figure 4 shows the effect of a change of ϕ_x on the determinacy area where ϕ_π and ψ_r change in the case of $\gamma = 0.5$. In contrast to the case of $\gamma = 0.1$, a negative response of the output gap almost results in multiple equilibria. In addition, even in the case of no output gap response in the Taylor rule, the determinacy regions are smaller in the case of $\gamma = 0.5$ than in the case of $\gamma = 0.1$.

Furthermore, in the case of $\phi_x = 0.5$, the severe upper bound for inflation stabilization is introduced when $\gamma = 0.5$. On the other hand, it follows from Figure 4 that the lower bound for it seems to be eased in that case. Interestingly, compared to the case of $\gamma = 0.1$, the determinacy regions expand as the parameter ϕ_x takes a larger value. Even in the case of a larger value of ψ_r, the unique REE is guaranteed if the central bank puts a sufficiently larger weight on inflation stabilization in the Taylor rule.

In sum, in contrast to the benchmark case, a smaller weight on the cross-check monetary policy is likely to generate equilibrium indeterminacy in the case of the cost channel. Regardless of the value of γ, in contrast to the result of Llosa and Tuesta (2009), for a given value of ϕ_x, an increase in ϕ_π expands the determinacy regions as long as the cost channel has a negligible effect on the economy (i.e., a smaller value of ψ_r). The parameter γ matters considerably, however, because even for a strong response of inflation in the Taylor rule a larger value of ϕ_x is likely to make the REE indeterminate as ψ_r takes a larger value. This result appears to be more severe in the case of a smaller value of γ.

5 Forward-looking instrument rule and cross-checking optimal monetary policy

This section examines how the determinacy of REE is affected when the central bank, which cross-checks its monetary policy, employs the following forward-looking Taylor rule:

$$r_t^T = \phi_\pi E_t \pi_{t+1} + \phi_x E_t x_{t+1}. \tag{19}$$

Bullard and Mitra (2002) show that in the standard new Keynesian model, $\phi_\pi > 1$, whereas too large a value of ϕ_π makes the REE indeterminate. In addition, as shown by

Llosa and Tuesta (2009), an introduction of the cost channel makes the determinacy conditions under a forward-looking Taylor rule more complicated. Our question is whether the presence of the forward-looking Taylor rule when the central bank cross-checks its monetary policy helps ease the indeterminacy problem.

The cross-checking optimal monetary policy rule becomes[14]

$$r_t = \frac{1-\gamma}{\gamma}[\kappa(1-\sigma\psi_r)\pi_t + \lambda x_t] + \phi_\pi E_t \pi_{t+1} + \phi_x E_t x_{t+1}. \tag{20}$$

Compared to the rule (13), the coefficient $\tilde{A}$ disappears because the forward-looking macro-variables in equation (19) are given when the central bank implements a discretionary policy. In contrast to rules suggested in numerous previous studies, this rule contains not only the contemporaneous response of endogenous variables but also the forward-looking response of those. To best of my knowledge, none of the studies attempts to assess this type of the rule (7). Our question is whether this type of the rule can attain a desirable outcome in that it uniquely achieves the REE.

To answer this question, substituting the optimal policy rule (20) into equation (1) and using equation (11), the system of two endogenous variables x_t and π_t are written as follows:

$$X_t = P E_t X_{t+1} + \mu r_t^n, \tag{21}$$

where

$$P = \frac{1}{\Gamma_f}\begin{bmatrix} \frac{\sigma(1-\kappa\psi_r)(1-\gamma)\kappa^2(1-\sigma\psi_r)^2}{\gamma} - \phi_x & 1 - \frac{(\beta+\kappa\psi_r)(1-\gamma)\kappa^2(1-\sigma\psi_r)^2}{\gamma} - \phi_\pi \\ \frac{\sigma\kappa(1+\psi_r)(1-\gamma)\lambda}{\gamma} - \kappa(1-\sigma\psi_r)\phi_x & \kappa + \sigma\beta + \frac{(\kappa\psi_r+\beta)(1-\gamma)\lambda}{\gamma} - \kappa(1-\sigma\psi_r)\phi_\pi \end{bmatrix},$$

with

$$\Gamma_f = \sigma + \frac{(1-\gamma)\lambda}{\gamma} + \frac{(1-\gamma)\kappa^2(1-\sigma\psi_r)^2}{\gamma}.$$

As mentioned repeatedly, since x_t and π_t are endogenous jump variables, both eigenvalues of P should be inside the unit circle for determinacy.[15]

[14] This paper examines the determinacy condition under the forward-looking rule in the case of the cost channel. The result of this rule in the benchmark case is available on request.

[15] Unfortunately, this paper has difficulty in analytically deriving the determinacy condition under Equation (21). Deriving the analytical expressions will be future works.

Figure 5 shows the the effect of a change of ϕ_x on the determinacy area where ϕ_π and ψ_r change in the case of $\gamma = 0.1$. Compared to the case of the contemporaneous Taylor rule, the result appears to remain unchanged when $\phi_x = 0$. However, as ϕ_x takes a larger value, the determinacy regions shrink as the stronger cost channel is introduced into the economy.

[Insert Figure 5 around here]

Figure 6 shows the the effect of a change of ϕ_x on the determinacy area where ϕ_π and ψ_r change in the case of $\gamma = 0.5$. In contrast to the case of Figure 4, the determinacy regions shrink when $\phi_x = 0$. This is also comparable with the case of $\gamma = 0.1$. In the case of the forward-looking rule under cross-checking policy, for given $\phi_x = 0$, a larger value of γ drastically narrows the determinacy regions for a larger value of ψ_r.

[Insert Figure 6 around here]

Compared to the cases of Figures 4 and 5, the determinacy regions expands again when the central bank employs a stronger response to inflation as ψ_r takes a larger value. This prescription no longer holds, however, when ϕ_x takes a larger value. As shown in Figures 3 and 4, even under the forward-looking cross-checking rule, a larger value of ϕ_x is likely to attain the real indeterminacy as the degree of the cost channel is amplified.

6 Conclusions

How should the central bank implement optimal monetary policy with the consideration of the robustness of the model? As suggested by Walsh (2017b) and Tillmann (2012), the central bank may refer to monetary policy rules that are robust to the misspecification of the model because it faces the model uncertainty that the true model is not known with certainty. This assertion justifies the significance of cross-checking optimal monetary policy. These studies may address the effectiveness of cross-checking optimal monetary policy, whereas it is unclear whether such a rule can lead to a unique REE.

This paper has studied equilibrium determinacy under cross-checking optimal monetary policy with discretion. The main message of this paper is twofold. First, this paper shows the condition that a unique REE is achieved under a cross-checking optimal policy rule in the standard new Keynesian model. Interestingly, a decrease in a positive weight on a cross-checking rule expands the determinacy regions. Surprisingly, when the central bank puts a considerably smaller weight on the information from the Taylor rule, the REE is always determinate when it employs a stronger response to the output gap. There are cases that the REE is always determinate even if the central bank sets the coefficient for inflation stabilization to zero in the Taylor rule. This result is the natural extension of Tillmann (2012) and Walsh (2017b).

Second, this paper has extended the determinacy condition of cross-checking optimal monetary policy rule in the case of the cost channel. The central bank, which employs cross-checking optimal monetary policy rule, can achieve the unique REE so long as the degree of a cost channel is small. Surprisingly, this result is overturned when a strong cost channel is introduced into the economy, a cross-checking optimal rule fails to attain the unique REE. In the case of the cost channel the REE is likely to be indeterminate as the central bank puts on a higher weight on the stabilization of the output gap. This result is more severe when the central bank puts a smaller weight on the cross-checking term. The result of this paper implies that as a smaller weight on the RBP term leads to an attenuated response to the inflation rate, such a policy results in equilibrium indeterminacy even if the instrument rule satisfies the Taylor principle.

Finally, suggestions for future research are included. This paper has examined the optimal monetary policy rule with cross-checking in a closed economy. We may consider the role of cross-checking optimal monetary policy in a small-open economy (c.f., Clarida, Gali, and Gertler , 2002; Gali and Monacelli, 2005). If the cross-checking rule contains the rules, such as exchange rate targeting, how does the determinacy condition change (c.f., Linnemann and Schabart, 2006)? While the determinacy analyses under cross-checking optimal monetary policy have been the focus, it is unclear whether the expectations of the private sector, which can be regarded as the so-called E-stability condition, is also stable even when a cross-checking optimal policy rule leads to the unique REE (c.f., Evans and Honkaphoja, 2001; Evans and Honkapohja, 2003; Llosa and Tuesta, 2009). Moreover, we may also apply the cross-checking monetary policy to several cases, such as asset price fluctuations, interest rate smoothing, and so on. Does the cross-checking optimal monetary policy always achieve the unique REE under such conditions? Does the cross-checking optimal policy leads to the unique REE even in the medium-scale new Keynesian model (Christiano, Eichenbaum and Evans, 2005; Smets and Wouters, 2007)? These topics may be interesting subjects for future research.

A Appendix

A.1 Appendix A: Proof of Proposition 1

The characteristic equation of polynomial is given by $P(\lambda) = \lambda^2 - tr(M)\lambda + det(M)$, where

$$
\begin{aligned}
tr(M) &= \frac{\sigma + \kappa + \beta(\sigma + \phi_x + (1-\gamma)\gamma^{-1}A^{-1}\lambda)}{\sigma + \kappa\phi_\pi + \phi_x + (1-\gamma)\gamma^{-1}A^{-1}(\kappa^2 + \lambda)}, \\
det(M) &= \frac{\sigma\beta}{\sigma + \kappa\phi_\pi + \phi_x + (1-\gamma)\gamma^{-1}A^{-1}(\kappa^2 + \lambda)},
\end{aligned}
$$

Both eigenvalues of M are inside the unit circle if both of the following conditions hold:

$$
\begin{aligned}
&(i)\ |det(M)| < 1, \\
&(ii)\ |tr(M)| < 1 + det(M).
\end{aligned}
$$

Condition (i) leads to the following inequality:

$$
\kappa\phi_\pi + \phi_x + \frac{1-\gamma}{\gamma A}(\kappa^2 + \lambda) + (1-\beta)\sigma > 0,
$$

which is trivially satisfied since $\beta < 1$. Condition (ii) implies equation (9).

A.2 Proof of Proposition 3

The characteristic equation of polynomial is given by $P(\lambda) = \lambda^2 - tr(\tilde{M})\lambda + det(\tilde{M})$, where

$$tr(\tilde{M}) = \frac{\sigma[1 - \kappa\psi_r(\phi_\pi + (1-\gamma)\gamma^{-1}\tilde{A}^{-1}\kappa)] + \kappa(1+\psi_r)(\phi_x + (1-\gamma)\gamma^{-1}\tilde{A}^{-1}\lambda) + \kappa + \sigma\beta}{\sigma + \kappa(1-\psi_r\sigma)(\phi_\pi + (1-\gamma)\gamma^{-1}\tilde{A}^{-1}\kappa) + \phi_x + (1-\gamma)\gamma^{-1}\tilde{A}^{-1}\lambda},$$

$$det(\tilde{M}) = \frac{\sigma\beta}{\sigma + \phi_x + \kappa(1-\psi_r\sigma)\phi_\pi + (1-\gamma)\gamma^{-1}\tilde{A}^{-1}[\kappa(1-\psi_r\sigma) + \lambda]},$$

Both eigenvalues of $\tilde{M}$ are inside the unit circle if both of the following conditions hold:

$$(i)\ |det(\tilde{M})| < 1,$$
$$(ii)\ |tr(\tilde{M})| < 1 + det(\tilde{M}).$$

Condition (i) leads to equation (16). Since $\beta < 1$ and $0 < \gamma < 1$, this condition is always satisfied as long as $1/\sigma > \psi_r$.

To check whether the condition (ii) is guaranteed, we must confirm it under the following cases:

(Case 1) $\sigma + \kappa + \sigma\beta + (\beta + \kappa\psi_r)(\phi_x + (1-\gamma)\gamma^{-1}\tilde{A}^{-1}\lambda) > \sigma\kappa\psi_r(\phi_\pi + (1-\gamma)\gamma^{-1}\tilde{A}^{-1}\kappa)$
Under this inequality, the condition (ii) leads to equation (17).

(Case 2) $\sigma + \kappa + \sigma\beta + (\beta + \kappa\psi_r)(\phi_x + (1-\gamma)\gamma^{-1}\tilde{A}^{-1}\lambda) < \sigma\kappa\psi_r(\phi_\pi + (1-\gamma)\gamma^{-1}\tilde{A}^{-1}\kappa)$
Under this inequality, the condition (ii) leads to equation (18), which is always satisfied as long as $1/2\sigma > \psi_r$.

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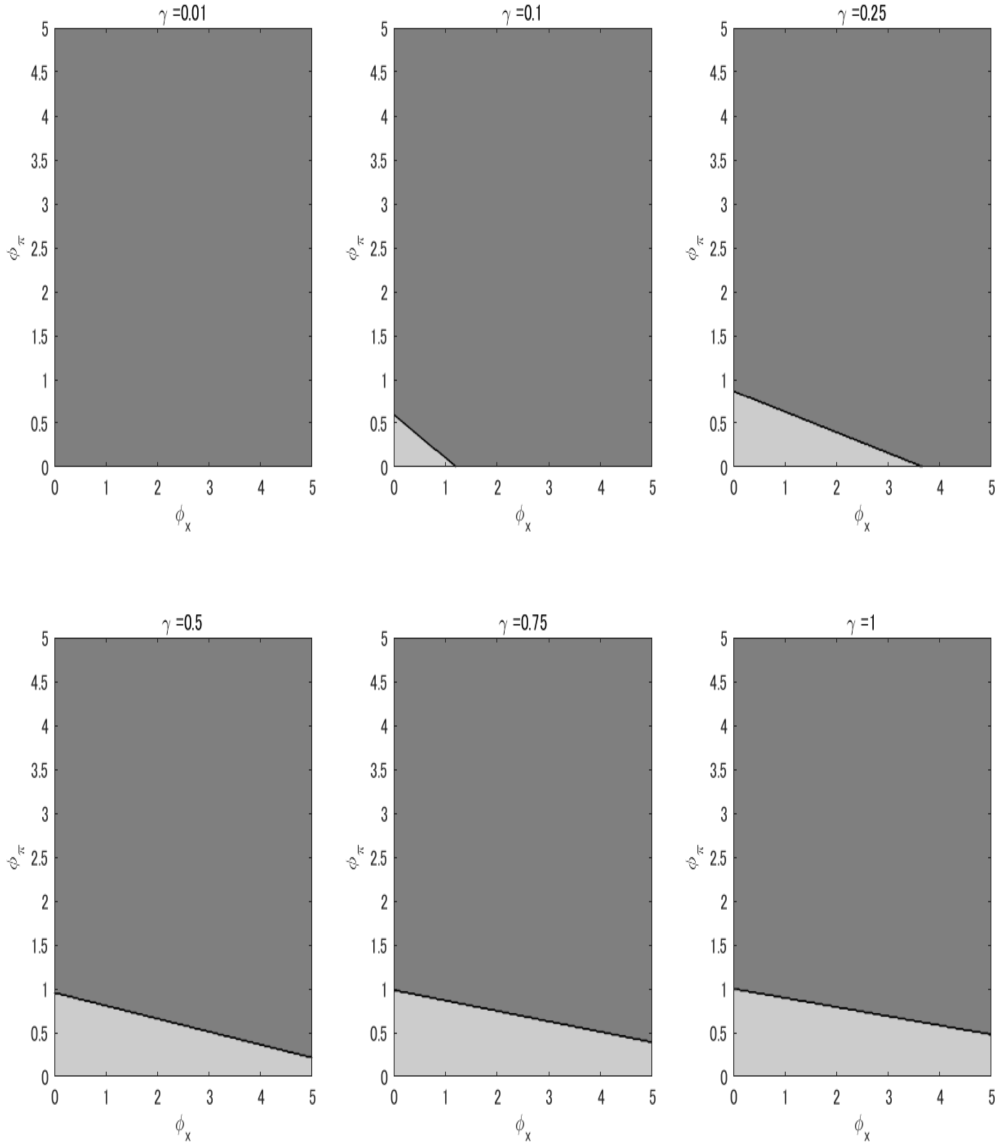
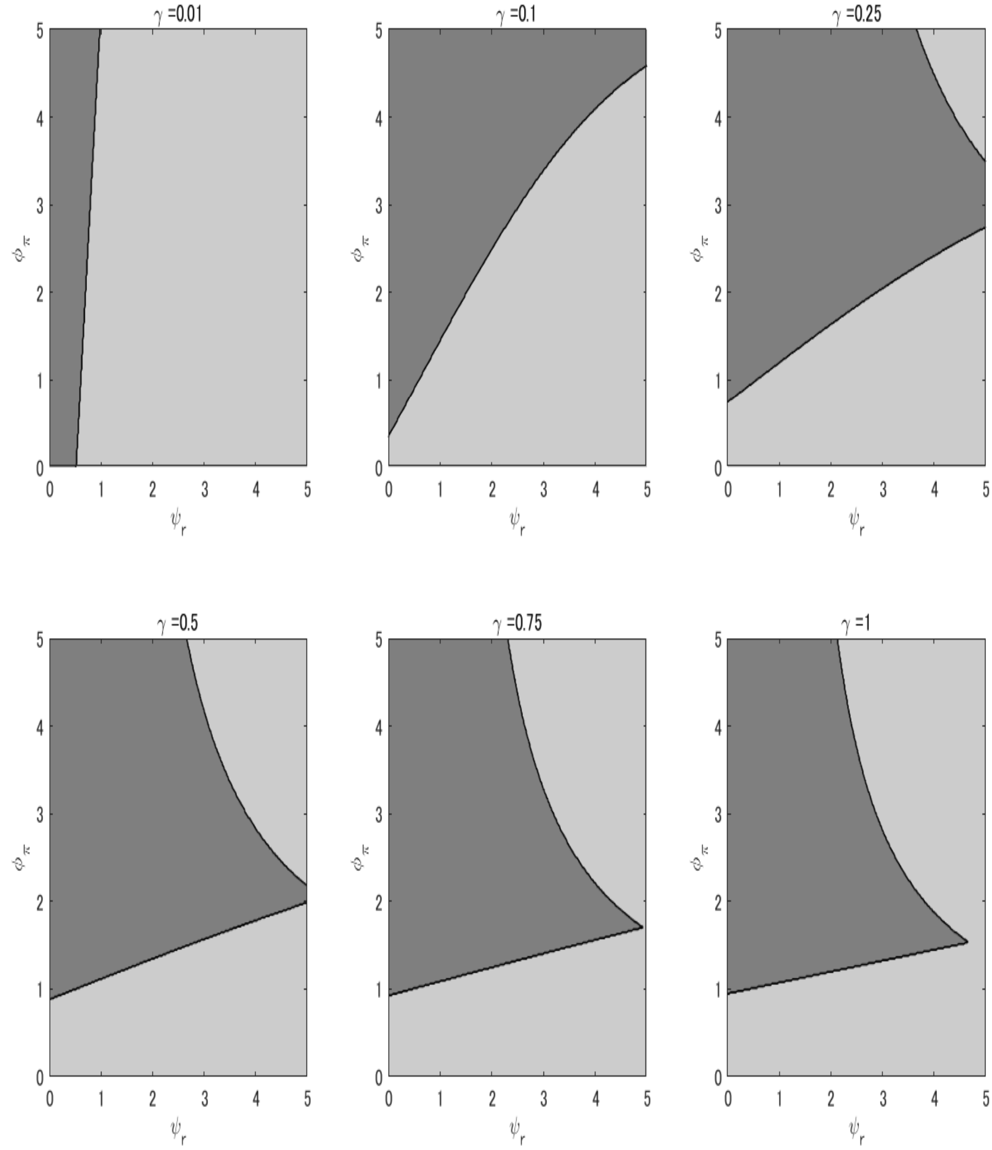
Figure 1: Determinacy and cross-checking monetary policy



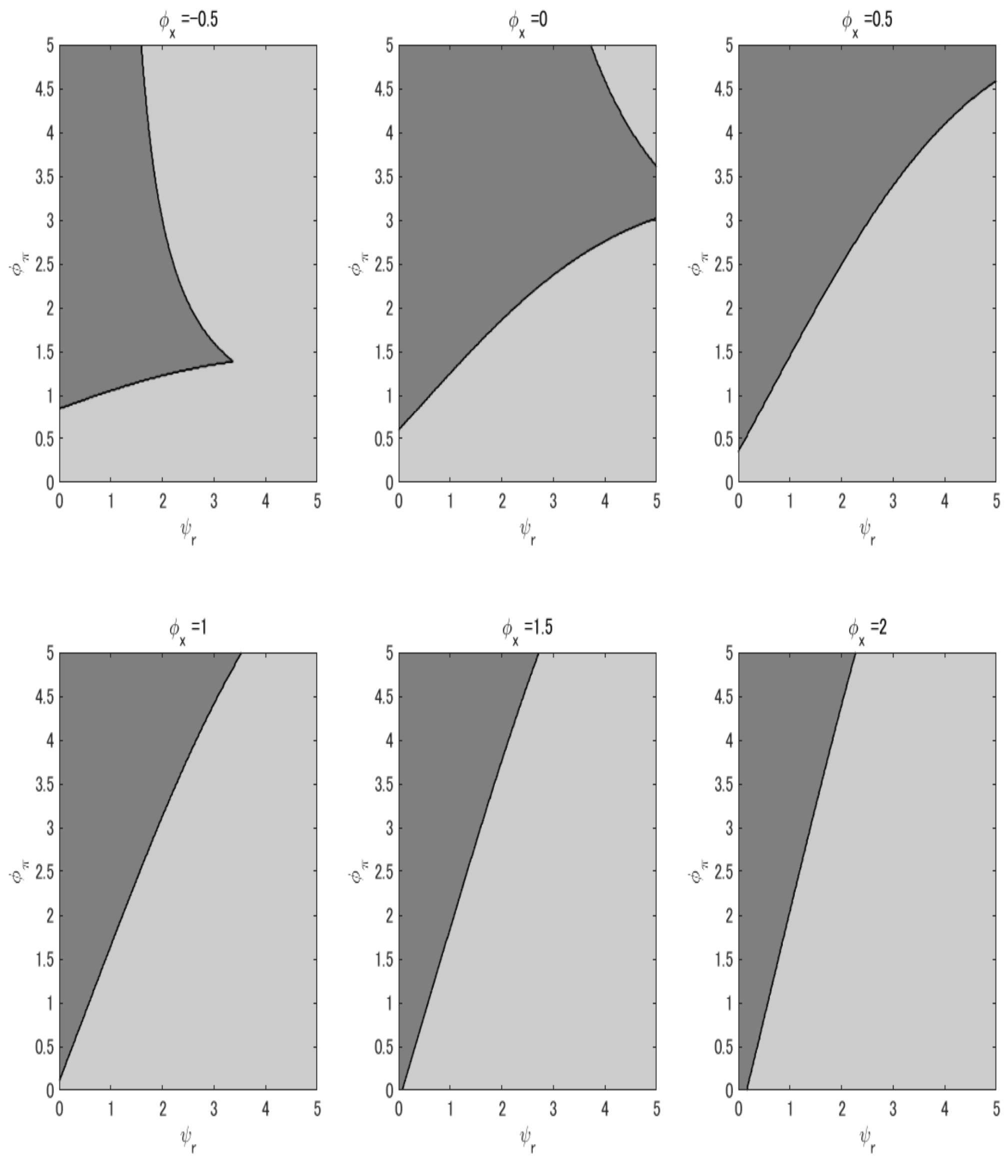
Note: Dark-shaded areas: determinacy regions; Light-gray areas: indeterminacy regions

Figure 2: Determinacy and cross-checking monetary policy in the case of a cost channel



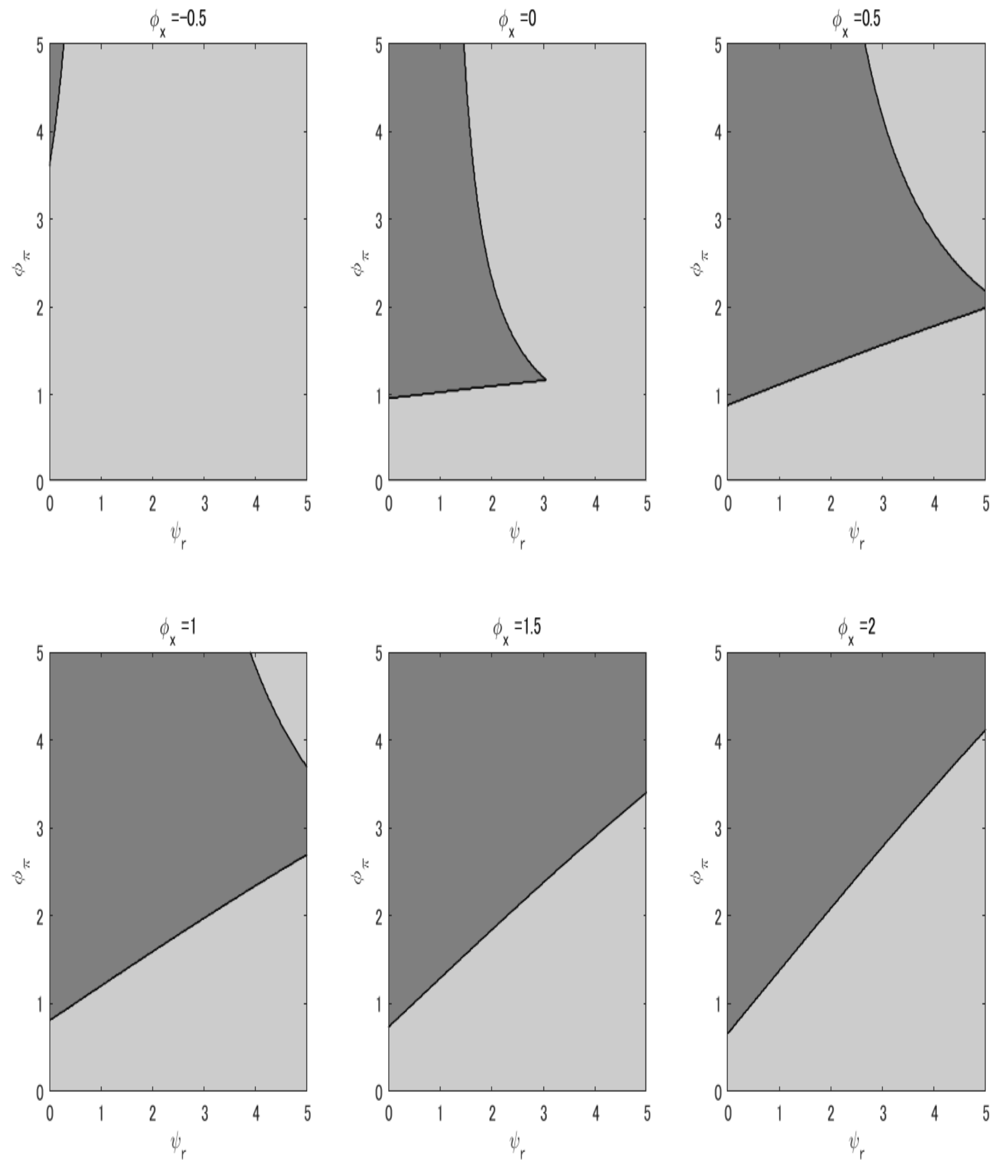
Note: Dark-shaded areas: determinacy regions; Light-gray areas: indeterminacy regions

Figure 3: Determinacy and cross-checking monetary policy in the case of the cost channel: A change of ϕ_x when $\gamma = 0.1$



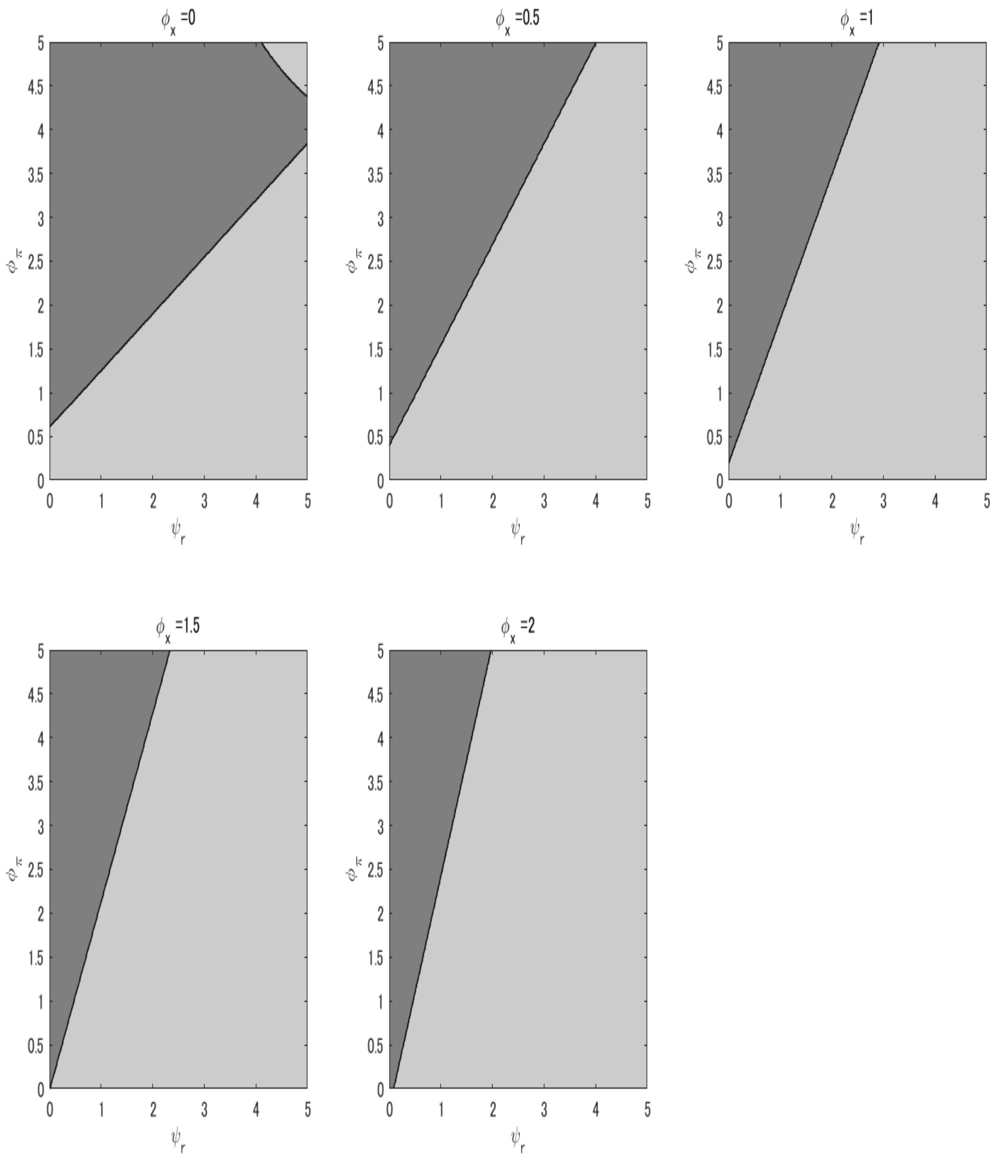
Note: Dark-shaded areas: determinacy regions; Light-gray areas: indeterminacy regions

Figure 4: Determinacy and cross-checking monetary policy in the case of the cost channel: A change of ϕ_x when $\gamma = 0.5$



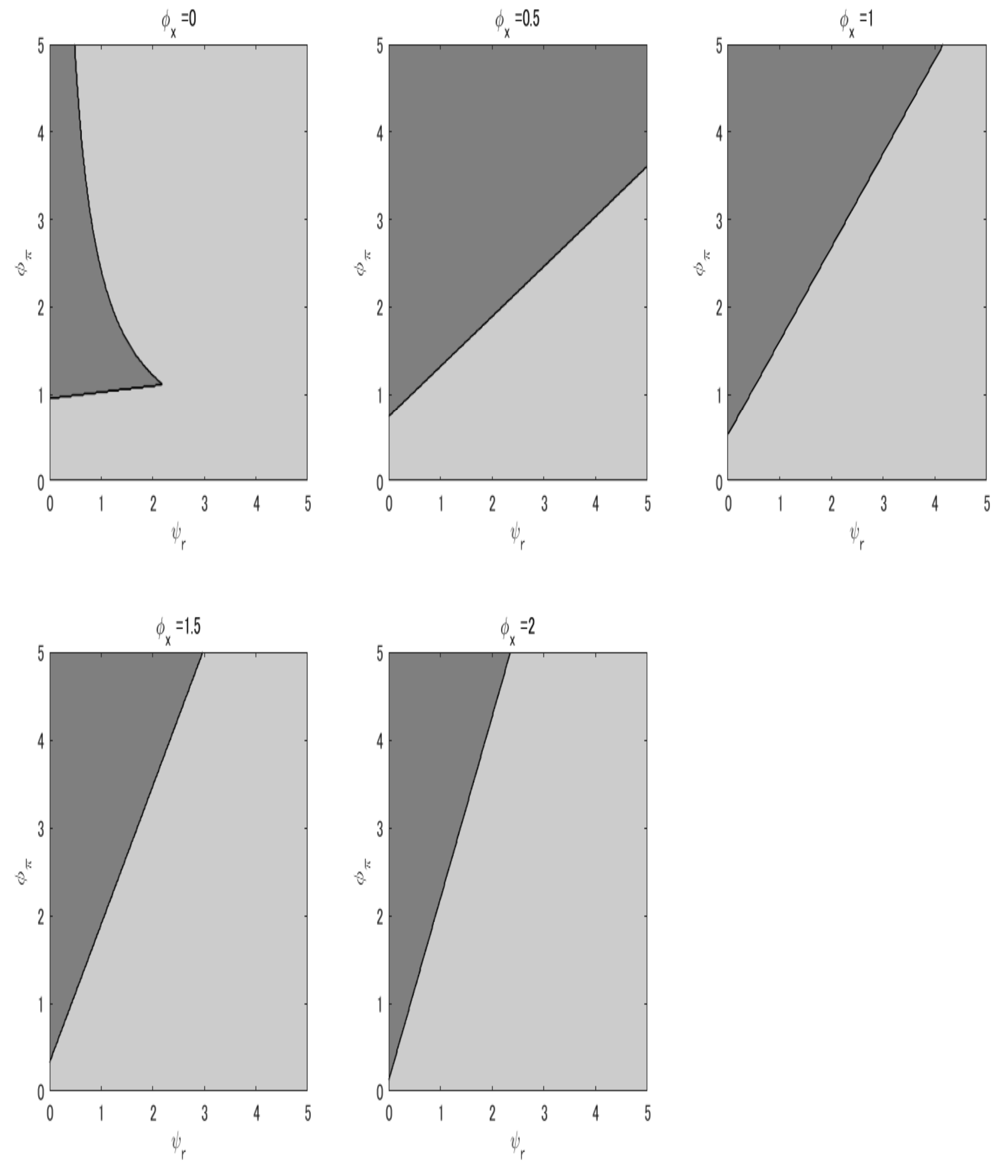
Note: Dark-shaded areas: determinacy regions; Light-gray areas: indeterminacy regions

Figure 5: Determinacy and cross-checking monetary policy in the case of the cost channel: Forward-looking Taylor rule when $\gamma = 0.1$



Note: Dark-shaded areas: determinacy regions; Light-gray areas: indeterminacy regions

Figure 6: Determinacy and cross-checking monetary policy in the case of the cost channel: Forward-looking Taylor rule when $\gamma = 0.5$



Note: Dark-shaded areas: determinacy regions; Light-gray areas: indeterminacy regions