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Liquidity constraints and optimal monetary policy in a currency union

Daisuke Ida

ida-dai@andrew.ac.jp

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桃山学院大学
Momoyama Gakuin University

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Liquidity constraints and optimal monetary policy in a currency union

Daisuke Ida*

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Abstract

This paper examines optimal monetary policy in a two-country new Keynesian (NK) model with liquidity constraints. We focus on the case of a currency union. We derive a well-defined loss function derived using a quadratic approximation of the household utility function. The result demonstrates that given liquidity constraints in the home country, optimal monetary policy implies that the response of macroeconomic variables within a currency union to a foreign structural shock is dampened as foreign liquidity constraints tighten. We argue that the costs of discretionary policy are generally not large in a currency union in which liquidity constraints exist in both countries. This result does not change unless nominal wages remain considerably sticky in both countries.

JEL codes: E52; E58; F41

Keywords: Optimal monetary policy; liquidity constraints; currency union; nominal wage rigidity; two-country new Keynesian (NK) model

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*Faculty of Economics, Momoyama Gakuin University, 1-1, Manabino, Izumi, Osaka 594-1198, Japan. Tel.: +81-725-54-3131. E-mail: ida-dai@andrew.ac.jp

1 Introduction

The purpose of this study is to investigate the impact of liquidity constraints on optimal monetary policy in a currency union model. While the standard new Keynesian (NK) model has been analyzed with the assumption of representative households (Woodford, 2003), the importance of liquidity constraints has been pointed out in several studies (Campbell and Mankiw, 1989, Bilbiie, 2008, Galí, López-Salido and Javier, 2004, 2007). Liquidity constraints refer to the fact that households with limited access to financial assets are unable to intertemporally behave according to an optimal decision based on a consumption Euler equation. Therefore, their consumption is constrained by their disposable income in the current period. In this paper, I refer to these households as "non-Richardian households" whereas those with free access to financial markets are referred to as "Richardian households".[1] If optimal intertemporal consumption behavior cannot be achieved, then a household's current consumption will be tied to its current disposable income. Therefore, the spillover path of monetary policy into the real economy naturally differs in the presence of liquidity constraints. The impact of liquidity constraints on monetary policy has been discussed in a closed economy model, but not in an open economy model.

Our motivation is based on the empirical fact that the share of non-Ricardian households differs across the member countries in the Euro area. For instance, Kaplan, Violante and Weidner (2014) pointed out that for the United States (US), Canada, Australia, the United Kingdom (UK), Germany, France, Italy, and Spain, the share of households that face liquidity constraints is between 20% and 35%. Almgren, Gallegos, Kramer and Lima (2019) reported that the fraction of households that faces liquidity constraints ranges from 10% in Malta to almost 65% in Latvia. We relate the motivation of this paper to these empirical findings in terms of deriving the optimal monetary policy for a currency union. We also focus on the role of nominal wage rigidity in a currency union. Several studies have examined the role of nominal wage rigidity in a dynamic stochastic general equilibrium (DSGE) model (Christiano, Eichenbaum and Evans, 2005). Recently, Iwasaki, Muto and Shintani (2021) argued the significant role of wage inflation in Europe using a non-linear DSGE model. In addition, Palomino, Rodríguez and Sebastian

[1] Following Bilbiie (2008), unless otherwise noted, I refer to households that do not have free access to financial assets as "non-Ricardian households". Several naming schemes for liquidity-constrained households have been proposed. For instance, Galí *et al.* (2007) refer to liquidity-constrained households as "rule of thumb households".

(2020) focused on the effect of the lockdown and social distancing measures precipitated by the coronavirus disease 2019 (COVID-19) pandemic on wage inequality in Europe. These studies enable me to address the role of nominal wage rigidity in a currency union.

This study examines optimal monetary policy in a two-country NK model under a currency union considering both liquidity constraints and nominal wage rigidity. Specifically, the model focuses on a currency union in the standard two-country model developed by Clarida, Galí and Gertler (2002) and Benigno (2004) and introduces liquidity constraints. As mentioned earlier, the fact that liquidity constraints differ between the two countries is the motivation for this paper. We also examine different degrees of wage stickiness in Europe and introduce wage stickiness to the two countries based on Erceg, Henderson and Levin (2000). To the best of our knowledge, no study has attempted such an extension.

The contributions of this study are as follows. First, we demonstrate that consumption risk-sharing in the two countries is affected by their differing degrees of liquidity constraint. Despite the fact that in this paper, Ricardian households have complete risk-sharing both domestically and internationally, the level of aggregate consumption in one country is affected by the share of liquidity-constrained households in both countries.

Second, in analyzing the optimal monetary policy, a well-defined loss function is derived by a quadratic approximation of the household utility function. According to this, in addition to the traditional targets of inflation and the output gap, the stabilization of wage inflation by wage stickiness is added to the loss function of each country. In addition, the stabilization of the real wage gap needs to be considered since non-Ricardian households coexist with wage stickiness in both countries. Therefore, since the policy tool in a currency union is a single union-wide nominal interest rate, it is shown that stabilization policy, among various macroeconomic variables, must be sought using this tool.

Third, given the share in liquidity constraints in the home country, optimal monetary policy implies that, under a currency union model, the response of macroeconomic variables to foreign structural shocks is attenuated as foreign liquidity constraint tightens. Since liquidity constraints suppress the path to the real economy through intertemporal changes in consumption (i.e., the interest rate channel), optimal monetary policy that introduces a large degree of inertia to the commitment solution is generally not introduced.

Fourth, importantly, the costs from discretionary policy are not generally greater in a currency union with liquidity constraints than those under a standard NK model. This result is in stark contrast to previous studies that have indicated substantial costs arising from discretionary policy. We actually calculated the cost of implementing discretionary policy in this model. As the share of non-Ricardian households increases in both countries, the costs from discretionary policy also increase. However, the costs are not as great as those claimed in the closed economy NK model with heterogeneous agents, with a benchmark of, at most, 6%.

Fifth, in terms of wage stickiness, we also find that nominal wage stickiness in both countries plays a significant role in determining the costs arising from discretion. In particular, unlike previous studies that have pointed out the substantial costs of discretion, the costs of discretionary policy are not greater than those of commitment policy, as long as nominal wages are highly sticky in both countries. However, the costs of discretionary policy become large under a reasonable calibration of liquidity constraints in both countries, if the home country's nominal wage rigidity is more flexible and the foreign country's nominal wage is stickier.

The structure of this paper is as follows. Section 2 briefly reviews the related literature. Section 3 provides a description of the two-country NK model with liquidity constraints. This paper focuses on the derivation of the two-country model under a currency union. Section 4 discusses optimal monetary policy in the model. Section 5 explains the deep parameters calibrated in the paper and reports the main results. Section 6 provides deeper insights into the role of liquidity constraints and wage stickiness in a currency union. Section 7 briefly concludes. The Appendix provides a detailed derivation of the central bank's loss function, derived by a second-order approximation of the household utility function in a currency union.

2 Related literature

This section briefly reviews previous studies in terms of the effect of liquidity constraints on monetary policy in an NK model. More concretely, in Section 2.1, we review the impact of liquidity constraints on monetary policy. Section 2.2 discusses the role of liquidity constraints in open economy models. Section 2.3 addresses the contribution of this paper to previous studies.

2.1 Impact of liquidity constraints on monetary policy

The purpose of this paper is to show how the existence of non-Ricardian households theoretically affects optimal monetary policy in a currency union model. First, the impact of liquidity constraints on monetary policy has been examined by several previous studies (Airaudo, 2013, Amato and Labach, 2003, Bilbiie, 2008, Galí *et al.*, 2004). Unlike the standard NK model, the presence of liquidity constraints significantly affects the uniqueness of the rational expectations equilibrium (Airaudo, 2013, Amato and Labach, 2003, Bilbiie, 2008, Galí *et al.*, 2004) and optimal monetary policy (Amato and Labach, 2003, Bilbiie, 2008). In addition, Galí *et al.* (2007) demonstrated a mechanism by which the fiscal policy puzzle-which implies that an increase in government spending leads to a decline in aggregate consumption-is eliminated by assuming the presence of non-Ricardian households in the standard NK model. Colciago (2011) focused on the role of nominal wage rigidity in an NK model with liquidity constraints. Areosa and Areosa (2016) examined optimal monetary policy when the presence of liquidity constraints generated income inequality as measured by the Gini coefficient. Ascari, Colciago and Rossi (2017) explored the effect of liquidity constraints on optimal monetary policy in an economy with nominal price and wage rigidity.

2.2 The role of liquidity constraints in an open economy

How important are liquidity constraints considered to be in an open economy? A number of studies have analyzed optimal monetary policy in a two-country NK model (Benigno, 2004, Benigno and López-Salido, 2006, Clarida *et al.*, 2002, Pappa, 2004). In an open economy, the key to international monetary transmission can be captured by the presence of international consumption risk-sharing. Benigno (2004) and Benigno and López-Salido (2006) examined optimal monetary policy in a currency union. Groll and Monacelli (2020) explored the cost of implementing a currency union compared to implementing exchange rate float in a two-country NK model.[2] Clarida *et al.* (2002) showed that, given the existence of a complete market in

[2] Groll and Monacelli (2020) compared the costs of a currency union with the costs of an exchange rate float in both the discretionary and commitment cases. When the terms of trade are characterized by inertial behavior, the home and foreign central banks that conduct monetary policy with discretion can derive welfare benefits by adopting a currency union. This comparison is a very interesting topic, but the issue is beyond the scope of this paper.

which the home and foreign households can trade state-contingent securities domestically and internationally, the home and foreign countries will have identical consumption, under the assumption of producer currency pricing and no home bias. They demonstrated the gains from international policy coordination when home and foreign central banks jointly implemented an optimal discretionary policy. Pappa (2004) examined the gain from policy coordination in a two-country NK model in which the consumption basket was constructed by a constant elasticity of substitution (CES) aggregate. Campolmi (2014) and Rhee and Turdaliev (2013) focused on the role of nominal wage rigidity in a small-open NK model. They showed that stabilizing consumer price index (CPI) inflation led to outcomes preferable to those under producer price inflation (PPI) inflation targeting.[3]

To the best of my knowledge, no previous studies have focused on the role of non-Ricardian households in an open economy model. The related studies are referred to by Bhattarai, Lee and Park (2015) and Palek and Schwanebeck (2019). Following Cúrdia and Woodford (2016), Bhattarai *et al.* (2015) examined optimal monetary policy in a two-country model with a currency union in which the incomplete consumption risk-sharing condition held in international asset markets. Palek and Schwanebeck (2019) examined optimal monetary policy in a currency union model in which the bank lending channel played a significant role in considering the performance of optimal monetary policy. Our goal is to bridge the gap between these previous studies and the empirical facts supporting the existence of liquidity constraints in aspects of an open economy.

2.3 Contribution of this paper to previous studies

The main contribution of this paper is to thoroughly examine the impact of liquidity constraints on optimal monetary policy in an NK model under a currency union. As mentioned previously, the problem of liquidity constraints has not been addressed in the two-country model, to the best of our knowledge. In fact, it does not explicitly derive a well-defined loss function and calculate the welfare costs of discretionary policy. The results of this analysis show that the costs of discretionary policy are not generally great in a currency union in which liquidity constraints exist in both countries. This result is captured by country-specific differences in

[3]Note that since I focus on the case for optimal monetary policy, I do not consider specifically which inflation central banks should target using simple rules.

the share of non-Ricardian households in the currency union.

Our paper is deeply related to Campolmi (2014), Galí and Monacelli (2016), Groll and Monacelli (2020), and Rhee and Turdaliev (2013). Campolmi (2014) examined optimal monetary policy in a small-open economy with nominal price and wage rigidity. He showed that stabilizing CPI inflation led to outcomes preferable to those under PPI inflation targeting. As pointed out by Rhee and Turdaliev (2013), the terms of trade depend on the real wage gap in an open economy. Groll and Monacelli (2020) argued the importance of terms-of-trade inertia in a two-country NK model with complete wage flexibility. However, I underscore the role of liquidity constraints and therefore emphasize the substantial impact of the real wage gap on the welfare costs arising from discretion in our model. This is because in the two-country model, the terms of trade change in response to fluctuations in domestic and foreign output. Moreover, inertial changes in wage rates now translate into changes in the terms of trade. This mechanism was not addressed in the aforementioned studies. As a result, the existence of nominal wage rigidity in both countries significantly impacts the structural equations, and the presence of liquidity constraints significantly affects the deep parameters upon which the structural equations are constructed. While Galí and Monacelli (2016) focused on the role of wage flexibility in a currency union, they did not examine the impact of liquidity constraints on optimal monetary policy in a two-country NK model.

Our study is also related to Ascari *et al.* (2017) and Bilbiie (2008), who analyzed liquidity constraints and optimal monetary policy, but in a closed economy model. Bhattarai *et al.* (2015) and Palek and Schwanebeck (2019) introduced heterogeneity by incorporating financial friction, but did not consider the role of liquidity constraints in a currency union. In their model, there was heterogeneity among households and financial institutions. In contrast to these studies, this paper shows that if liquidity constraints in both countries tighten, the intertemporal channel is weakened in a currency union through a change in the union-wide policy rate.

Finally, I would like to comment on the heterogeneous agent NK (HANK) model, which could be used to analyze optimal monetary policy by considering the existence of more heterogeneous households (Kaplan *et al.*, 2014). The two-country model in this paper is based on the two-agent NK (TANK) model because this is impossible for the following reasons.[4] First,

[4]See Kaplan and Violante (2018) and Galí (2018) for detailed explanations of the difference between HANK and TANK.

it is difficult to intuitively understand the structure of the economy in this model, since the policy objectives of the central bank cannot be explicitly derived. In the TANK model, the log-linearized structural equations are derived more easily and intuitively than in the HANK model. Second, since the model is a currency union and its size is larger than the closed NK model, taking HANK into account would be computationally burdensome, and optimal monetary policy may not be derived, even with a numerical solution. Third, it is difficult to appropriately evaluate the optimal monetary policy for a closed economy. Fourth, according to Debortoli and Galí (2017), while there is no clear difference between HANK and TANK in terms of aggregate shocks, we can adopt the TANK model as an approximation in this study.

3 The model

As mentioned earlier, we incorporate nominal wage rigidity and liquidity constraints into the two-country NK model developed by Clarida *et al.* (2002). As in Benigno (2004), we focus on the case for a currency union.[5] Our model differs from the existing literature in that we investigate optimal monetary policy in a currency union with liquidity constraints.

Consider an economy with two large symmetrical countries: home and foreign. The economy sizes for home and foreign are $1-\gamma$ and γ, respectively. Each country has two production sectors: a final goods sector characterized by perfect competition, and an intermediate goods sector in which firms face monopolistic competition and set prices according to Calvo (1983)-type nominal price rigidity.

In each country, households obtain utility from consumption and disutility from supplying labor. A fraction of $1-\lambda$ is comprised of Ricardian households that can freely access financial markets in each country. Ricardian households in each country have access to a complete set of state-contingent securities that are traded both domestically and internationally. On the other hand, in each country, a fraction of λ represents non-Ricardian households that cannot trade financial assets in the financial market. Following Ascari *et al.* (2017), we assume that there exists a labor union that determines the nominal wage. Each union faces Calvo-type nominal

[5] As noted in the conclusion, if we introduce an exchange rate float into the two-country model with liquidity constraints, we cannot explicitly derive the central bank's loss function by implementing a second-order approximation of the household utility function. In this case, we have to solve optimal monetary policy under non-linear Ramsey policy. While this is beyond the scope of this paper, I would like to consider this issue as a future work.

wage rigidity.[6]

Finally, unless otherwise noted, analogous equations hold for the foreign country. Foreign variables are represented by asterisks.

3.1 Households

Consumption preferences

Preferences for consumption in the home country are given by:

$$C_t \equiv C_{H,t}^{1-\gamma} C_{F,t}^{\gamma}, \tag{1}$$

where $C_{H,t}$ is the consumption of domestic goods and $C_{F,t}$ is the consumption of foreign goods. The price index in the home country is given by:

$$P_t = \kappa^{-1} P_{H,t}^{1-\gamma} P_{F,t}^{\gamma}, = \kappa^{-1} P_{H,t} S_t^{\gamma}, \tag{2}$$

where $\kappa \equiv (1-\gamma)^{(1-\gamma)}\gamma^{\gamma}$, $P_{H,t}$ is the price of domestic goods, and $P_{F,t}$ is the price of foreign goods. In addition, S_t represents the terms of trade, which is given by:

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}}. \tag{3}$$

Labor market

Following Ascari *et al.* (2017), we assume that the nominal wage that each household earns is set by the labor type-specific union, indexed by j ($j \in [0,1]$). The nominal wage is fixed by union j. With this setup, the labor supply L_t^j is given by:

$$L_t^j = \left(\frac{W_t^j}{W_t}\right)^{-\theta_w} L_t^d, \tag{4}$$

where L_t^j denotes the labor supply in union j, W_t^j is the nominal wage set by union j, and the parameter θ_w is the elasticity of substitution for individual labor. Following Ascari *et al.* (2017), since we assume that L_t^j is identical for Ricardian and non-Ricardian households, we can eliminate the discrepancy in the wage difference between households as follows:

$$L_t = \int_0^1 L_t^j dj. \tag{5}$$

[6]See also Erceg *et al.* (2000) for a detailed discussion of nominal wage rigidity as well as nominal price rigidity.

Therefore, we obtain the following common labor income:

$$\int_0^1 W_t^j L_t^j dj = L_t^d \int_0^1 W_t^j \left(\frac{W_t^j}{W_t}\right)^{-\theta_w} dj. \tag{6}$$

Ricardian households

A fraction of $1-\lambda$ is comprised of Ricardian households that can freely access financial markets. The intertemporal utility of Ricardian households is as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(C_{o,t}) - V(L_{o,t}) \right\},$$

where $C_{o,t}$ denotes Ricardian household's consumption, and $L_{o,t}$ is the Ricardian household's labor supply. We assume that the utility function, $u(\cdot)$, is strictly concave and continuously differentiable, and the disutility of supplying labor, $V(\cdot)$, is strictly convex and continuously differentiable. The representative household maximizes the above utility function, subject to the following budget constraint:

$$P_t C_{o,t} + E_t[Q_{t,t+1} B_{o,t+1}] = B_{o,t} + L_t^d \int_0^1 W_t^j \left(\frac{W_t^j}{W_t}\right)^{-\theta_w} dj + \Gamma_{o,t} - T_{o,t},$$

where $B_{o,t}$ represents nominal bonds held for one period, and $\Gamma_{o,t}$ denotes the dividends earned from domestic firms. $T_{o,t}$ denotes a lump-sum tax. We assume that Ricardian households can access state-contingent bonds traded in a complete market domestically and internationally and introduce the following stochastic discount factor:

$$E_t(Q_{t,t+1}) = \frac{1}{1+r_t}, \tag{7}$$

where $Q_{t,t+1}$ denotes a stochastic discount factor, and r_t is the union-wide, risk-free, short-term nominal interest rate.

From the optimal condition, we obtain the familiar Euler consumption equation.

$$E_t[Q_{t,t+1}] = \frac{1}{1+r_t} = \beta E_t \left[\frac{u_c(C_{o,t+1})}{u_c(C_{o,t})} \frac{P_t}{P_{t+1}} \right]. \tag{8}$$

Since Ricardian households in the foreign country can also trade state-contingent bonds both domestically and internationally, the consumption Euler equation holds for foreign Ricardian households as follows:

$$E_t[Q_{t,t+1}] = \frac{1}{1+r_t} = \beta E_t \left[\frac{u_c(C^*_{o,t+1})}{u_c(C^*_{o,t})} \frac{P^*_t}{P^*_{t+1}} \right], \tag{9}$$

where an asterisk denotes a foreign variable. In this paper, international consumption risk-sharing is not feasible at the aggregate consumption level because liquidity-constrained households exist in both countries. However, at the level of Ricardian households, there are complete markets for state-contingent nominal bonds both domestically and internationally, so complete consumption risk-sharing can be established between Ricardian households in the two countries. Thus, we obtain the following:

$$C_{o,t} = C^*_{o,t}.$$

As I show in Sections 5 and 6, this result leads to significant policy implications for optimal monetary policy in a currency union. If liquidity constraints are not present in both countries, the risk-sharing condition corresponds to the case presented by Benigno (2004) and Clarida *et al.* (2002). In addition, $P_t = P^*_t$ holds in a currency union.

Non-Ricardian households

Following Galí *et al.* (2007), Bilbiie (2008), and Ascari *et al.* (2017), this paper assumes that a fraction of $1 - \lambda$ cannot access financial markets in each country. Therefore, non-Ricardian households that face the same periodic utility function as Ricardian households,[7] but their budget constraints are given by:

$$P_t C_{r,t} = L^d_t \int_0^1 W^j_t \left(\frac{W^j_t}{W_t} \right)^{-\theta_w} dj. \tag{10}$$

Thus, in contrast to Ricardian households, since non-Ricardian households cannot implement intertemporal consumption smoothing by trading state-contingent bonds in bond markets, they end up consuming their entire disposable income every period.

Wage determination

Ricardian and non-Ricardian households delegate the role of wage determination to a labor union.[8] Following Ascari *et al.* (2017), the wage setting is subject to Calvo-type staggered wage contracts. Thus, a fraction of $1 - \zeta$ can change nominal wages in its union, whereas a

[7]Thus, the properties of the Ricardian household's utility function are carried over to the non-Ricardian household.

[8]See Ascari *et al.* (2017) and Galí *et al.* (2007) for a detailed discussion of the labor union specification in the model.

remaining fraction of ζ cannot do so. With this setting, the labor union for each country solves the following maximization problem:

$$\underset{\tilde{W}_t}{max}\ E_t \sum_{s=0}^{\infty} (\zeta\beta)^s \Big[(1-\lambda)u(C_{o,t+s}) + \lambda u(C_{r,t+s}) - V(L_{t+s})\Big].$$

The first-order condition of this problem is given by:

$$E_t \sum_{s=0}^{\infty} (\zeta\beta)^s V_l(L_{t+s}) L_{t+s}^d W_{t+s}^{\theta_w} \left[\left(\frac{\lambda}{MRS_{o,t+s}} + \frac{1-\lambda}{MRS_{r,t+s}}\right)\frac{\tilde{W}_t}{P_{t+s}} - \mu_w\right] = 0. \tag{11}$$

where $\mu_w = \theta_w/(\theta_w - 1)$ and $\tilde{W}_t$ denotes the optimal nominal wage. The aggregate wage index is defined as follows:

$$W_t = \left(\frac{1}{1-\gamma}\int_0^{1-\gamma} W_t(j)^{1-\theta_w} dj\right)^{1/(1-\theta_w)}. \tag{12}$$

Under a Calvo-type wage setting, this equation can be rewritten as follows:

$$W_t = \left[(1-\zeta)\tilde{W}_t^{1-\theta_w} + \zeta W_{t-1}^{1-\theta_w}\right]^{1/(1-\theta_w)}. \tag{13}$$

3.2 Firms

Each country has two production sectors. The first is the final goods sector, which produces final goods using intermediate goods and is characterized by perfect competition. The second is the intermediate goods sector, in which firms face monopolistic competition and Calvo (1983)'s nominal price rigidity.

Final goods firms

The final goods sector is perfectly competitive, and producers use inputs that are produced in the intermediate goods sector. Final goods are produced according to the following CES aggregate:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta_{p,t}-1}{\theta_{p,t}}} di\right]^{\frac{\theta_{p,t}}{\theta_{p,t}-1}}, \tag{14}$$

where Y_t is aggregate output, $Y_t(i)$ is demand for intermediate goods produced by firm i, and $\theta_{p,t}$ is the elasticity of substitution, which is time-varying, as assumed by Steinsson (2003).[9] Note that both variables are normalized by $1-\gamma$.

[9] Clarida *et al.* (2002) presumed a time-varying wage markup to introduce an exogenous cost-push shock. In this study, instead of introducing a wage markup, a price markup derived from a time-varying elasticity of substitution $\theta_{p,t}$ is introduced to examine the effect of a cost-push shock on optimal monetary policy. See Clarida *et al.* (2002) for a detailed discussion of the presence of a wage markup shock.

Under the CES aggregate, the demand function is given by:

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\theta_{p,t}} Y_t, \tag{15}$$

and the domestic price level is defined as follows:

$$P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\theta_{p,t}} di\right]^{\frac{1}{1-\theta_{p,t}}}, \tag{16}$$

where $P_{H,t}(i)$ is the price for intermediate goods produced by firm i. Note that these variables are also normalized by $1-\gamma$.

Intermediate goods firms

The intermediate goods sector is characterized by monopolistic competition, and each firm produces a differentiated intermediate good. Firm i's production function is given by:

$$Y_t(i) = A_t N_t(i), \tag{17}$$

where A_t denotes an aggregate productivity disturbance, which follows an autoregressive (AR)(1) process given by $\log A_t = \rho_a \log A_{t-1} + \epsilon_t^a$ with $0 \leq \rho_a < 1$, where ϵ_t^a is an independent and identically distributed (i.i.d.) shock with constant variance σ_a^2.

As in Clarida *et al.* (2002), the intermediate firm's real marginal cost is given as follows:

$$\varphi_t = (1-\tau)\frac{W_t}{P_{H,t}}\frac{1}{A_t}, \tag{18}$$

where τ denotes the subsidies that eliminate the price markup distortion arising from the presence of monopolistic competition in the intermediate goods sector.

Following Calvo (1983), we assume that price rigidity is present in the intermediate goods sector. The following explanation focuses on the home country, whereas we can also consider the case for the foreign country. Thus, a fraction $1-\omega$ of all firms adjusts their price, while the remaining fraction of firms ω does not.

We now consider the intermediate firms that can adjust their prices. When revising their prices, these firms account for the uncertainty regarding when they will next be able to adjust prices. In this case, the intermediate firm's optimization problem for the home country is given by:

$$E_t \sum_{t=0}^{\infty} \omega^j Q_{t,t+j} Y_{t+j}(i)(P_{H,t}^{opt} - P_{H,t+j}\varphi_{t+j}), \tag{19}$$

where $P^{opt}_{H,t}$ is the firm's optimal price. The first-order condition of this maximization problem is as follows:

$$E_t \sum_{t=0}^{\infty} \omega^j Q_{t,t+j} Y_{t+j}(i)(P^{opt}_{H,t} - \mu^p_{t+j} P_{H,t+j} \varphi_{t+j}) = 0. \tag{20}$$

where the variable $\mu_{p,t} = \theta_{p,t}/(\theta_{p,t} - 1)$ is the time-varying price markup.

3.3 Equilibrium

We now describe the equilibrium conditions in an open economy. The equilibrium conditions for the goods market are given as follows:

$$(1 - \gamma)Y_t = (1 - \gamma)C_{H,t} + \gamma C^*_{H,t}, \tag{21}$$

$$\gamma Y^*_t = (1 - \gamma)C_{F,t} + \gamma C^*_{F,t}. \tag{22}$$

Under the Cobb-Douglas consumption index, we obtain the following demand function for $C_{H,t}$ and $C_{F,t}$:

$$C_{H,t} = (1 - \gamma)\left(\frac{P_{H,t}}{P_t}\right)^{-1} C_t, \tag{23}$$

$$C_{F,t} = \gamma\left(\frac{P_{F,t}}{P_t}\right)^{-1} C_t. \tag{24}$$

Substituting these equations into (21) yields:

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-1} C^u_t, \tag{25}$$

where $C^u_t = (1 - \gamma)C_{H,t} + \gamma C_{F,t}$. Similarly,

$$Y^*_t = \left(\frac{P_{F,t}}{P_t}\right)^{-1} C^u_t. \tag{26}$$

At this point, the domestic terms of trade are represented by the ratio of domestic output to foreign output as follows:

$$S_t = \frac{Y_t}{Y^*_t}. \tag{27}$$

Equation (27) indicates that holding domestic output constant, an increase in foreign output leads to an appreciation of the domestic terms of trade.

Since the Ricardian households in each country can trade state-contingent bonds both domestically and internationally, the market-clearing conditions for the bond markets are as follows:

$$B_{o,t} = (1-\lambda)B_t,$$
$$B^*_{o,t} = (1-\lambda^*)B^*_t,$$
$$(1-\gamma)B_t + \gamma B^*_t = 0.$$

Aggregate consumption and dividends are given as follows:

$$C_t = (1-\lambda)C_{o,t} + \lambda C_{r,t}, \tag{28}$$
$$C^*_t = (1-\lambda^*)C^*_{o,t} + \lambda^* C^*_{r,t}, \tag{29}$$
$$\Gamma_{o,t} = (1-\lambda)\Gamma_t,$$
$$\Gamma^*_{o,t} = (1-\lambda^*)\Gamma^*_t.$$

3.4 Log-linearization

3.4.1 The effect of liquidity constraints in a currency union

This section provides the log-linearized equations derived in the previous section. In this section, we mainly focus on how the presence of liquidity constraints in both countries affects the aggregate macroeconomic dynamics. First, $\bar{H}$ represents the value of the steady state, H^n_t is the value of the efficiency level. We define $\hat{H}_t = \log(H_t/\bar{H})$ as the deviation of H_t from the steady state.

The log-linearization of equations (28) and (29) leads to the following:

$$\hat{C}_t = (1-\lambda)\hat{C}_{o,t} + \lambda \hat{C}_{r,t}, \tag{30}$$
$$\hat{C}^*_t = (1-\lambda^*)\hat{C}^*_{o,t} + \lambda^* \hat{C}^*_{r,t}. \tag{31}$$

Since Ricardian households in both countries can trade state-contingent bonds both domestically and internationally, international consumption risk-sharing holds for Ricardian households in each country as follows:

$$\hat{C}_{o,t} = \hat{C}^*_{o,t}. \tag{32}$$

Importantly, this paper addresses the fact that even if consumption risk-sharing holds for Ricardian households in each country, it cannot hold at the aggregate consumption level. More precisely, log-linearizing Equation (25) and the corresponding equation for the foreign country and using (32), yields the consumption level for Ricardian households as follows:

$$\hat{C}_{o,t} = \frac{1}{\Omega}\Big[(1-\gamma)(1-\lambda\hat{Y}_t - \lambda(1-\gamma)(\hat{w}_t - A_t) + \gamma(1-\lambda^*)\hat{Y}_t^* - \lambda^*\gamma(\hat{w}_t^* - A_t^*)\Big], \tag{33}$$

$$\hat{C}_{o,t}^* = \frac{1}{\Omega}\Big[\gamma(1-\lambda^*)\hat{Y}_t^* - \lambda^*\gamma(\hat{w}_t^* - A_t^*) + (1-\gamma)(1-\lambda)\hat{Y}_t - \lambda(1-\gamma)(\hat{w}_t - A_t)\Big]. \tag{34}$$

where $\Omega = (1-\gamma)(1-\lambda) + \gamma(1-\lambda^*)$. If not, the consumption of non-Ricardian households in the home and foreign countries will increase, and home Ricardian households' consumption will decrease. In addition, non-Ricardian households' consumption will increase if real wages rise. The key to understanding this mechanism is that the home country's Ricardian consumption is reduced not only by the home country's real wage, but also by the increase in the foreign country's real wage. When liquidity constraints are present, non-Ricardian households cannot risk-share using financial assets, which is a factor of incomplete risk-sharing, so the fact that $C_{o,t} = C_{o,t}^*$ holds does not mean that $C_t = C_t^*$ also holds.

As the share of non-Ricardian households in a country increases, the share of consumption by Ricardian households declines in that country. Therefore, the effectiveness of the interest rate channel, which is the path through which the real interest rate changes such that the policy rate affects the household's intertemporal optimal consumption allocation, is weakened.

3.4.2 Equilibrium under flexible prices and wages

When prices and wages are flexible in both countries, the natural rate of output for each country is given as follows:

$$\begin{aligned}(\eta+\sigma+D_0-D_1)\hat{Y}_t^n + (D_1-D_0)\hat{Y}_t^{n,*} &= (1+\eta+D_0)A_t - D_0\hat{w}_t^n \\ &\quad + \frac{\lambda^*(1-\lambda)\gamma\sigma}{\Omega}(\hat{w}_t^{n,*} - A_t^*), \end{aligned} \tag{35}$$

$$\begin{aligned}(\eta+\sigma+D_0^*-D_1^*)\hat{Y}_t^n + (D_1^*-D_0^*)\hat{Y}_t^{n,*} &= (1+\eta+D_0^*)A_t^* - D_0^*\hat{w}_t^{n,*} \\ &\quad + \frac{\lambda(1-\lambda^*)(1-\gamma)\sigma}{\Omega}(\hat{w}_t^n - A_t), \end{aligned} \tag{36}$$

where

$$\begin{aligned} &1-\psi = \frac{(1-\gamma)(1-\lambda)}{\Omega},\ \psi = \frac{\gamma(1-\lambda^*)}{\Omega}, \\ &D_1 = \sigma\psi - \gamma,\ D_0 = \sigma\psi\lambda, \\ &D_1^* = \sigma(1-\psi) - (1-\gamma),\ D_0^* = \sigma(1-\psi)\lambda^*, \end{aligned}$$

The open economy effect is captured by the parameter D_1, which consists of both the terms-of-trade effect and the international risk-sharing effect of Ricardian households in both countries. Importantly, unlike the standard two-country NK model (Clarida *et al.*, 2002), D_1 is now affected by ψ, which directly contains the effect of liquidity constraints for both countries. The share of liquidity constraints in each country has a significant impact on the value of D_0. Importantly, even when non-Ricardian households completely disappear in the home country, ψ does not correspond to $1-\gamma$, which holds for a two-country model without liquidity constraints, as long as the share of non-Ricardian households in the foreign country is not zero. Consequently, a share of non-Ricardian households affects the home output and real wages under flexible price and wage equilibrium. Note that the case in which $\lambda = \lambda^* = 0$ corresponds to the standard two-country NK model (Clarida *et al.*, 2002).

The intuition is as follows. In the case in which liquidity constraints do not exist in both countries, home and foreign natural output depend only on productivity shocks in each country (Clarida *et al.*, 2002). However, when liquidity constraints exist in both countries, real wages at the flexible price level in both countries will affect home and foreign natural output. For example, the home country's natural output level rises with an increase in the natural level of the home country's real wage, and it falls with the rise in the foreign country's natural level of real wages, given the foreign country's natural output and productivity shocks in both countries.

Finally, if liquidity constraints are absent in both countries, Equations (35) and (36) reduce to the following:

$$
\begin{aligned}
&[\eta + \sigma + \gamma(\sigma - 1)]\hat{Y}_t^n + \gamma(\sigma - 1)\hat{Y}_t^{n,*} = (1 + \eta)A_t, \\
&(1 - \gamma)(\sigma - 1)\hat{Y}_t^n + [\eta + \sigma + (1 - \gamma)(\sigma - 1)]\hat{Y}_t^{n,*} = (1 + \eta)A_t^*,
\end{aligned}
$$

which corresponds to the results derived by Clarida *et al.* (2002).

3.4.3 Equilibrium under sticky prices

Aggregate employment in the home country can be rewritten in terms of price and wage dispersion as follows:

$$
L_t = \int_0^1 L_t^j dj = \frac{\Delta_{p,t}\Delta_{w,t}Y_t}{A_t}. \tag{37}
$$

Price dispersion is defined as follows:

$$\Delta_{p,t} \equiv \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\theta_{p,t}} di, \tag{38}$$

$$\Delta_{w,t} \equiv \int_0^1 \left(\frac{W_t^j}{W_t}\right)^{-\theta_w} dj. \tag{39}$$

At a sticky price equilibrium, the real marginal cost is expressed by the "gap" term as follows:

$$\hat{\varphi}_t = (\eta + \sigma + D_0 - D_1)x_t + (D_1 - D_0)x_t^* + D_0\tilde{w}_t - \frac{\sigma\gamma(1-\lambda)\lambda^*}{\Omega}\tilde{w}_t^*, \tag{40}$$

where x_t is the home output gap, x_t^* is the foreign output gap, $\tilde{w}_t$ is the home real wage gap, and $\tilde{w}_t^*$ is the foreign real wage gap. It follows fro this equation that the home real marginal cost is characterized by the inclusion of not only the home country's real wage, but also the foreign country's real wage gap. The home country's real wage gap positively affects the home country's marginal costs. The first channel is that the home country's real wages pushes up the real marginal cost by increasing the home country's cost of production. The second channel is the effect of rising consumption in the foreign country through risk-sharing by non-Ricardian households in the foreign country. Therefore, as the share of liquidity constraints rises in the foreign country, the volatility of non-Ricardian consumption increases in that country. However, it also works to lower the real marginal cost in the home country through Ricardian households' risk-sharing. Since the first channel is always dominated by the second channel, the home country's real marginal cost will respond positively to the home country's real wage. On the other hand, the real marginal cost of the foreign country always has a negative effect on the marginal cost of the home country.

As noted earlier, the open economy effect is captured by the parameter D_1, which is affected by ψ, which directly contains the effect of liquidity constraints in both countries. As noted earlier, the share of liquidity constraints in each country has a significant impact on the value of D_0. Even when non-Ricardian households completely disappear in the home country, the model does not correspond to the standard two-country NK model developed by Clarida *et al.* (2002) as long as the share of non-Ricardian households in the foreign country is not zero. Equation (40) reduces to one when liquidity constraints are absent in both countries (Clarida *et al.*, 2002).

Log-linearizing the firm's optimal price-setting rule (20) leads to the following price new

Keynesian Phillips curve (PNKPC), expressed by the real marginal cost as follows:

$$\pi_t = \beta E_t \pi_{t+1} + \delta_p \hat{\varphi}_t + u_t, \tag{41}$$

where $\pi_t = \log(P_{H,t}/P_{H,t-1})$ and u_t is the price markup shock associated with a time-varying elasticity of substitution. In addition, $\delta_p = (1-\omega)(1-\omega\beta)/\omega$. Substituting Equation (18) into Equation (41) yields the following PNKPC in terms of the "gap" term as follows:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_1 x_t + \kappa_2 x_t^* + \kappa_3 \tilde{w}_t - \kappa_4 \tilde{w}_t^* + u_t, \tag{42}$$

where

$$\kappa_1 = \delta_p(\eta + \sigma + D_0 - D_1),\ \ \kappa_2 = \delta_p(D_1 - D_0),$$
$$\kappa_3 = \delta_p D_0,\ \ \kappa_4 = \frac{\delta_p \sigma \gamma (1-\lambda)\lambda^*}{\Omega}.$$

In the two-country model, home inflation is affected by the foreign output gap due to the existence of the terms of trade (Benigno, 2004, Clarida *et al.*, 2002, Pappa, 2004).[10] Whether the foreign output gap has a positive or negative impact on the home country depends on the magnitude of D_1 and D_0. Unlike Benigno (2004) and Clarida *et al.* (2002), since our model characterizes the effect of liquidity constraints on the structural equations in a currency union, the difference from a standard two-country model is that these coefficients are affected by home and foreign liquidity constraints (i.e., liquidity constraints affect D_1 and D_0). In addition, the existence of liquidity constraints in both countries causes the real wage gap in both countries to affect the home inflation rate. Specifically, the home country's real wage gap and the foreign country's real wage gap have positive and negative effects on inflation in the home country, respectively. It is important to note that there is no path of the real wage gap to inflation, only in the absence of liquidity constraints in both countries.

Since the model in this paper has the effect of sticky wages, the wage New Keynesian Phillips curve (WNKPC) is also derived as follows:

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \kappa_{w,1} x_t + \kappa_{w,2} x_t^* + \kappa_{w,3} \tilde{w}_t - \kappa_{w,4} \tilde{w}_t^*, \tag{43}$$

[10] See Clarida *et al.* (2002) for an intuitive discussion of international spillover through the terms of trade and international consumption risk-sharing.

where $\delta_w = (1-\zeta)(1-\zeta\beta)/\zeta$. Also,

$$\kappa_{w,1} = \delta_w(\eta + \sigma + D_0 - D_1),\ \kappa_{w,2} = \delta_w(D_1 - D_0),$$
$$\kappa_{w,3} = \delta_w D_0,\ \kappa_{w,4} = \frac{\delta_w \sigma\gamma(1-\lambda)\lambda^*}{\Omega}.$$

The WNKPC in this paper differs from Erceg *et al.* (2000) in that home wage inflation depends on the foreign output gap and the foreign real wage gap. The former is associated with the impact of the terms of trade and the latter with the presence of foreign liquidity constraints. As in the PNKPC, the impact of the foreign output gap and real wage gap on home wage inflation depends on the magnitude of D_1 and D_0.

In this economy, I show that movements in the real wage rate substantially impact optimal monetary policy. The law of motion for the definition of the home real wage evolves as follows:

$$w_t = \frac{W_t}{P_t} = \frac{W_t\ S_t^{\gamma}}{P_{H,t}}. \tag{44}$$

As pointed out in Campolmi (2014) and Rhee and Turdaliev (2013), the terms of trade affect the real wage rates, and this mechanism is characterized in an open economy model. Unlike the aforementioned studies, since our model is based on a two-country model under a currency union, the home and foreign real wage rates also affect their interdependence through a change in the terms of trade. Log-linearizing Equation (44) leads to the following law of motion for the home real wage as follows:

$$\hat{w}_t = \hat{w}_{t-1} + \gamma(\hat{S}_t - \hat{S}_{t-1}) + \pi_t^w - \pi_t. \tag{45}$$

Thus, since the terms of trade are characterized by the ratio of home output to foreign output, this implies that the foreign output gap indirectly affects the home real wage gap through the law of motion of home real wages due to changes in the terms of trade.

Finally, using home and foreign consumption Euler equations, the definition of the terms of trade, and the relationship $x_t^u = (1-\gamma)x_t + \gamma x_t^*$, the union-wide investment-saving equation is derived as follows:

$$\begin{aligned} x_t^u =& E_t x_{t+1}^u + \vartheta_1 E_t \Delta x_{t+1}^* + \vartheta_2 E_t \Delta x_{t+1} - \sigma_u(\hat{r}_t - E_t\pi_{t+1}^u - rr_{t+1}^u) \\ & - \vartheta_3 E_t \Delta \tilde{w}_{t+1} - \vartheta_4 E_t \Delta \tilde{w}_{t+1}^*, \end{aligned} \tag{46}$$

where variables x_t^u and π_t^u denote the union-wide output gap and inflation, respectively. In addition, the parameters are defined as follows:

$$\vartheta_1 = \frac{(1-\gamma)(\sigma\psi - \gamma)}{\sigma_0}, \ \vartheta_2 = \frac{\gamma(\sigma(1-\psi) - (1-\gamma))}{\sigma_0^*},$$
$$\sigma_0 = \sigma(1-\psi) - \gamma, \ \sigma_0^* = \sigma(1-\psi) - (1-\gamma),$$
$$\sigma_u = \frac{\sigma_0 + \sigma_0^*}{\sigma_0 \sigma_0^*}, \ \vartheta_3 = \frac{\sigma_u \sigma (1-\gamma)\lambda}{\Omega}, \ \vartheta_4 = \frac{\sigma_u \sigma \gamma \lambda^*}{\Omega}.$$

Since the model in this paper is built under a currency union, the two countries have a common policy rate, which is given by $\hat{r}_t$. The central bank under the currency union determines the path of the worldwide policy rate by implementing optimal monetary policy.

4 Optimal monetary policy

This section examines the optimal monetary policy in a currency union with liquidity constraints. In Section 4.1, I derive the central bank's loss function by calculating a second-order approximation of the household utility function around the efficient steady state. Section 4.2 explores the optimal monetary policy under a commitment policy.

4.1 The central bank's loss function

I must derive a well-defined loss function with a micro-foundation to investigate the optimal monetary policy in an NK model. Woodford (2003) shows that the second-order approximation of the household utility function corresponds to the central bank's loss function. In a two-country framework, Clarida *et al.* (2002) and Engel (2011) derived the central bank's loss function using a second-order approximation of the utility function.

To derive a well-defined loss function for the central bank from a quadratic approximation of the household utility function, we must perform a Taylor expansion around an efficient steady state. In this paper, an efficient steady state that removes the following distortions is necessary to derive the central bank's loss function. In each country, there is a distortion of markups arising from monopolistic competition by intermediate goods firms and a distortion of wage markups by labor unions. As shown in the Appendix, the optimal subsidies that attain an efficient steady state are given as follows:[11]

$$\tau = \tau^* = 1 - \frac{1}{\kappa \mu_p \mu_w}.$$

In deriving these subsidies, it is assumed that the government in each country will calculate the optimal subsidy. If a distorted steady state is not offset by these subsidies, I have to use the second-order approximation of both the PNKPC and WNKPC to obtain a well-defined central bank loss function in this economy (Benigno and Woodford, 2005).[12] In fact, except for the presence of nominal wage rigidity and liquidity constraints, Benigno and Benigno (2006) derived the central bank's loss function by calculating the second-order conditions of the PNKPC in a two-country economy with a distorted steady state.[13] However, this task is beyond the scope of this paper. Therefore, I assume that these subsidies offset the distortions caused by the respective markups on prices and wages due to monopolistic competition between intermediate firms and labor unions.

With the presence of optimal subsidies, I derive the central bank's policy objective in a currency union by implementing the second-order approximation of the following household utility function.

$$\begin{aligned}\mathcal{W}_t \simeq \mathcal{W} - \frac{u_c C \Omega}{2} \sum_{t=0}^{\infty} \Bigg\{ &(1-\psi)\left[\alpha_\pi^p \pi_t^2 + \alpha_\pi^w (\pi_t^w)^2 + \alpha_x x_t^2 + \alpha_w \tilde{w}_t^2\right] \\ &+ \psi\left[\alpha_\pi^{p,*}(\pi_t^*)^2 + \alpha_\pi^{w,*}(\pi_t^{w,*})^2 + \alpha_x^*(x_t^*)^2 + \alpha_w^*(\tilde{w}_t^*)^2\right] \\ &- 2\Lambda_1 x_t x_t^* - 2\Lambda_2 \tilde{w}_t \tilde{w}_t^* - 2\Lambda_3(\tilde{\lambda}\tilde{w}_t - \tilde{\lambda}^* \tilde{w}_t^*)(x_t - x_t^*)\Bigg\} + t.i.p. + O(||\xi||^3). \qquad (47)\end{aligned}$$

[11]See also Ascari *et al.* (2017) for a detailed discussion about optimal subsidies in an economy with liquidity constraints and nominal price and wage rigidity.

[12]Fujiwara and Wang (2017) obtained a well-defined central bank loss function in a two-country NK model with local currency pricing.

[13]To assess the gain from international monetary policy coordination in a two-country model under producer currency pricing, Benigno and Benigno (2006) derived the central bank's loss function by calculating a second-order approximation of the PNKPC under both policy coordination and no policy coordination.

where

$$
\begin{aligned}
&\alpha_\pi^p = \frac{\omega\theta_p}{(1-\omega)(1-\omega\beta)},\ \alpha_\pi^w = \frac{\zeta\theta_w}{(1-\zeta)(1-\zeta\beta)},\\
&\alpha_x = \sigma + \eta - \frac{(1-\sigma)(1-\psi(1-\lambda))}{1-\lambda},\ \alpha_w = (\sigma-1)\lambda\Phi,\\
&\alpha_\pi^{p,*} = \frac{\omega^*\theta_p}{(1-\omega^*)(1-\omega^*\beta)},\ \alpha_\pi^{w,*} = \frac{\zeta^*\theta_w}{(1-\zeta^*)(1-\zeta^*\beta)},\\
&\alpha_x^* = \sigma + \eta - \frac{(1-\sigma)(1-(1-\psi)(1-\lambda^*))}{1-\lambda^*},\ \alpha_w^* = (\sigma-1)\lambda^*\Phi^*,\\
&\Lambda_1 = (1-\sigma)\psi(1-\psi),\ \Lambda_2 = \frac{(1-\sigma)\psi(1-\psi)\lambda\lambda^*}{(1-\lambda)(1-\lambda^*)},\ \Lambda_3 = \frac{1-\sigma}{\Omega},\\
&\Phi = \frac{(1-\gamma)\lambda + (1-\lambda)\Omega}{(1-\lambda)\Omega},\ \Phi^* = \frac{\gamma^*\lambda^* + (1-\lambda^*)\Omega}{(1-\lambda^*)\Omega}.
\end{aligned}
$$

In addition, $t.i.p$ includes terms that are independent of monetary policy, and $O(||\xi||^3)$ indicates the terms of third or higher orders. The Appendix provides a detailed derivation of the central bank's loss function by calculating the second-order approximation of the household utility function.

As in the standard two-country NK model, our loss function contains several standard policy objectives. Standard inflation and output gap objectives are assigned to both the home and foreign countries. The first objective implies the presence of price dispersions caused by nominal price rigidity. In addition, the loss functions for each country contain the wage inflation stabilization associated with wage stickiness. This objective is associated with the presence of wage dispersion caused by staggered wage contracts (Erceg *et al.*, 2000). As shown in Ascari *et al.* (2017), our loss function includes the stabilization of the real wage gap due to the presence of non-Ricardian households.

In contrast to the central bank loss function derived from the standard two-country NK model (Clarida *et al.*, 2002), our loss function contains several additional terms, as follows.

First, a stabilizing term for the co-variation of the home and foreign economies, adjusted for the weighting of each country's liquidity-constrained households, appears under a currency union. While this is similar to Clarida *et al.* (2002), the difference is that it is adjusted for the weight of the liquidity constraint ratio in both countries.

Second, liquidity constraints in the home and foreign countries have a co-movement effect through the risk-sharing conditions of Ricardian households. The difference from Clarida *et al.* (2002) is that real wages in both countries responded negatively to international risk-sharing conditions for consumption due to Ricardian consumption when liquidity constraints existed in both countries. The magnitude of this weight is greater if liquidity constraints are higher in both countries. However, as long as non-Ricardian households are absent in either country, this term becomes zero.

Third, I focus on the effect of the co-movement of the difference between home and foreign weight-adjusted wages and the difference in the output gap (terms of trade) between the two countries. This is because the term that the interaction between business cycle fluctuations and non-Ricardian households' consumption co-movement is also associated with the international risk-sharing condition of Ricardian consumption. In other words, it can be interpreted as the co-movement of real wages and the terms of trade in the home country. The existence of non-Ricardian households will increase the output gap in the home country associated with consumption of non-Ricardian consumption in the home country. It is further classified as an impact through the foreign output gap associated with risk-sharing of Ricardian consumption. Therefore, the weight on real wages will be larger due to greater liquidity constraints.

Fourth, with the exception of the stabilizing coefficients of price and wage inflation, all coefficients of its loss function are affected by home and foreign liquidity constraints. I emphasize the fact that the impact of the share of liquidity constraints in both countries on the stabilization parameters of the loss function is more complex than that considered in Ascari *et al.* (2017). For example, as long as $\sigma > 1$, the coefficient of output gap stabilization in the home country rises as the home country's liquidity constraint tightens. This is because an increase in the share of non-Ricardian households in the home country will cause fluctuations in the output gap through changes in consumption. Tighter liquidity constraints in the home country also increase the stabilizing weight of the real wage gap, as long as $\sigma > 1$. The same is true for the foreign country. It is noteworthy that the share of non-Ricardian households in both countries affects the stabilizing weight of the cross term between home and foreign variables in currency integration.

Finally, in addition to the absence of nominal wage rigidity, the aforementioned loss function corresponds to that derived by Clarida *et al.* (2002) when liquidity constraints are not present in both countries (i.e., $\lambda = \lambda^* = 0$).[14]

[14] See Clarida *et al.* (2002) for a detailed discussion of the central bank's loss function in an open economy.

4.2 The central bank's optimization problem

This paper firstly considers the commitment solution under a currency union. Under a currency union, the benevolent central bank chooses the union-wide interest rate path by intertemporally solving the minimization problem of the central bank's loss function. I focus on the commitment solution as a benchmark case because no studies have investigated the performance of a commitment policy in a currency union under liquidity constraints. However, since the commitment solution is a powerful tool for determining an optimal stabilization policy (McCallum and Nelson, 2004, Woodford, 2003), I also explore a comparison of the commitment and discretionary solutions in Section 5.3.

Under a commitment policy, central banks can commit to future monetary policy stances in the current period. This paper regards the commitment policy as optimal from a timeless perspective as suggested by Woodford (2003).[15] If central banks commit to a policy, they can introduce policy inertia into the economy. This policy inertia then enables central banks to influence the private sector's expectations.

Combining the first-order conditions of the central bank's optimization problem with structural equations, we obtain the following:

$$A_0 X_t = A_1 X_{t-1} + B_1 R_t + \Gamma_t, \tag{48}$$

with

$$\begin{aligned} X_t &= [X_{1t+1} \ E_t X_{2t+1}]', \\ X_{1t} &= [A_t \ A_t^* \ u_t \ u_t^* \ \phi_{1t} \ \phi_{2t} \ \phi_{3t} \ \phi_{4t}]', \\ X_{2t} &= [\tilde{w}_t \ \tilde{w}_t^* \ \pi_t^w \ \pi_t^{w,*} \ \pi_t \ \pi_t^* \ x_t \ x_t^* \ \phi_{5t} \ \phi_{6t}]'. \end{aligned}$$

A_0, A_1, and B_1 are coefficient matrices constructed by deep parameters. X_{1t} denotes the vector for the predetermined and state variables, X_{2t} represents that for the jump variables, R_t is the vector of the central bank's instrumental variables, and Γ_t denotes the vector of structural shocks. ϕ_{1t}, and ϕ_{2t} denote the Lagrange multipliers for the PNKPCs, ϕ_{3t}, and ϕ_{4t} represent those for the WNKPCs. Finally, ϕ_{5t} and ϕ_{6t} are the Lagrange multipliers for the identities

[15] See Chapter 7 in Woodford (2003) for a detailed discussion of optimal monetary policy from a timeless perspective.

for the real wage. The properties of optimal monetary policy are simulated using the Dynare software package.[16]

5 Quantitative results

This section provides the main results. Section 5.1 provides the calibrated deep parameters used in this paper. Section 5.2 presents the results of impulse response analysis under optimal commitment policy for the cases of productivity and price markup shocks occurring in the foreign country. In Section 5.3, I report welfare losses under optimal monetary policy caused by the presence of non-Ricardian households in each country, and we calculate the costs of discretionary policy under several parameterizations of λ and λ^*. Finally, in Section 5.4, I provide several intuitive discussions about the results obtained in this section.

5.1 Calibration

This section describes the deep parameters used in this paper. The discount factor is set to 0.99, and the relative risk aversion coefficient for consumption, σ, is 2.0, based on Pappa (2004). In addition, this paper assumes that parameter η equals 1.0. Following Pappa (2004), the elasticity of substitution between goods, θ_p, is set to 7.88. The elasticity of substitution among labor varieties is set to 5.0. Parameter γ, which denotes the degree of openness, is set to 0.4 as a baseline parameter value. The degree of nominal price rigidity for both countries is set to 0.75. The degree of nominal wage rigidity for both countries is set to 0.5 as a benchmark parameter value.

We provide the calibrated value of λ for both countries. We select four candidates for λ: 0.01, 0.25, 0.5, and 0.75.[17] The first value implies the standard two-country NK economy without liquidity constraints. The second value is based on the estimated value reported by Kaplan *et al.* (2014) and Almgren *et al.* (2019). The third and fourth values correspond to economies such as Latvia in which the share of liquidity constraint is considerably high (Almgren *et al.*, 2019). We examine the impulse response and welfare costs under several combinations of λ and λ^*.

[16] Dynare is available at http://www.dynare.org/.

[17] Section 6 provides a robustness check.

Finally, the standard deviations of a productivity shock, σ_a, and price and wage markup shocks, σ_u, are both set to 0.02. Analogous values are assumed for the foreign country. Regarding the autoregressive coefficients for structural shocks, we set ρ_a and ρ_u to 0.8 and 0, respectively. These calibrated values for economic shocks are presumed to be the same in the foreign country.

In the following, we focus only on foreign structural shocks when conducting impulse response and welfare analysis.

5.2 Optimal monetary policy under commitment

Figure 1 shows the impulse response function under the commitment solution when a foreign productivity shock occurs. In this impulse response analysis, the calibrated value for home liquidity constraints is fixed at 0.25. This shock causes output to decline in the foreign country, and the output gap is negative because inflation does not rise in output as much as potential output. On the other hand, in the foreign country, wage inflation declines in proportion to the decline in the output gap, and the real wage gap also responds negatively.

[Figure 1 around here]

International consumption risk-sharing between home and foreign Ricardian households and changes in the terms of trade cause foreign productivity shocks to spill over into the home country. This encourages a decline in the terms of trade in the home country. However, since $\lambda = 0.25$ and the share of non-Ricardian households is higher in the home country than in the foreign country, the output gap rises in response to the fall in terms of trade. A rise in the output gap in the home country increases inflation in the home country through the NKPC. In addition, wage inflation and real wages will also rise. In other words, a productivity shock in the foreign country leads to an asymmetrical optimal monetary policy response in the home country and the foreign country.

Let us now consider the case in which the liquidity constraints increase in the foreign country. When the liquidity constraints in the home country and the foreign country are the same, the impulse response is the same as that described above. However, when the liquidity constraints are tighter in the foreign country than those in the home country, several interesting results can be obtained. First, when liquidity constraints are greater than 0.5 in the foreign country, the output gap and inflation are attenuated in the home country, which, in turn, counteracts the reduction in wage inflation and the real wage gap. On the other hand, for the home country, an increase in liquidity constraints in the foreign country also alleviates inflation, wage inflation, and real wage gap in the home country. In addition, the output gap in the home country is characterized by a negative response. This suggests that the home country's output gap contributes negatively to the decline in real marginal cost, as the effect of the increased foreign real wage gap dominates the restraining effect of home increased real wages and an attenuated rise in inflation.

Next, let's look at the reaction to a price markup shock in the foreign country. For this one, we check the case of $\lambda^* = 0.01$. In the foreign country, the trade-off between price inflation and the output gap is observed as in the canonical NK model (Woodford, 2003). It also induces a trade-off between higher wage inflation and a reduction in the real wage gap. This is a different response from the productivity shock. Remarkably, the trade-off between wage inflation and the real wage gap is more severe than the trade-off between price inflation and the output gap.

[Figure 2 around here]

In the home country, a foreign markup shock increases the output gap through higher terms of trade, and price inflation also rises through the PNKPC. In other words, the trade-off between price inflation and the output gap is not observed in the home country. However, wage inflation dominates the effect of the decline in the foreign output gap by the rise in the foreign real wage and the decline in the home output gap, so that the home country faces a trade-off between rising wage inflation and falling real wages. The trade-off becomes more severe when $\lambda^* = 0.01$. Let us now consider the case in which foreign liquidity constraints become tighter. Interestingly, foreign liquidity constraints have no effect on the change in the trade-off between price inflation and the output gap in the home country.

Given the value of liquidity constraints in the home country, an increase in liquidity constraints in the foreign country reduces macroeconomic volatility in both countries, even in the event of a shock in the foreign country.

5.3 Liquidity constraints and costs of discretion

We have focused on the case of an optimal commitment policy. In a commitment policy, the central bank minimizes its loss function at some initial point and commits to a time-contingent strategy to influence the expectations of the private sector. In a purely forward-looking model, several studies have examined the performance of a commitment policy versus that of a purely discretionary policy. In contrast to a commitment policy, a discretionary policy enables the central bank to re-optimize its loss function in every period, given the future expectations of the private sector. The welfare loss under a commitment policy is less than that under a discretionary policy (McCallum and Nelson, 2004, Woodford, 2003, Walsh, 2010).[18] Moreover, Pappa (2004) pointed out that a commitment policy can eliminate the deflationary bias induced by a discretionary policy, which is a source of additional gain from commitment associated with an open economy.

It is generally believed that it is difficult for central banks to implement time-consistent commitment solutions. There seems to be a practical aspect of implementing discretionary solutions, because it is difficult for a central bank to make a strong commitment to the private sector.[19] It would also be considered a difficult challenge for the central bank to make a strong commitment to the private sector in a framework such as the two-country model.

Are the costs of implementing a discretionary policy in an economy with a currency union high? And how does the share of non-Ricardian households in both countries affect this cost? Ascari *et al.* (2017) examined welfare losses under commitment, but did not calculate the cost of discretionary policy in an NK model with liquidity constraints. Galí and Monacelli (2016) also calculated the gain from commitment in a currency union model with nominal wage rigidity. Moreover, Groll and Monacelli (2020) compared the cost of discretionary policy in both cases of an exchange rate float and a currency union. However, these studies did not consider the effect of liquidity constraints on the costs from discretion in a two-country NK model. In this paper, we focus on the cost of discretionary policy according to the degree of liquidity constraint in the two-country NK model with a currency union. We focus on the fact that it is unclear whether the costs of discretionary policy are negligible when liquidity constraints play a significant role in a currency union.

[18] If we introduce endogenous persistence into the forward-looking model, then the gain from commitment decreases (Steinsson, 2003).

[19] When the central bank cannot implement a commitment policy, the government may delegate the loss function with policy inertia to it. See Bilbiie (2014) and Walsh (2003) for a detailed discussion of this delegation problem. See also Ida and Okano (2020) for a detailed discussion of the delegation problem in a small-open economy.

Table 1 calculates the effect of a change in non-Ricardian households on the costs from discretionary policy. The welfare loss in each policy regime is expressed in consumption-equivalent units. In calculating the costs of discretionary policy, the welfare losses of both commitment and discretionary policy for each case are expressed according to the relative losses normalized by the commitment solution in the absence of liquidity constraints (i.e., the case for $\lambda = \lambda^* = 0$). To do so, we can assess how the costs from discretion increase in accordance with a change in the share of liquidity constraints in both countries. The percentage differences in Table 1 report the costs from a discretionary policy.

[Table 1 around here]

We obtain the following results. First, importantly, given the share of non-Ricardian households in the home country, the welfare losses of both the commitment solution and the discretionary solution are improved relative to the benchmark case in which non-Ricardian households in the foreign country increase. Second, compared to the case in which $\lambda = \lambda^* = 0.01$, the welfare loss is greater when the share of non-Ricardian households in the home country increases, given the share of non-Ricardian households in the foreign country. Third, the cost of discretionary policy is not great in the NK model under a currency union with non-Ricardian households. For example, in Table 1(a), the cost from discretionary policy is approximately 3.7 % in the case in which $\lambda = 0.01$ and $\lambda^* = 0.75$. Table 1(d) reports the case in which $\lambda = \lambda^* = 0.75$, but, even in that case, the cost from discretionary policy is approximately 6 %. In other words, this result indicates that the costs from discretionary policy are not great compared to the standard two-country NK model.

We also confirm the above result in the impulse response function. Figure 3 plots the responses of the commitment and discretionary solutions to foreign productivity shocks. In Figure 3, we set $\lambda = \lambda^* = 0.25$. We observe that the performance of the commitment and discretionary solutions are not significantly different. However, while there is no difference between the two in terms of the foreign country macroeconomic variables, there is a slight difference for the home country variables. Specifically, in contrast to the discretionary policy, the commitment policy succeeds in introducing policy inertia to the home country macroeconomic variables. However, since the difference is negligible, we can say that, as a result, the costs from discretionary policy are not great in this economy.

[Figure 3 around here]

Next, I show the impulse responses of the commitment and discretionary solutions to a foreign cost-push shock. Figure 4 shows that, as in the case of the productivity shock in Figure 3, there is no difference between the two policies in terms of foreign macroeconomic variables. However, compared to Figure 3, a more pronounced difference can be observed for the home macroeconomic variables in the case of a supply shock. In other words, optimal monetary policy aims to reduce the volatility of the output gap and real wage gap by introducing inertia into price and wage inflation in the home country.

[Figure 4 around here]

The findings from Figures 3 and 4 are summarized as follows. On the one hand, there is no difference in the performance of the commitment and discretionary solutions in the countries in which the shocks occurred. On the other hand, foreign shocks spill over into the home country, and the difference between them is quantitatively small, although the performance of the commitment and discretionary solutions differs in the home country. These results are consistent with the results presented in Table 1, in which the cost of the discretionary solution is low.

5.4 Discussion

This paper demonstrates that considering the difference in the share of liquidity constraints provides important implications for optimal monetary policy in a currency union.

First, as the share of non-Ricardian households increases in both countries, so does social welfare in the currency union. Why do higher liquidity constraints in both countries increase economic welfare? This is a very interesting result that has not been considered in previous studies. For example, in the case of the impulse response function under the commitment solution, we find that, given a share of liquidity constraints in the home country, macroeconomic fluctuations in the home and foreign countries are attenuated as the share of the foreign liquidity constraint increases. In our model, there is rigidity in both prices and nominal wages. Specifically, the existence of nominal wage rigidity implies that real wages are characterized by endogenous inertia. If liquidity constraints become tighter in one country, the central bank that implements the commitment solution will be less likely to adjust the real interest rate through the Phillips curve or Euler equation at different points in time. However, the existence of real wage inertia may enable the central bank to enhance social welfare without forcing the economy to introduce inertia through the commitment solution. Our results show that this is especially true in cases in which the share of non-Ricardian households is predominately large. This naturally implies that the central bank can achieve a substantial improvement in social welfare when the proportion of non-Ricardian households is high in both countries.

Second, why is the performance of the commitment solution and the discretionary solution almost equivalent in the absence of liquidity constraints? The key to understanding this result is the existence of nominal wage stickiness. The fact that nominal wages are sticky indicates that there is endogenous inertia in the economy due to real wages. In other words, if this endogenous inertia through real wages exists in the economy, the central bank will not be able to effectively manipulate the policy inertia path through the history-dependent monetary policy of the commitment solution.

In addition, when liquidity constraints are high in both countries, the central bank is less likely to achieve intertemporal adjustment of the real interest rate through the Euler equation by implementing the commitment solution. Conversely, a change in the nominal interest rate will immediately affect the consumption of non-Ricardian households, implying that the performance of the discretionary policy will be closer to the commitment policy. This indicates that the cost from discretion is only slightly greater, about 6% at most, when liquidity constraints become more severe.

Several previous studies have reported that the cost of discretionary policy is significant (McCallum and Nelson, 2004). On the other hand, in some NK models, such as those with strong inflation inertia and consumption habit formation, there may be no difference in the performance of discretionary and commitment policies (Amato and Labach, 2003, Steinsson, 2003). The contribution of this paper is to show that in a currency union, there is no difference in the performance of discretionary and commitment policies, even in the absence of endogenous inertia with respect to inflation and output. Ascari *et al.* (2017) showed that higher liquidity constraints lead to greater welfare losses under a commitment policy. However, they did not consider the impact of increased liquidity constraints on a discretionary policy. Groll and Monacelli (2020) argued for the importance of endogenous inertia in the terms of trade in a two-country NK model with full wage flexibility. However, they did not consider the role of liquidity constraints in a two-country model. This paper demonstrates that, in a currency union model, given liquidity constraints in the home country, higher foreign liquidity constraints reduce global welfare losses. It also shows that the cost of discretionary policy is generally not great in a currency union with liquidity constraints. To the best of our knowledge, these are our most important contributions to previous studies.

6 Liquidity constraints, openness, and wage stickiness in a currency union

In this section, we perform several robustness checks on the results obtained in Section 5.3. First, we examine the impact of economic openness and the share of home and foreign liquidity constraints on the cost of discretionary policy in a currency union. Second, given various combinations of the shares of home and foreign liquidity constraints, we explore the role of nominal wage rigidity in each country to assess the cost of discretionary policy.

6.1 Share of non-Ricardian households and the degree of openness

In previous studies, a wide range of estimates of the share of non-Ricardian households has been reported in the European region. For example, as noted above, Almgren *et al.* (2019) found that the share of non-Ricardian households is around 10% in Malta, while it is 65% in Latvia. Furthermore, Kaplan *et al.* (2014) reported that the proportion of non-Ricardian households ranges from 20%-35%, mainly in developed countries. Based on these results, we confirm the robustness of how home and foreign liquidity constraints affect the cost of discretionary policy. In addition to a change in the share of non-Ricardian households in each country, we focus on the openness parameter, γ, which plays a significant role in the two-country NK model. In this exercise, we select three values: 0.01, 0.2, and 0.4. The $\gamma = 0.4$ case corresponds to the calibrated benchmark value used in Section 5.

Figure 5 shows the costs from discretionary policy under various parameters of home and foreign liquidity constraints in the case of $\gamma = 0.01$. This case corresponds to a closed economy. This exercise is useful for assessing the role of liquidity constraints in a two-country framework. Note, again, that in calculating the cost from this discretionary policy, the welfare loss in each case is normalized by the loss of the commitment solution in the absence of non-Ricardian households in both countries. Figure 5 shows that regardless of the size of the home country's liquidity constraints, the cost of discretionary policy increases with an increase in the foreign liquidity constraints, but the magnitude is negligible. The maximum value is approximately 2.5 % at $\lambda^* = 0.8$. This result simply reflects the reason that all of the structural shocks generated in this model are foreign-oriented.

[Figure 5 around here]

Figure 6 illustrates the welfare costs from a discretionary policy in the case of $\gamma = 0.4$. Naturally, since the scale of the home country increases, the international spillover effect is larger for $\gamma = 0.4$ than for $\gamma = 0.01$. In Figure 6, compared to the case of Figure 5, the costs from discretion becomes non-negligible as λ and λ^* increase. The costs are around 6% at the maximum level. This result is consistent with the results obtained in the previous section.

[Figure 6 around here]

Therefore, as long as structural shocks occur only in the foreign country, welfare costs under discretion are unaffected by changes in the share of liquidity constraints in the home country. Even if the share of liquidity constraints increases in the foreign country, the costs from discretion are not generally great compared to the commitment case when liquidity constraints are not present in both countries. In fact, in the case of $\gamma = 0.01$, the costs from discretion are approximately 2.5% when the share of liquidity constraints is 0.8 in the foreign country. Even in the case of $\gamma = 0.4$, the costs are approximately 6% at the maximum level. This result indicates that the welfare cost of discretion is generally not greater in the currency union model with liquidity constraints than in the standard two-country NK model.

Summing up, the results obtained in Section 5.3 are confirmed to be robust to changes in the share of liquidity constraints in both countries under a given degree of openness. In other words, this exercise demonstrates the importance of whether the presence of liquidity constraints can significantly affect the performance of optimal monetary policy in a currency union. It also confirms that although the cost of discretion is less than in a two-country NK model without liquidity constraints, the degree of openness, γ, significantly affects the performance of discretionary policy in a currency union with liquidity constraints.

6.2 Share in non-Ricardian households and the degree of nominal wage rigidity

It has been established that the existence of wage stickiness significantly affects optimal monetary policy (Erceg *et al.*, 2000). In other words, whether nominal wages are sticky is important for optimal monetary policy. In particular, it is useful to consider the relationship between liquidity constraints and wage stickiness in a currency union model. Branten, Lamo and Room (2018) and Fabiani, Kwapil, Rõõm, Galuscak and Lamo (2010) examined the severity of wage rigidity in the European region. Fabiani *et al.* (2010) confirmed the existence of downward rigidity in nominal and real wages in the European economy. In addition, Galí (2011) pointed out the importance of the wage version of the new Keynesian Phillips curve (NKPC) using data from the US economy. Galí (2013) examined the impact of wage stickiness on social welfare. Galí and Monacelli (2016) analyzed how the degree of wage flexibility affected optimal monetary policy in a small-open NK model. These studies imply the importance of considering the effect of nominal wage rigidity on optimal monetary policy in a currency area in which liquidity constraints exist. This section examines how liquidity constraints and wage stickiness in both countries affect the cost of discretionary solutions.

Figure 7 shows the welfare costs from a discretionary policy, given values of $\lambda = 0.01$ and $\lambda^* = 0.25$. In Figures 7 and 8, the welfare costs from discretion are normalized by the loss of the commitment solution in the cases of $\lambda = \lambda^* = 0$ and $\zeta = \zeta^* = 0$. To do so, given values for λ and λ^*, we can evaluate the costs arising from discretion under several parameterizations of ζ and ζ^*. This simulation reveals how the degree of wage stickiness in both countries affects the welfare costs of discretion when the share of liquidity constraints is higher in the foreign country than in the home country. Figure 7 shows that regardless of the magnitude of home country's wage stickiness, the higher the nominal wage stickiness is in the foreign country, the greater the welfare costs that arise from discretionary policy. Similar to the results we obtained, the costs are not great, even in this exercise. At the maximum level, 5% of welfare cost is generated when $\zeta^* = 0.8$, given the value of ζ. Thus, as long as the share of liquidity constraints is less than 0.25 in both countries, the costs from discretion are negligible.

[Figure 7 around here]

Figure 8 shows how changes in wage stickiness in both countries affect the cost of discretionary policy when home and foreign liquidity constraints are set to 0.25. These parameter values imply realistic values of liquidity constraints in the euro area and are based on observations reported by Kaplan *et al.* (2014) and Almgren *et al.* (2019). This shows that, similar to the result depicted in Figure 7, the effect of changes in home wage stickiness on the cost of discretionary policy is negligible under a currency union with liquidity constraints of the same size in both countries. However, when foreign wage stickiness is sufficiently high, the cost of discretionary policy exceeds 10%. This means that when liquidity constraints are of the same magnitude in both countries in a currency union, the cost of discretionary policy is not negligible when foreign wage stickiness is sufficiently high. The shape of the graph is similar to when the share of non-Ricardian households is small in both countries. Thus, Figure 8 can be interpreted as depicting the case that when liquidity constraints tighten in both countries, the resultant increase in wage stickiness in the foreign country in which the structural shock occurs pushes the cost of discretionary policy up.

[Figure 8 around here]

This exercise implies that it is robust for the costs of discretionary policies to be negligible when wage stickiness is not extremely high in either country. Needless to say, given the greater value of wage stickiness for the foreign country, we can easily conjecture that the welfare costs from discretion are great for any value of λ and λ^* above 0.25.

While the impact of wage stickiness on welfare loss has been examined by several studies, the impact of liquidity constraints on costs arising from discretion was not considered by (Ascari *et al.*, 2017, Erceg *et al.*, 2000, Galí, 2013, Galí and Monacelli, 2016). In addition, most of the previous studies were based on closed economy models and did not point out the importance of wage stickiness in an open economy.

We also demonstrate that the effect of wage stickiness on welfare loss depends on the magnitude of home and foreign liquidity constraints and the country in which the shock occurs. As noted earlier, Groll and Monacelli (2020) argued for the importance of inertia in the terms of trade in a two-country NK model with full wage flexibility. They did not consider the role of liquidity constraints in a two-country model. In their model, the adoption of a currency union had inherent benefits for the social welfare of both countries. However, this paper addresses the role of liquidity constraints, which leads to a potential source of reducing the costs arising from discretion. The key to understanding this reason is to consider the substantial impact of the real wage gap on social welfare. The terms of trade shift in response to changes in home and foreign output. Moreover, persistent changes in real wage rates lead to endogenous inertial changes in the terms of trade. Unlike previous studies that have pointed out the substantial costs of discretion, this paper establishes the fact that in a currency union with liquidity constraints, the cost of discretionary policy is not greater than that of adopting a commitment policy, as long as nominal wages are highly sticky in both countries.

7 Concluding remarks

This paper examined optimal monetary policy in a two-country new Keynesian model with liquidity constraints. We explored the properties of optimal monetary policy under a currency union because we focused on the empirical fact that the share of non-Ricardian households differs across the member countries of the European economy. We relate the motivation of this paper to these empirical findings in terms of deriving an optimal monetary policy for a currency union.

The contributions of this research to previous studies are as follows. First, we showed that the economic structure differs from the standard two-country model. In particular, it was demonstrated that the consumption risk-sharing in the two countries is affected by the different degrees of liquidity constraints in the two countries. Second, in analyzing the optimal monetary policy, the well-defined loss function was analytically derived by a quadratic approximation of the household utility function. Third, given the share of liquidity constraints in the home country, optimal monetary policy implies that the response of the macroeconomic variables within the currency union to a foreign structural shock is suppressed as liquidity constraints tighten in the foreign country. Fourth, the costs arising from a discretionary policy are not generally greater than those under a commitment policy in our model. We actually calculated the cost of implementing discretionary policy in this model. Fifth, in terms of wage stickiness, we also found that nominal wage stickiness in both countries plays a significant role in determining the costs arising from discretion. This result will not change unless nominal wages remain considerably sticky in both countries.

Finally, I would like to mention a few extensions to this paper. The model in this paper focuses on a currency union. This restriction was required to successfully derive a log-linearized structural model and a loss function for the central bank. However, it would be interesting to analyze how the presence of liquidity constraints interacts between two countries under an exchange rate float. Are the gains from the cooperative solution greater in the presence of liquidity constraints in both countries? This topic is interesting, but beyond the scope of this paper, so it will be left for future work.

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A Appendix : The derivation of the central bank's loss function

A.1 Preliminaries

We derive the loss function approximated around a steady state. Before doing so, I define the relevant notation. First, $\bar{H}$ denotes the value of the steady state, and H_t^n is the value of the efficiency level. Second, as in the main text, I define $\hat{H}_t = \log(H_t/\bar{H})$ as the deviation of H_t from the steady state. To implement the second-order approximation of the planner's objective function, I introduce the following equation:

$$H_t - \bar{H} = \bar{H}\left(\frac{H_t}{\bar{H}} - 1\right) \approx \hat{H}_t + \frac{1}{2}\hat{H}_t^2. \tag{A.1}$$

A.2 Optimal subsidies

The planner's objective function under policy coordination is given by:

$$\begin{aligned}\mathcal{W} =& (1-\gamma)[(1-\lambda)u(C_o) + \lambda u(C_r) - (1-\lambda)V(L_o) - \lambda V(L_r)] \\ &+ \gamma[(1-\lambda^*)u(C_o^*) + \lambda^* u(C_r^*) - (1-\lambda^*)V(L_o^*) - \lambda^* V(L_r^*)]. \end{aligned} \tag{A.2}$$

The planner maximizes this objective function, subject to the following resource constraint:

$$Y = (1-\lambda)L_o + \lambda L_r = \kappa^{-1}C_u,$$
$$Y^* = (1-\lambda^*)L_o^* + \lambda^* L_r^* = \kappa^{-1}C_u.$$

From the first-order conditions, we obtain the following:

$$\frac{V_l(L_o)}{u_c(C_o)} = \frac{V_l(L_r)}{u_c(C_r)}, \tag{A.3}$$

$$\frac{V_l(L_o^*)}{u_c(C_o^*)} = \frac{V_l(L_r^*)}{u_c(C_r^*)}. \tag{A.4}$$

Since, for any time, t $C_{o,t} = C_{o,t}^*$ holds under international consumption risk-sharing between home and foreign Ricardian households, $C_o = C_o^*$ holds at the steady state. Therefore, we obtain the following:

$$C_o = C_o^*,$$

$$C = C_r = C_o = C_o^* = C_r = C^*,$$

$$L = Y = C = C^* = Y^* = L^*.$$

On the one hand, from the optimization problem in intermediate goods firms in each country, we obtain the following labor demand condition at the steady state:

$$w = \frac{1}{\kappa\mu_p(1-\tau)} MPL.$$

On the other hand, the labor supply relationship at the steady state is given by:

$$w = \mu_w MRS.$$

Using the above relationship between labor demand and supply at the steady state, we finally obtain the optimal subsidy as follows:

$$\tau = \tau^* = 1 - \frac{1}{\kappa\mu_p\mu_w}, \tag{A.5}$$

Here, we use the fact that $MRS = MPL = 1$ holds at the steady state.

A.3 Derivation of the central bank's loss function

I now provide the second-order approximation of the household utility function. The derivation of the loss function in the main text is based on the idea of Clarida *et al.* (2002). Thus, I implement the second-order approximation of the planner's objective function around their efficient price and wage equilibrium. The following step in the derivation of the loss function is mainly based on Clarida *et al.* (2002) and Engel (2011).

The home utility function is given by:

$$\mathcal{W}_{H,t} = (1-\lambda)[u(C_{o,t}) - V(L_{o,t})] + \lambda[u(C_{r,t}) - V(L_{r,t})], \tag{A.6}$$

and the corresponding foreign one is given by:

$$\mathcal{W}_{F,t} = (1-\lambda^*)[u(C^*_{o,t}) - V(L^*_{o,t})] + \lambda[u(C^*_{r,t}) - V(L^*_{r,t})]. \tag{A.7}$$

The social planner's objective under a currency union is defined as follows:

$$\mathcal{W}_t = (1-\gamma)\mathcal{W}_{H,t} + \gamma\mathcal{W}_{F,t}. \tag{A.8}$$

Here, as in Ascari *et al.* (2017), since the presence of a labor union in each country leads to $L_{o,t} = L_{r,t} = L_t$ and $L^*_{o,t} = L^*_{r,t} = L^*_t$, Equations (A.6) and (A.7) can be rewritten as follows:

$$\mathcal{W}_{H,t} = (1-\lambda)u(C_{o,t}) + \lambda u(C_{r,t}) - V(L_t), \tag{A.9}$$

$$\mathcal{W}_{F,t} = (1-\lambda^*)u(C^*_{o,t}) + \lambda u(C^*_{r,t}) - V(L^*_t). \tag{A.10}$$

The first and second terms of the right-hand side of Equations (A.9) and (A.10) are given by:

$$u(C_{o,t}) \approx u(C) + u_c C\left[\tilde{c}_{o,t} + \frac{(1-\sigma)}{2}\tilde{c}^2_{o,t} + (1-\sigma)\hat{C}^n_{o,t}\tilde{c}_{o,t}\right] + t.i.p. + O(||\xi||^3), \tag{A.11}$$

$$u(C_{r,t}) \approx u(C) + u_c C\left[\tilde{c}_{r,t} + \frac{(1-\sigma)}{2}\tilde{c}^2_{r,t} + (1-\sigma)\hat{C}^n_{r,t}\tilde{c}_{r,t}\right] + t.i.p. + O(||\xi||^3), \tag{A.12}$$

$$u(C^*_{o,t}) \approx u(C^*) + u_c C\left[\tilde{c}^*_{o,t} + \frac{(1-\sigma)}{2}(\tilde{c}^*_{o,t})^2 + (1-\sigma)\hat{C}^{n,*}_{o,t}\tilde{c}^*_{o,t}\right] + t.i.p. + O(||\xi||^3), \tag{A.13}$$

$$u(C^*_{r,t}) \approx u(C^*) + u_c C\left[\tilde{c}^*_{r,t} + \frac{(1-\sigma)}{2}(\tilde{c}^*_{r,t})^2 + (1-\sigma)\hat{C}^{n,*}_{r,t}\tilde{c}^*_{r,t}\right] + t.i.p. + O(||\xi||^3), \tag{A.14}$$

where a variable with a tilde denotes a gap variable that represents the logarithmic deviation of the variable from the one under a flexible price equilibrium. I also used the following equations:

$$u_c c^n_{o,t} = u_c C + u_c C(1-\sigma)c^n_{o,t} + O(||\xi||^2),$$

$$u_c c^n_{r,t} = u_c C + u_c C(1-\sigma)c^n_{r,t} + O(||\xi||^2),$$

$$u_c c^{n,*}_{o,t} = u_c C + u_c C(1-\sigma)c^{n,*}_{o,t} + O(||\xi||^2),$$

$$u_c c^{n,*}_{r,t} = u_c C + u_c C(1-\sigma)c^{n,*}_{r,t} + O(||\xi||^2).$$

The second-order approximation of the second term of the right-hand side of the planner's objective function is as follows:

$$V(L_t) \approx V(L) + V_l L\left[\tilde{l}_t + \frac{(1+\eta)}{2}\tilde{l}^2_t + (1+\eta)\hat{L}^n_t\tilde{l}_t\right] + t.i.p. + O(||\xi||^3). \tag{A.15}$$

In this derivation, I used the following relationship: $V_l(L) = V_l(L) + V_l(L)(1+\eta)\hat{L}_t^n$. Similarly, the approximation of the third term of the right-hand side is given by:

$$V(L_t^*) \approx V(L^*) + V_l L\left[\tilde{l}_t^* + \frac{(1+\eta)}{2}(\tilde{l}_t^*)^2 + (1+\eta)\hat{L}_t^{n,*}\tilde{l}_t^*\right] + t.i.p. + O(||\xi||^3). \tag{A.16}$$

In this derivation, I used the following relationship: $V_l(L) = V_l(L) + V_l(L)(1+\eta)\hat{L}_t^{n,*}$.

Here, the price dispersion terms are derived as follows:

$$\begin{aligned} L_t &= \int_0^1 L_t(i)di \\ &= \int_0^1 \int_0^1 L_t(j)djdi \\ &= \int_0^1 L_t(i) \int_0^1 \left(\frac{L_t(j)}{L_t(i)}\right) dj\ di \\ &= \int_0^1 L_t(i) \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} djdi \\ &= \int_0^1 L_t(i)di\ \Delta_{w,t}, \end{aligned} \tag{A.17}$$

where $\Delta_{w,t}$ denotes wage dispersion, which is defined as follows:

$$\Delta_{w,t} \equiv \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w}. \tag{A.18}$$

Moreover, using a production function, we can rewrite the above equation as follows:

$$\begin{aligned} \int_0^1 L_t(i)di &= \int_0^1 \left(\frac{Y_t(i)}{Y_t}\frac{Y_t}{A_t}\right) di \\ &= \left(\frac{Y_t}{A_t}\right) \int_0^1 \left(\frac{Y_t(i)}{Y_t}\right) di, \\ &= \left(\frac{Y_t}{A_t}\right) \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon_{p,t}} di \\ &= \left(\frac{Y_t}{A_t}\right) \Delta_{p,t}. \end{aligned} \tag{A.19}$$

Price dispersion is defined as follows:

$$\Delta_{p,t} \equiv \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon_{p,t}} di \tag{A.20}$$

Therefore,

$$L_t = \left(\frac{Y_t}{A_t}\right) \Delta_{p,t}\ \Delta_{w,t}, \tag{A.21}$$

Note that the analogous equations for Equations (A.17) - (A.19) hold for the foreign country.

Next, approximating the production function up to the first order is given by:

$$\tilde{l}_t = x_t + O(||\xi||^2). \tag{A.22}$$

This approximation is expressed by the gap term. Its second-order approximation is derived as follows:

$$\tilde{l}_t = x_t + \frac{\theta_p}{2} var(p_{H,t}) + \frac{\theta_w}{2} var(W_t) + t.i.p. + O(||\xi||^3), \tag{A.23}$$

where

$$\log \Delta_{p,t} \approx \frac{\theta_p}{2} var(p_{H,t}).$$

Similarly, we also derive the second-order approximation of the wage dispersion as follows:

$$\log \Delta_{w,t} \approx \frac{\theta_w}{2} var(W_t).$$

This implies

$$\tilde{l}_t^2 = x_t^2 + t.i.p. + O(||\xi||^3). \tag{A.24}$$

Using these equations, we can rewrite Equations (A.15) and (A.16) as follows:

$$\begin{aligned} V(L_t) \approx V(L) + V_l L \bigg[& x_t + \frac{\theta_p}{2} var(p_{H,t}) + \frac{\theta_w}{2} var(W_t) + \frac{(1+\eta)}{2}(x_t)^2 \\ & + (1+\eta)\hat{L}_t^n \tilde{l}_t \bigg] + t.i.p. + O(||\xi||^3). \end{aligned} \tag{A.25}$$

Similarly, the approximation of the third term of the right-hand side is given by:

$$\begin{aligned} V(L_t^*) \approx V(L^*) + V_l L \bigg[& x_t^* + \frac{\theta_p}{2} var(p_{F,t}^*) + \frac{\theta_w}{2} var(W_t^*) + \frac{(1+\eta)}{2}(x_t^*)^2 \\ & + (1+\eta)\hat{L}_t^{n,*} \tilde{l}_t^* \bigg] + t.i.p. + O(||\xi||^3). \end{aligned} \tag{A.26}$$

Here

$$\tilde{c}_{r,t} = \tilde{w}_t + \tilde{l}_t = \tilde{l}_t + x_t + O(||\xi||^2). \tag{A.27}$$

Therefore, squaring (A.27) implies the following:

$$\tilde{c}_{r,t}^2 = \tilde{w}_t^2 + 2\tilde{w}_t x_t + x_t^2 + O(||\xi||^3). \tag{A.28}$$

Moreover, the second order-approximation of aggregate consumption expressed in the gap term is given by:

$$\begin{aligned}\tilde{c}_{o,t}^2 =&\frac{1}{\Omega^2}\Big[(1-\gamma)^2(1-\lambda)^2 x_t^2 + \gamma^2(1-\lambda^*)^2(x_t^*)^2 + (1-\gamma)^2\lambda^2\tilde{w}_t^2 + \gamma^2\lambda^{*,2}(\tilde{w}_t^*)^2\\&- 2(1-\gamma)^2\lambda(1-\lambda)x_t\tilde{w}_t + 2(1-\gamma)\gamma(1-\lambda)(1-\lambda^*)x_t x_t^*\\&- 2(1-\gamma)\gamma(1-\lambda)\lambda^*\tilde{w}_t^* x_t - 2\gamma^2\lambda^*(1-\lambda^*)x_t^*\tilde{w}_t^* - 2\gamma(1-\gamma)\lambda(1-\lambda^*)\tilde{w}_t x_t^*\\&2(1-\gamma)\gamma\lambda\lambda^*\tilde{w}_t\tilde{w}_t^*\Big] + t.i.p. + O(||\xi||^3). \qquad (A.29)\end{aligned}$$

Similarly, the second-order approximation of the Ricardian households' consumption is derived in the foreign country as follows:

$$\begin{aligned}(\tilde{c}_{o,t}^*)^2 =&\frac{1}{\Omega^2}\Big[(1-\gamma)^2(1-\lambda)^2 x_t^2 + \gamma^2(1-\lambda^*)^2(x_t^*)^2 + (1-\gamma)^2\lambda^2\tilde{w}_t^2 + \gamma^2\lambda^{*,2}(\tilde{w}_t^*)^2\\&- 2(1-\gamma)^2\lambda(1-\lambda)x_t\tilde{w}_t + 2(1-\gamma)\gamma(1-\lambda)(1-\lambda^*)x_t x_t^*\\&- 2(1-\gamma)\gamma(1-\lambda)\lambda^*\tilde{w}_t^* x_t - 2\gamma^2\lambda^*(1-\lambda^*)x_t^*\tilde{w}_t^* - 2\gamma(1-\gamma)\lambda(1-\lambda^*)\tilde{w}_t x_t^*\\&+ 2(1-\gamma)\gamma\lambda\lambda^*\tilde{w}_t\tilde{w}_t^*\Big] + t.i.p. + O(||\xi||^3). \qquad (A.30)\end{aligned}$$

Rearranging Equation (A.29) leads to[20]

$$\begin{aligned}\tilde{c}_{o,t}^2 =&(1-\psi)^2 x_t^2 + \psi^2(x_t^*)^2 + \frac{(1-\gamma)^2\lambda^2}{\Omega^2}\tilde{w}_t^2 + \frac{\gamma^2\lambda^{*,2}}{\Omega^2}(\tilde{w}_t^*)^2 - 2\frac{(1-\gamma)\lambda(1-\psi)}{\Omega}x_t\tilde{w}_t\\&+ 2(1-\psi)\psi x_t x_t^* - 2\frac{\gamma(1-\psi)\lambda^*}{\Omega}\tilde{w}_t^* x_t - 2\frac{\gamma\lambda^*\psi}{\Omega}\tilde{w}_t^* x_t^* - 2\frac{\lambda(1-\gamma)\psi}{\Omega}\tilde{w}_t x_t^*\\&+ 2\frac{(1-\gamma)\gamma\lambda\lambda^*}{\Omega^2}\tilde{w}_t\tilde{w}_t^* + t.i.p. + O(||\xi||^3). \qquad (A.31)\end{aligned}$$

Similarly, the foreign country's counterpart is given by:

$$\begin{aligned}(\tilde{c}_{o,t}^*)^2 =&(1-\psi)^2 x_t^2 + \psi^2(x_t^*)^2 + \frac{(1-\gamma)^2\lambda^2}{\Omega^2}\tilde{w}_t^2 + \frac{\gamma^2\lambda^{*,2}}{\Omega^2}(\tilde{w}_t^*)^2 - 2\frac{(1-\gamma)\lambda(1-\psi)}{\Omega}x_t\tilde{w}_t\\&+ 2(1-\psi)\psi x_t x_t^* - 2\frac{\gamma(1-\psi)\lambda^*}{\Omega}\tilde{w}_t^* x_t - 2\frac{\gamma\lambda^*\psi}{\Omega}\tilde{w}_t^* x_t^* - 2\frac{\lambda(1-\gamma)\psi}{\Omega}\tilde{w}_t x_t^*\\&+ 2\frac{(1-\gamma)\gamma\lambda\lambda^*}{\Omega^2}\tilde{w}_t\tilde{w}_t^* + t.i.p. + O(||\xi||^3). \qquad (A.32)\end{aligned}$$

Substituting Equations (A.30) and (A.31) into Equations (A.11) and (A.13), and substituting Equations (A.27) and (A.28) and their counterparts for the foreign country into Equations

[20]The parameters are defined in the main text.

(A.12) and (A.14), after several calculations, we obtain the following:

$$
\begin{aligned}
\mathcal{W}_t \approx \mathcal{W} - \frac{u_c C\Omega}{2}\Bigg\{ & \frac{(1-\gamma)(1-\lambda)}{\Omega}\Bigg[\bigg(\sigma+\eta-\frac{(1-\sigma)(1-\psi(1-\lambda))}{1-\lambda}\bigg)x_t^2 + (\sigma-1)\lambda\Phi\tilde{w}_t^2 \\
& \theta_p var(p_{H,t}) + \theta_w var(W_t)\Bigg] + \frac{\gamma(1-\lambda^*)}{\Omega}\Bigg[\bigg(\sigma+\eta+\frac{(1-\sigma)((1-\psi)(1-\lambda^*)-1)}{1-\lambda^*}\bigg)(x_t^*)^2 \\
& + (\sigma-1)\lambda\Phi\tilde{w}_t^2 + \theta_p var(p_{F,t}^*) + \theta_w var(W_t^*)\Bigg] - 2(1-\psi)\psi(1-\sigma)x_t x_t^* \\
& - 2\frac{(1-\sigma)\psi(1-\psi)\lambda\lambda^*}{(1-\lambda)(1-\lambda^*)}\tilde{w}_t\tilde{w}_t^* - 2\frac{(1-\sigma)}{\Omega}(\tilde{\lambda}\tilde{w}_t - \tilde{\lambda^*}\tilde{w}_t^*)(x_t - x_t^*)\Bigg\} + t.i.p. + O(||\xi||^3).
\end{aligned} \tag{A.33}
$$

Rearranging this equation yields:

$$
\begin{aligned}
\mathcal{W}_t \approx \mathcal{W} - \frac{u_c C\Omega}{2}\Bigg\{ & (1-\psi)\Bigg[\bigg(\sigma+\eta-\frac{(1-\sigma)(1-\psi(1-\lambda))}{1-\lambda}\bigg)x_t^2 + (\sigma-1)\lambda\Phi\tilde{w}_t^2 \\
& \theta_p var(p_{H,t}) + \theta_w var(W_t)\Bigg] + \psi\Bigg[\bigg(\sigma+\eta+\frac{(1-\sigma)((1-\psi)(1-\lambda^*)-1)}{1-\lambda^*}\bigg)(x_t^*)^2 \\
& + (\sigma-1)\lambda\Phi\tilde{w}_t^2 + \theta_p var(p_{F,t}^*) + \theta_w var(W_t^*)\Bigg] - 2(1-\psi)\psi(1-\sigma)x_t x_t^* \\
& - 2\frac{(1-\sigma)\psi(1-\psi)\lambda\lambda^*}{(1-\lambda)(1-\lambda^*)}\tilde{w}_t\tilde{w}_t^* - 2\frac{(1-\sigma)}{\Omega}(\tilde{\lambda}\tilde{w}_t - \tilde{\lambda^*}\tilde{w}_t^*)(x_t - x_t^*)\Bigg\} + t.i.p. + O(||\xi||^3).
\end{aligned} \tag{A.34}
$$

Now, we define the following:

$$
\begin{aligned}
\Delta_{p,t} &= var(P_{H,t}), \\
\Delta_{p,t}^* &= var(P_{F,t}^*), \\
\Delta_{w,t} &= var(W_t), \\
\Delta_{w,t}^* &= var(W_t^*).
\end{aligned}
$$

Here, following the proposition of Woodford (2003, Chap 6), we can obtain the following conditions for the price dispersion and wage dispersion of both countries. We only derive the second-order approximation for price dispersion in the home country.

$$
\begin{aligned}
\Delta_{p,t} &= var_i[\log(p_{H,t}(i)) - \bar{P}_{H,t-1}], \\
&= E_i[\log(p_{H,t}(i)) - \bar{P}_{H,t-1}]^2 - [E_i \log p_{H,t}(i) - \bar{P}_{H,t-1}]^2, \\
&= \omega E_i[\log(p_{H,t-1}(i)) - \bar{P}_{H,t-1}]^2 + (1-\omega)(\log p_{H,t}^{opt} - \bar{P}_{H,t-1})^2 - (\bar{P}_{H,t} - \bar{P}_{H,t-1})^2.
\end{aligned} \tag{A.35}
$$

Here,

$$
\begin{aligned}
P_{H,t} &= (1-\omega)\log P_{H,t}^{opt} + \omega \bar{P}_{H,t-1},\\
\bar{P}_{H,t} - \bar{P}_{H,t-1} &= (1-\omega)\log P_{H,t}^{opt} - (1-\omega)\bar{P}_{H,t-1},\\
\log P_{H,t}^{opt} - \bar{P}_{H,t-1} &= \frac{1}{1-\omega}(\bar{P}_{H,t} - \bar{P}_{H,t-1}).
\end{aligned}
$$

Then, from Equation (A.35), we obtain the following:

$$
\begin{aligned}
\Delta_{p,t} &= \omega\Delta_{p,t-1} + \frac{1}{1-\omega}(\bar{P}_{H,t} - \bar{P}_{H,t-1})^2 - (\bar{P}_{H,t} - \bar{P}_{H,t-1})^2 + O(||\xi||^3),\\
&= \omega\Delta_{p,t-1} + \frac{\omega}{1-\omega}(\bar{P}_{H,t} - \bar{P}_{H,t-1})^2 + O(||\xi||^3).
\end{aligned}
$$

Since $\bar{P}_{H,t} = \log P_{H,t} + O(||\xi||^2)$.,

$$
\Delta_{p,t} = \omega\Delta_{p,t-1} + \frac{\omega}{1-\omega}\pi_t^2 + O(||\xi||^3)
$$

Then,[21]

$$
\sum_{t=0}^{\infty}\beta^t\Delta_{p,t} = \frac{\omega}{(1-\omega)(1-\omega\beta)}\sum_{t=0}^{\infty}\beta^t\pi_t^2 + t.i.p. + O(||\xi||^3). \tag{A.36}
$$

The analogous equations hold for π_t^*, π_t^w, and $\pi_t^{w,*}$.

$$
\sum_{t=0}^{\infty}\beta^t\Delta_{p,t}^* = \frac{\omega^*}{(1-\omega^*)(1-\omega^*\beta)}\sum_{t=0}^{\infty}\beta^t(\pi_t^*)^2 + t.i.p. + O(||\xi||^3), \tag{A.37}
$$

$$
\sum_{t=0}^{\infty}\beta^t\Delta_{w,t} = \frac{\zeta}{(1-\zeta)(1-\zeta\beta)}\sum_{t=0}^{\infty}\beta^t(\pi_t^w)^2 + t.i.p. + O(||\xi||^3), \tag{A.38}
$$

$$
\sum_{t=0}^{\infty}\beta^t\Delta_{w,t}^* = \frac{\zeta^*}{(1-\zeta^*)(1-\zeta^*\beta)}\sum_{t=0}^{\infty}\beta^t(\pi_t^{w,*})^2 + t.i.p. + O(||\xi||^3). \tag{A.39}
$$

Substituting Equations (A.35)-(A.39) into the discounted sum of $\mathcal{W}_t$, we finally obtain the following:

$$
\begin{aligned}
\mathcal{W}_t - \mathcal{W} \approx -\frac{u_c C\Omega}{2}\sum_{t=0}^{\infty}\beta^t\Bigg\{&(1-\psi)\Bigg[\left(\sigma+\eta-\frac{(1-\sigma)(1-\psi(1-\lambda))}{1-\lambda}\right)x_t^2 + (\sigma-1)\lambda\Phi\tilde{w}_t^2\\
&\frac{\theta_p}{\nu_{H,p}}\pi_t^2 + \frac{\theta_w}{\nu_{H,w}}(\pi_t^w)^2\Bigg] + \psi\Bigg[\left(\sigma+\eta+\frac{(1-\sigma)((1-\psi)(1-\lambda^*)-1)}{1-\lambda^*}\right)(x_t^*)^2\\
&+(\sigma-1)\lambda\Phi\tilde{w}_t^2 + \frac{\theta_p}{\nu_H,p}(\pi_t^*)^2 + \frac{\theta_w}{\nu_{F,w}}(\pi_t^{w,*})^2\Bigg] - 2(1-\psi)\psi(1-\sigma)x_t x_t^*\\
&-2\frac{(1-\sigma)\psi(1-\psi)\lambda\lambda^*}{(1-\lambda)(1-\lambda^*)}\tilde{w}_t\tilde{w}_t^* - 2\frac{(1-\sigma)}{\Omega}(\tilde{\lambda}\tilde{w}_t - \tilde{\lambda^*}\tilde{w}_t^*)(x_t - x_t^*)\Bigg\} + t.i.p. + O(||\xi||^3).
\end{aligned} \tag{A.40}
$$

[21] See Woodford (2003) for a detailed derivation of the following equation.

where

$$\nu_{H,p} = \frac{\omega}{(1-\omega)(1-\omega\beta)},\ \nu_{F,p} = \frac{\omega^*}{(1-\omega^*)(1-\omega^*\beta)},$$
$$\nu_{H,w} = \frac{\zeta}{(1-\zeta)(1-\zeta\beta)},\ \nu_{F,w} = \frac{\zeta^*}{(1-\zeta^*)(1-\zeta^*\beta)}.$$

This equation corresponds to the loss function in the main text.

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Table 1: Welfare costs from discretionary policy

(a) $\lambda = 0.01$

λ^*	Commitment	Discretion	% difference
0.01	1.0000	1.0060	0.60
0.25	0.5235	0.5351	2.18
0.5	0.3281	0.3369	2.64
0.75	0.1786	0.1853	3.68

(b) $\lambda = 0.25$

λ^*	Commitment	Discretion	% difference
0.01	1.0862	1.0957	0.86
0.25	0.6188	0.6351	2.58
0.5	0.4035	0.4164	3.15
0.75	0.2271	0.2361	3.91

(c) $\lambda = 0.5$

λ^*	Commitment	Discretion	% difference
0.01	1.2509	1.2655	1.16
0.25	0.7753	0.8000	3.13
0.5	0.5346	0.5558	3.89
0.75	0.3194	0.3345	4.61

(d) $\lambda = 0.75$

λ^*	Commitment	Discretion	% difference
0.01	1.4927	1.5154	1.51
0.25	1.0606	1.1007	3.71
0.5	0.8054	0.8473	5.06
0.75	0.5465	0.5807	6.08

(*Note*) The values for both the commitment and discretion policies represent welfare losses under each regime. In calculating the costs of discretionary policy, the welfare losses of both commitment and discretionary policies for each case are expressed according to the relative losses normalized by the commitment solution in the absence of liquidity constraints (i.e., the case for $\lambda = \lambda^* = 0$). The percentage difference implies the costs from discretionary policy.

Figure 1: Impulse response to a productivity shock in country F

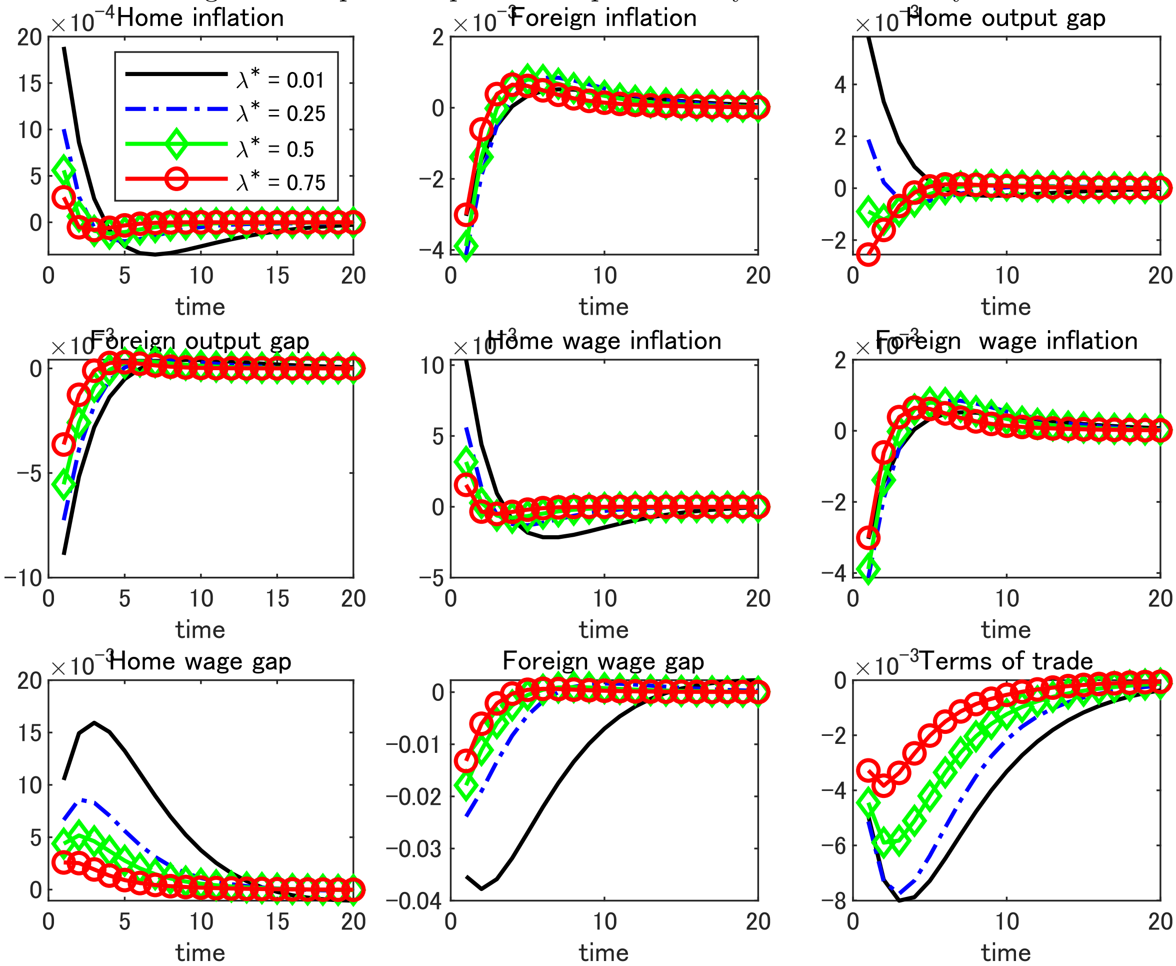


Figure 2: Impulse response to a price markup shock in country F

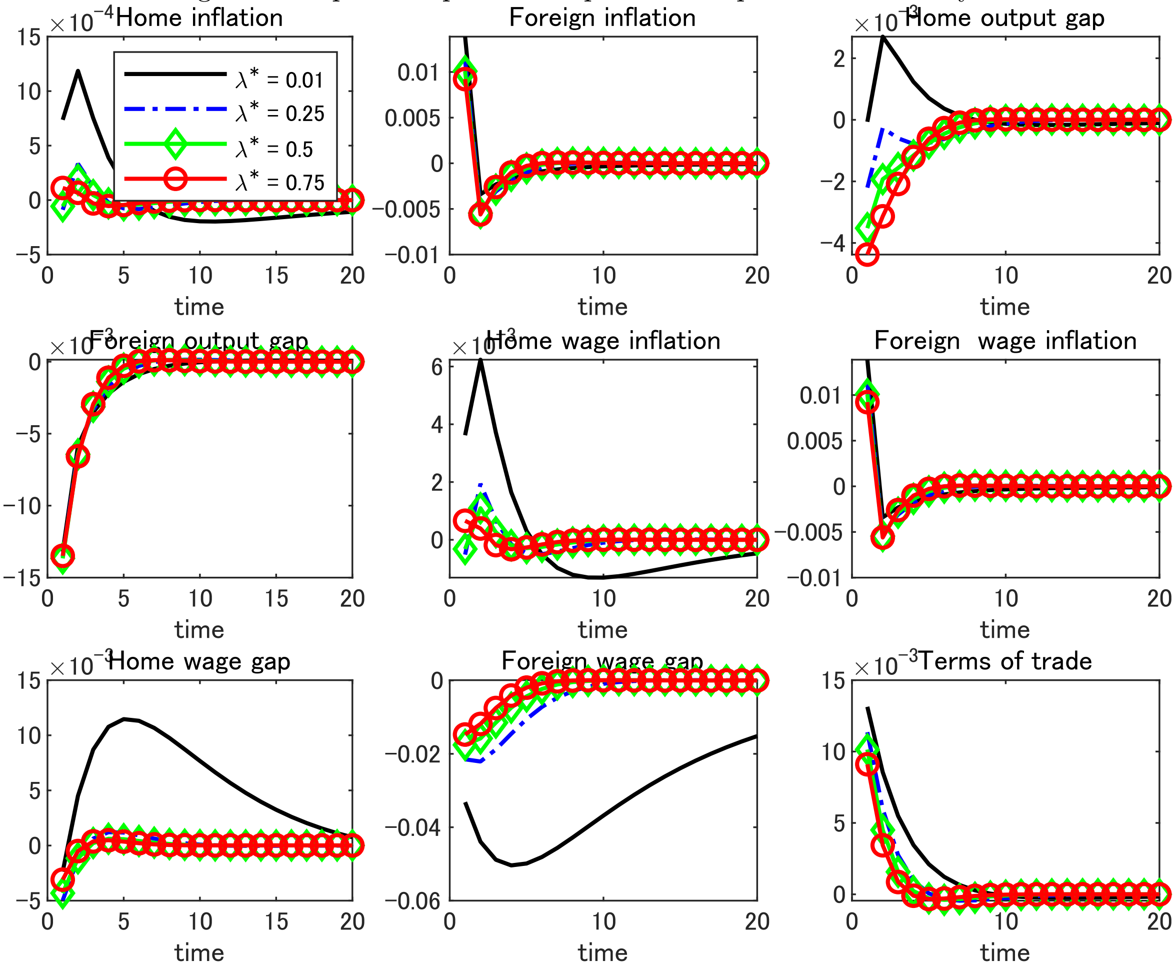
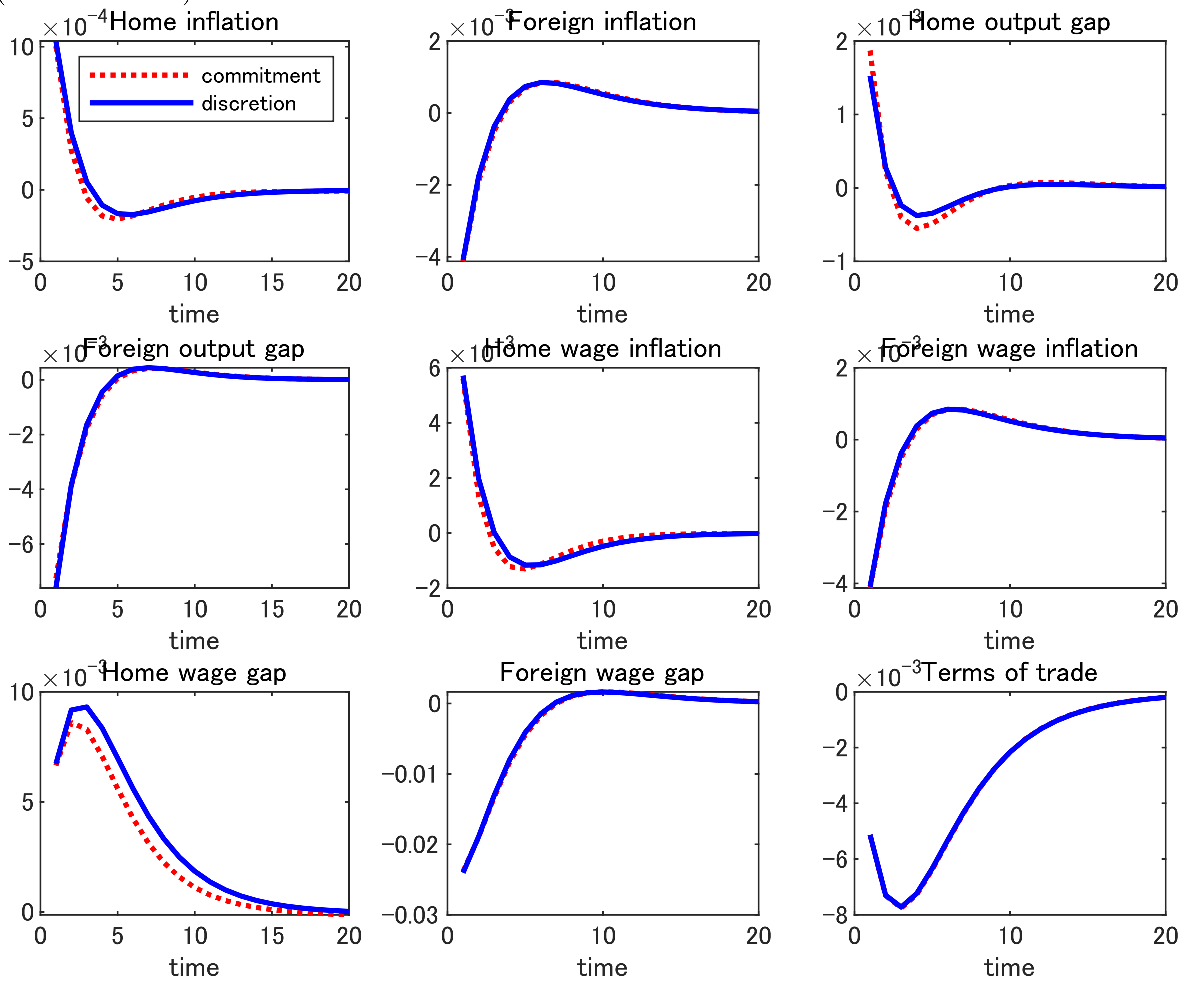


Figure 3: Impulse response to a productivity shock in country F: Commitment vs. Discretion ($\lambda = \lambda^* = 0.25$)



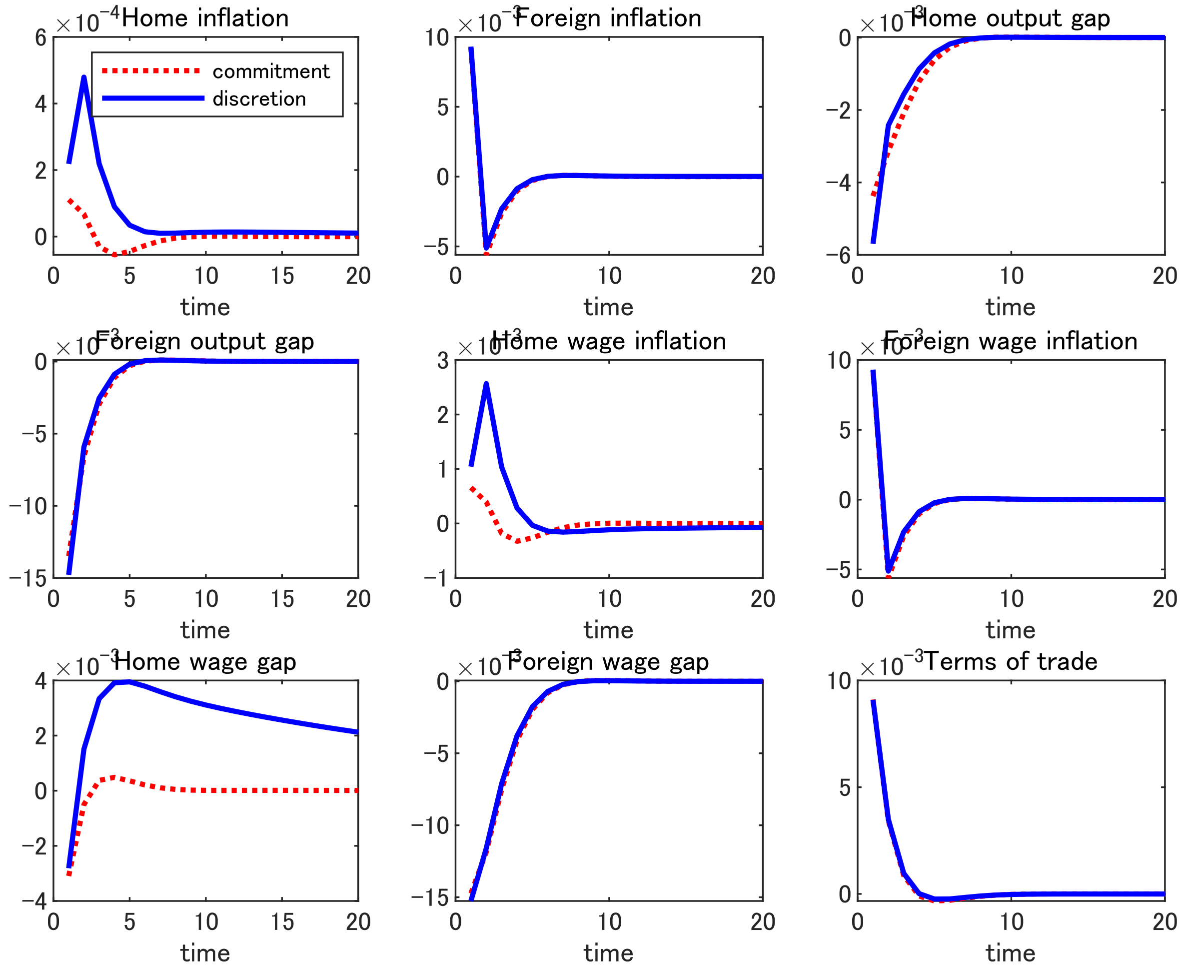
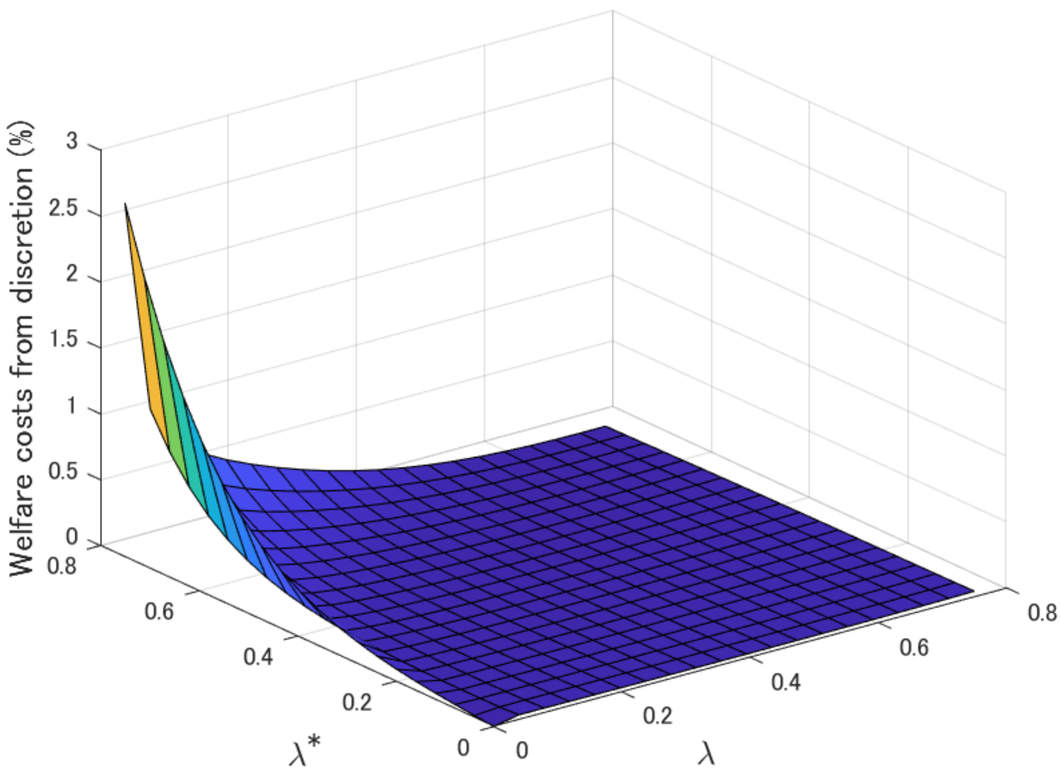


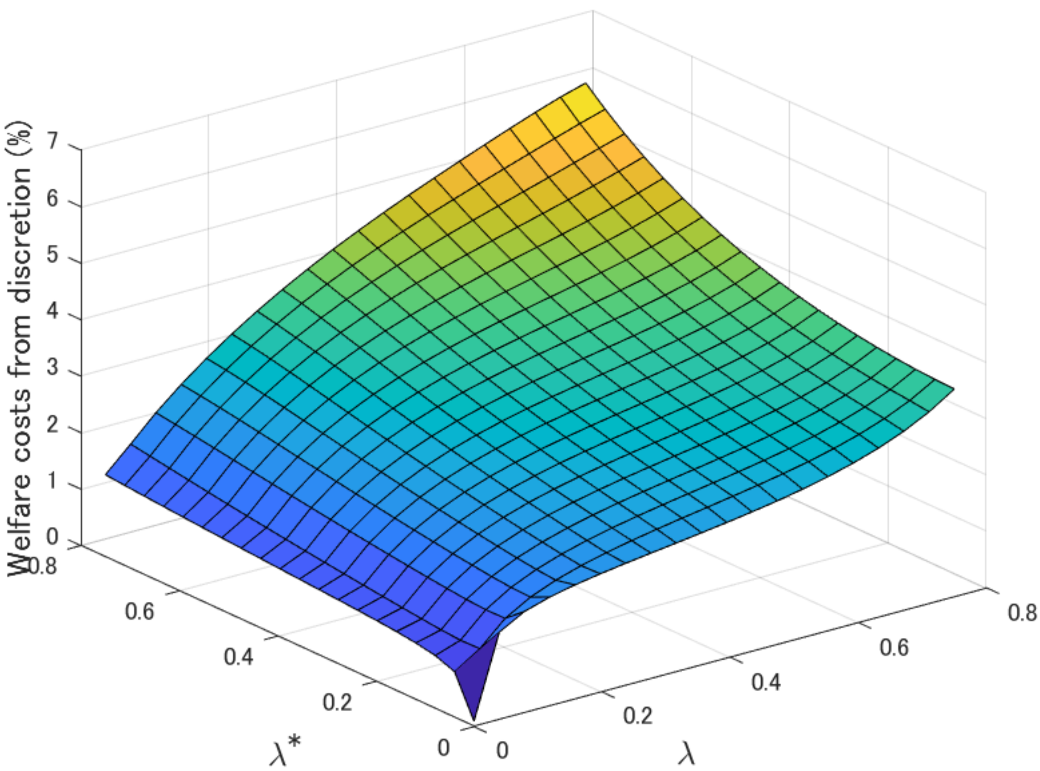
Figure 4: Impulse response to a cost-push shock in country F: Commitment vs. Discretion ($\lambda = \lambda^* = 0.25$)

Figure 5: Welfare costs from discretion ($\gamma = 0.01$)



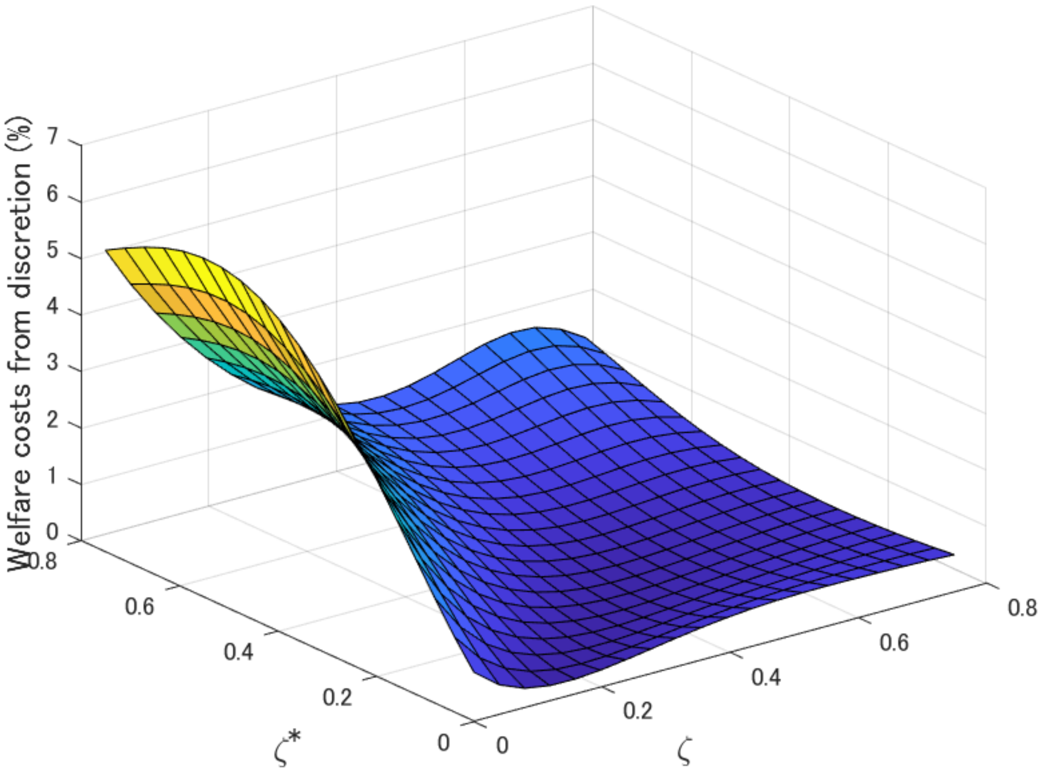
(*Note*) The values for both the commitment and discretion policies represent welfare losses under each regime. In calculating the costs of discretionary policy, the welfare losses of both commitment and discretionary policies for each case are expressed according to the relative losses normalized by the commitment solution in the absence of liquidity constraints (i.e., the case for $\lambda = \lambda^* = 0$). The percentage difference implies the costs from discretionary policy.

Figure 6: Welfare costs from discretion ($\gamma = 0.4$)



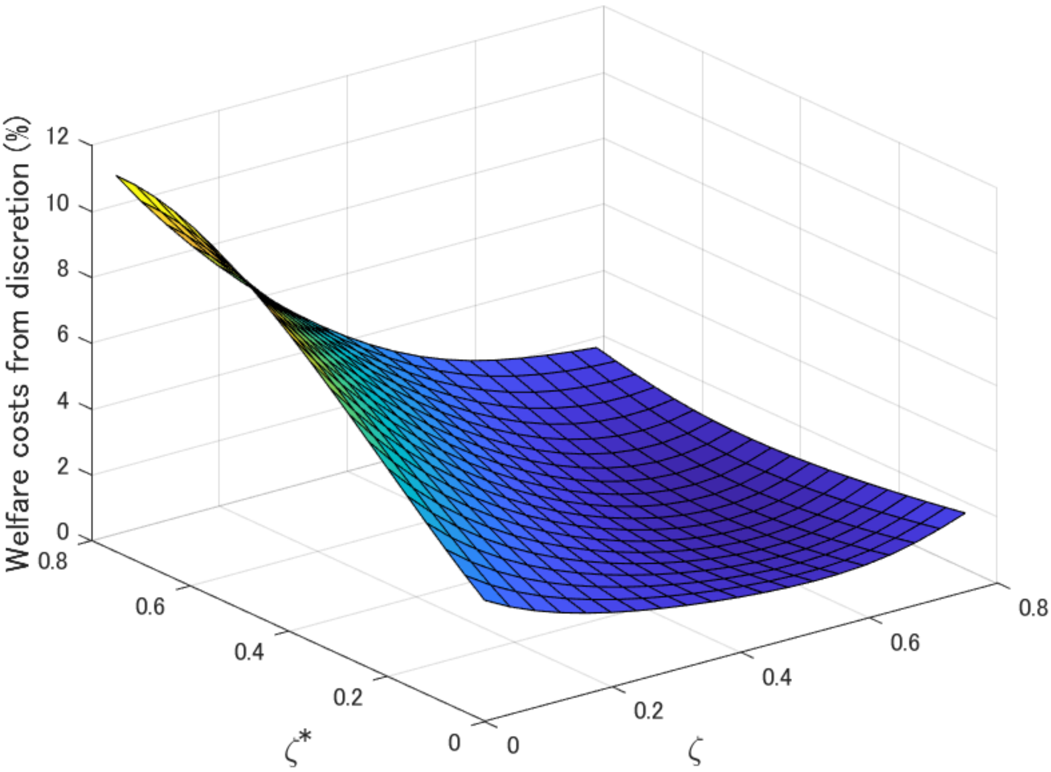
(*Note*) The values for both the commitment and discretion policies represent welfare losses under each regime. In calculating the costs of discretionary policy, the welfare losses of both commitment and discretionary policies for each case are expressed according to the relative losses normalized by the commitment solution in the absence of liquidity constraints (i.e., the case for $\lambda = \lambda^* = 0$). The percentage difference implies the costs from discretionary policy.

Figure 7: Welfare costs from discretion ($\lambda = 0.01$ and $\lambda^* = 0.25$)



(*Note*) The values for both the commitment and discretion policies represent welfare losses under each regime. In calculating the costs of discretionary policy, the welfare losses of both commitment and discretionary policies for each case are expressed according to the relative losses normalized by the commitment solution in the absence of liquidity constraints (i.e., the case for $\lambda = \lambda^* = 0$). The percentage difference implies the costs from discretionary policy.

Figure 8: Welfare costs from discretion ($\lambda = \lambda^* = 0.25$)



(*Note*) The values for both the commitment and discretion policies represent welfare losses under each regime. In calculating the costs of discretionary policy, the welfare losses of both commitment and discretionary policies for each case are expressed according to the relative losses normalized by the commitment solution in the absence of liquidity constraints (i.e., the case for $\lambda = \lambda^* = 0$). The percentage difference implies the costs from discretionary policy.