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The role of wage flexibility and optimal monetary policy in a two-country model*

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Abstract

This paper examines wage flexibility and optimal monetary policy in a two-country New Keynesian (NK) model. In contrast to the two-country model with no nominal wage rigidity, we address the fact that in a two-country model with nominal wage rigidity, the dynamics of the terms of trade significantly characterize the key mechanism of wage flexibility. In particular, when nominal wages in both countries are perfectly flexible, the welfare gains from a commitment policy are the greatest. When nominal wages in the foreign country are stickier; however, the gains from a commitment policy are predominantly reduced. This result is in contrast to the results obtained in previous studies.

JEL codes: E52; E58; F41

Keywords: Optimal monetary policy; Wage flexibility; Policy coordination; Commitment; Discretion

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1 Introduction

The purpose of this paper is to examine the effects of nominal wages and prices on optimal monetary policy in a two-country new Keynesian (NK) model. The importance of wage flexibility and employment fluctuations have been discussed in many previous studies. The importance of the impact of wage flexibility on monetary policy has also been pointed out by (Christiano, Eichenbaum and Evans, 2005, Smets and Wouters, 2003). In addition, as noted by Christiano, Eichenbaum and Evans (1999), the structural model that considers sticky prices and flexible wages might be inconsistent with the fact that monetary tightening causes too sharp a decline in real wages.[1] The existence of nominal wage rigidity causes fluctuations in employment, which, in turn, reduces social welfare. In this case, in addition to standard policy objectives, optimal monetary policy is needed to stabilize fluctuations in wage inflation (Erceg, Henderson and Levin, 2000, Galí, 2013). These studies underscore the importance of considering the role of wage rigidity in structural models. In addition, Palomino, Rodríguez and Sebastian (2020) pointed out the effect of the lockdown and social distancing measures prompted by coronavirus disease 2019 (COVID-19)-induced shocks on wage inequality in Europe.

The spillover mechanism of wage stickiness on international monetary policy has not been sufficiently discussed, however. For example, one may focus on the fact that changes in United States (US) employment levels have led to changes in international asset prices. Indeed, previous studies have pointed out that changes in US employment affect asset prices, such as stock prices and exchange rates (Ederington, Guan and Yang, 2019). Furthermore, since the 1980s, income inequality has emerged in the US, and unemployment characterized by wage rigidity has increased in Europe (Cahuc, Carcillo and Zylberberg, 2014). In fact, as noted by Galí and Monacelli (2016), a decline in the home country's wage leads to a depreciation in the terms of trade, which prompts increases in its output and employment. Recently, Iwasaki, Muto and Shintani (2021) explored an international comparison of the role of wage rigidity by using a non-linear dynamic stochastic general equilibrium model. In this study, we consider whether the mechanism suggested by Galí and Monacelli (2016) still holds for the case of a two-country economy model.

Why do policymakers and the private sector in countries such as the Eurozone and Japan pay so much attention to fluctuations in US employment? Needless to say, it is because policy-

[1] Colciago (2011) considered the role of wage flexibility in an economy with liquidity constraints.

makers in other countries may be concerned about recessions caused by the exporting of a large country's unemployment, as long as there are international links between financial markets (Ederington *et al.*, 2019). Several studies have explored the role of wage flexibility in the Eurozone (Branten, Lamo and Room, 2018, Fabiani, Kwapil, Rõõm, Galuscak and Lamo, 2010). In these cases, there may be room for both home and foreign policymakers to consider the possibility of international monetary policy coordination that might maximize global welfare losses by reducing the volatility in their respective employment levels. While this is an important topic, the international spillover mechanism, and its implications for monetary policy have not been adequately discussed in terms of optimal monetary policy.

In this paper, we construct a two-country NK model to examine the impact of wage flexibility on the international dimension of optimal monetary policy. Specifically, we construct a two-country NK model facing both nominal wage and price rigidity. We simply incorporate the idea suggested by Erceg *et al.* (2000) into the two-country NK model developed by Clarida, Galí and Gertler (2002). Specifically, we construct a two-country NK model in which nominal wage and price rigidity are asymmetric between the two countries. As pointed out by Rhee and Turdaliev (2013), the terms of trade depend on the real wage gap in an open economy. In our model, the key understanding is to consider how the real wage gap plays a significant role in the two-country model. This is because the terms of trade change in response to fluctuations in domestic and foreign output in the two-country model. Finally, we derive the loss function of the central bank by computing a quadratic approximation of the household utility function when international policy coordination between the home and foreign central banks is considered. This derivation is a natural extension of an NK model that incorporates nominal wage rigidity.

The main findings of our paper are summarized as follows. In contrast to the two-country model with no nominal wage rigidity, we address the fact that the dynamics of the terms of trade significantly characterize the key mechanism underlying wage flexibility. Given wage flexibility in the home country, we show that changes in the degree of foreign wage flexibility substantially impact the worldwide welfare losses and gains from commitment policy. Specifically, when nominal wages in both countries are perfectly flexible, the welfare gains from a commitment policy are the greatest. However, when nominal wages in the foreign country are stickier, the gains from commitment are predominantly reduced. This result is robust to any changes in several key parameters that play an important role in the two-country NK model. Therefore, we emphasize that some of the results obtained in this paper contrast with those obtained in previous studies.

The structure of this paper is as follows. Section 2 briefly reviews the related literature, and Section 3 provides a description of the two-country NK model in which nominal wage and price rigidity coexist. Section 4 discusses optimal monetary policy in the model. Section 5 reports the main results of the paper. In Section 6, we explore the gains from a commitment policy in our model. Section 7 briefly concludes. Appendix A provides a derivation of the macroeconomic variables under flexible prices and wages in our two-country model. In the online appendix, we drive the central bank's loss function by calculating the second-order approximation of the household utility function in our model.

2 Related literature

The objective of this section is to clarify previous papers related to this study. First, we briefly review the effect of nominal wage flexibility on optimal monetary policy. Erceg *et al.* (2000) introduced nominal wage rigidity into a standard NK model and examined its impact on optimal monetary policy. Their study demonstrated that a stabilizing term in wage inflation due to staggered changes in nominal wages added to the central bank's policy objective as well as stabilized inflation and output. Galí (2011) addressed the role of wage inflation in explaining actual wage inflation dynamics in the US economy. Galí (2013) examined the impact of nominal wage flexibility on optimal monetary policy in an NK model with nominal price and wage rigidity based on the observation that the wage Phillips curve could explain wage dynamics in the US.[2] He showed that higher wage flexibility does not necessarily lead to higher social welfare.[3] Ascari, Colciago and Rossi (2017) examined the role of liquidity constraints in an NK model with nominal price and wage rigidity. While these studies provide important

[2]Several studies have focused on the role of wage flexibility in examining monetary policy conducted under the zero lower bound (ZLB) on nominal interest rates (Glover, 2019, Shen and Yang, 2018). The topic of the ZLB is very important, but that investigation is beyond the scope of this paper. How does the ZLB affect the properties of optimal monetary policy in a two-country NK model with nominal wage rigidity? We would like to examine this issue as a future work.

[3]This argument is based on the results of the calculation of welfare losses under several parameterizations of nominal wage rigidity in the case of the Taylor rule.

contributions to the study of optimal monetary policy, we point out the limitation that they all focused on wage flexibility and optimal monetary policy in a closed economy.

Second, we focus on a literature review on the effect of wage flexibility on international aspects of optimal monetary policy. How does wage flexibility affect the international dimension of monetary policy? Several studies have emphasized the international dimension of optimal monetary policy, but the role of wage flexibility has not yet been incorporated into a standard two-country NK model (Clarida, Galí and Gertler, 2001, Clarida *et al.*, 2002, De Paoli, 2009b, Engel, 2011). Groll and Monacelli (2020) addressed the role of terms of trade dynamics under a commitment policy in a two-country model, but their insights were based on a fully flexible model of nominal wages. While Monacelli (2003) showed that welfare losses due to a discretionary policy were substantially greater than those due to a commitment policy, their results were based on those suggested for a small-open NK economy with flexible nominal wages.

Third, we address the contribution of this paper to previous studies. What role has been considered by previous studies for wage flexibility in terms of an open economy? Several studies have focused on wage flexibility in small-open NK models. Our study is related to Galí and Monacelli (2016), who examined the welfare gain from wage flexibility in a small-open NK model. They showed that in contrast to conventional wisdom, increased wage flexibility often worsens social welfare in a currency union. Our study is also related to Campolmi (2014) and Rhee and Turdaliev (2013), who argued the important role of consumer price index (CPI) inflation targeting when nominal wage rigidity was introduced in a small-open NK model. They derived a central bank's loss function for a small-open economy, but calculated it assuming that both the intertemporal substitution of consumption and substitution between home and foreign goods was unity. In contrast to their work, we derive the central bank's loss function without assuming a unitary elasticity of intertemporal substitution of consumption.[4] A key to understanding this argument is to observe the impact of real wage changes on the terms of trade, since a change in the terms of trade affects both producer price index (PPI) and CPI inflation.

While these studies addressed the role of wage flexibility in a small-open NK model or a

[4]The Cobb-Douglas consumption basket is assumed in our derivation of the central bank's loss function. See Pappa (2004) and Groll and Monacelli (2020) for a detailed derivation under a constant elasticity of substitution (CES)-type consumption basket.

currency union, they did not consider the interaction of wage flexibility through the terms of trade in a two-country model with an exchange rate float. Considering the role of wage flexibility in the international dimension of monetary policy is supported by the empirical facts mentioned in the Introduction. Our paper is deeply related to Groll and Monacelli (2020), who argued for the importance of endogenous inertia in the terms of trade in a two-country NK model with wage flexibility. In their model, the adoption of a currency union created inherent benefits for the social welfare of both countries. Ida (2021) considered the role of liquidity constraints in a currency union with nominal wage rigidity. However, their model did not consider the role of wage flexibility in a two-country NK model with an exchange rate float. Our study addresses the impact of endogenous inertia in the terms of trade through the real wage rigidity caused by staggered nominal wages. To the best of our knowledge, none of the previous studies emphasized the role of wage flexibility in a two-country NK framework.

Although our model is a natural extension of the two-country model developed by Clarida *et al.* (2002) to the case of nominal wage rigidity, we discuss the contribution of this paper in terms of several important results that were not obtained in previous studies. First, our study demonstrates that foreign wage flexibility significantly impacts worldwide welfare losses, depending on the degree of flexibility in the home country's wages. For example, while Galí and Monacelli (2016) showed how the degree of wage flexibility affected welfare loss in a currency union, this study focuses on how changes in wage flexibility in both countries affect worldwide welfare loss in a two-country model with an exchange rate float. More concretely, while they pointed out that wage flexibility negatively affects social welfare, our study shows that it does not always do so, from the perspective of a two-country NK model. Second, we demonstrate the welfare gains from commitment policy in a two-country NK model. Clarida *et al.* (2001), Ida and Okano (2020), and Monacelli (2003) argued that commitment is more beneficial than discretion in a small-open economy. While these studies focused on the gains from commitment, they did not consider the role of wage flexibility in accounting for the delegation problem under a discretionary policy. Our model illustrates the substantial effect of wage flexibility on the performance of commitment policy in both countries in a two-country model.

3 Model

We consider a two-country NK model with nominal wage rigidity. Consider an economy with two large symmetric countries: home and foreign. The size of the economy for home and foreign is $1 - \upsilon$ and υ, respectively. There are two production sectors in each country: a final goods sector that is characterized by perfect competition, and an intermediate goods sector in which firms face monopolistic competition and Calvo (1983)-type nominal price rigidity. We also assume that the degree of price stickiness differs in each country.

We assume that there are complete markets in both countries, and that only final goods are traded. The number of final goods producers is equal to the number of households in each country. Finally, unless otherwise noted, analogous equations hold for the foreign country. In deriving several equations, we express the log-deviations from the steady state using lower case letters. First, H represents the value of the steady state, and H_t^n is the value of the efficiency level. We define $h_t = \log(H_t/H)$ as the deviation of H_t from the steady state. Note that the variables for the foreign country are denoted by an asterisk.

Except for the introduction of nominal wage rigidity, the model we adopt is based on the standard two-country NK model developed by Clarida *et al.* (2002). Readers familiar with the two-country NK model can skip to Section 4.

3.1 Preferences

Preferences for consumption in the home country are given as follows:

$$C_t \equiv \Upsilon C_{H,t}^{1-\upsilon} C_{F,t}^{\upsilon}, \tag{1}$$

where $\Upsilon \equiv 1/\left(1-\upsilon\right)^{1-\upsilon} \upsilon^{\upsilon}$, $C_{H,t}$ is the consumption of domestic goods, and $C_{F,t}$ is the consumption of foreign goods.

The household's cost minimization yields the following:

$$P_{H,t} C_{H,t} = (1-\upsilon) P_t C_t, \tag{2}$$

$$P_{F,t} C_{F,t} = \upsilon P_t C_t, \tag{3}$$

where the price index in the home country is given by the following:

$$P_t \equiv P_{H,t}^{1-\upsilon} P_{F,t}^{\upsilon} = P_{H,t} \mathcal{S}_t^{\upsilon}, \tag{4}$$

where $P_{H,t}$ is the price of domestic goods, and $P_{F,t}$ is the price of foreign goods. S_t represents the terms of trade, which are given as follows:

$$\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}}. \tag{5}$$

3.2 Households

The intertemporal utility of an infinitely lived representative household is as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{1}{1+\varphi} \int_0^1 \mathcal{N}_t(j)^{1+\varphi} dj \right), \tag{6}$$

where β denotes the discount factor, C_t is consumption, and $N_t(j)$ is the household's type j's labor supply.[5] Parameter σ denotes the relative risk aversion coefficient for consumption, and φ is the inverse of the Frisch elasticity of the labor supply.

The representative household maximizes the above utility function subject to the following budget constraint as follows:

$$\int_0^1 P_{H,t}(i) C_{H,t}(i) di + P_{F,t} C_{F,t} + E_t \left\{ Q_{t,t+1} D_{t+1} \right\} \leq \int_0^1 W_t(j) \mathcal{N}_t(j) dj + D_t \Gamma_t - T_t \tag{7}$$

where D_t is nominal bonds held for one period, $W_t(j)$ denotes nominal wages, and Γ_t denotes the nominal wage and dividend, respectively, earned from domestic firms. T_t denotes a lump-sum tax. We assume that the households located in each country can access state-contingent bonds traded in a complete market both domestically and internationally and introduce the following stochastic discount factor as follows:

$$E_t(Q_{t,t+1}) = \frac{1}{1+i_t}, \tag{8}$$

where $Q_{t,t+1}$ denotes a stochastic discount factor, and i_t is the nominal interest rate.

The first-order condition of the household's utility maximization problem yields the following familiar consumption Euler equation as follows:

$$1 = \beta \left(1 + i_t\right) E_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{1/\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right\}. \tag{9}$$

where i_t is the nominal interest rate.

[5] We assume that there is a variety of labor $j \in [0, 1]$; we can also assume that each household has $j \in [0, 1]$ members of the labor force, as in Galí and Monacelli (2016).

Next, we consider a risk-sharing condition between countries. The Euler equation for foreign consumption denominated in the domestic currency is as follows:

$$1 = \beta \left(1 + i_t^*\right) E_t \left\{ \left(\frac{C_t^*}{C_{t+1}^*} \right)^{1/\sigma} \left(\frac{P_t/\mathcal{E}_t}{P_{t+1}/\mathcal{E}_{t+1}} \right) \right\}. \tag{10}$$

where i_t^* denotes the foreign nominal interest rate and $\mathcal{E}_t$ is the nominal exchange rate. By assuming that there exist state-contingent bonds that allow both domestic and foreign households to trade internationally, combining Equation (10) with the Euler equation for domestic consumption and the definition of the real exchange rate, the real exchange rate becomes the following:

$$C_t^\sigma = \vartheta C_t^{*\sigma} \left(\frac{\mathcal{E}_t P_t^*}{P_t} \right). \tag{11}$$

Since our model is based on the assumption of producer currency pricing, the law of one price holds. [6]

$$\frac{\mathcal{E}_t P_t^*}{P_t} = 1.$$

Therefore, taking $\vartheta = 1$ without any loss of generality, as shown in Clarida *et al.* (2002), international consumption risk-sharing leads to[7]

$$C_t = C_t^*, \tag{12}$$

for all t.

3.3 Wage determination

In each country, households delegate the role of wage determination to a labor union. Following Erceg *et al.* (2000), the wage setting is subject to Calvo-type staggered wage contracts. Thus, a fraction of $1 - \theta_w$ can change nominal wages in its union, whereas the remaining fraction of θ_w cannot do so. Under this setting, the labor union for each country solves the following

[6]When we assume local currency pricing, the following relationship does not hold. See Engel (2011) for a detailed discussion of this issue.

[7]In our model, we do not consider the presence of a consumption home bias. As shown in Pappa (2004) and Engel (2011), the real exchange rate adjusts the difference between home and foreign consumption in the presence of a home bias.

maximization problem:

$$E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k U\left(C_{t+k|t}, \mathcal{N}_{t+k|t}\right), \tag{13}$$

subject to

$$\mathcal{N}_{t+k|t} = \left(\frac{W_t^o}{N_{t+k}}\right)^{-\epsilon_w} N_{t+k}, \tag{14}$$

$$P_{t+k}C_{t+k} + E_t\left\{Q_{t+k,t+k+1}D_{t+k|t}\right\} = W_t^o \mathcal{N}_{t+k|t} + D_t - T_t + \Gamma_t,$$

where $C_{t+k|t}$ and $C_{t+k|t}$ denote consumption and labor, respectively, in period $t+k$ for a union that last reset its nominal wage in period t. Equation (14) denotes the demand for labor in period t for $t+k$ periods ahead, and ϵ_w is the elasticity of substitution for individual labor demand. W_t^o denotes the optimal nominal wage that the labor union chooses in the current period.

The first-order condition of this maximization problem yields the following:

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t\left\{N_{t+k|t}U_{c,t+k}\left(\frac{W_t^o}{P_{t+k}} - \mathcal{M}^w MRS_{t+k|t}\right)\right\} = 0, \tag{15}$$

where $\mathcal{M} \equiv \frac{\epsilon_w}{\epsilon_w - 1}$ and $MRS_{t+1|t} \equiv C_{t+k}^{\sigma}\mathcal{N}_{t+k|t}^{\varphi}$.

Log-linearizing this equation around the steady state yields the following:

$$w_t^o = \mu^w + (1-\beta\theta_w)\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t\left\{mrs_{t+k|t} + p_{t+k}\right\}. \tag{16}$$

$$W_t \equiv \left(\int_0^1 W_t(j)^{1-\epsilon_w} dj\right)^{1/(1-\epsilon_w)}. \tag{17}$$

Log-linearization of the wage index is given as follows:

$$w_t = \theta_w w_{t-1} + (1-\theta_w) w_t^o, \tag{18}$$

where $w_t \equiv \log W_t$. Substituting Equation (18) into Equation (16), we obtain the following wage new Keynesian Phillips curve (WNKPC) as follows:

$$\pi_t^w = \beta E_t\left\{\pi_{t+1}^w\right\} - \lambda_w\left(\mu_t^w - \mu^w\right), \tag{19}$$

where π_t^w ($\equiv w_t - w_{t-1}$) denotes wage inflation, and μ_t^w ($\equiv w_t - p_t - mrs_t$) denotes the average wage markup, which is related to the marginal rate of substitution between consumption and the labor supply. In addition,

$$\lambda_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)} > 0.$$

3.4 Firms

Each country has two production sectors. The first is the final goods sector, which produces final goods using intermediate goods and is characterized by perfect competition. The second is the intermediate goods sector, in which firms face monopolistic competition and Calvo-type nominal price rigidity.

Final goods firms

The final goods sector is perfectly competitive and producers use inputs that are produced in the intermediate goods sector. Specifically, final goods are produced according to the following CES aggregate:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\epsilon_{p,t}-1}{\epsilon_{p,t}}} di\right]^{\frac{\epsilon_{p,t}}{\epsilon_{p,t}-1}}, \tag{20}$$

where Y_t is aggregate output, $Y_t(i)$ is the demand for intermediate goods produced by firm i, and $\epsilon_{p,t}$ is the elasticity of substitution, which is time-varying, as assumed by Steinsson (2003).[8] Note that both variables are normalized by $1-\nu$.

Under the CES aggregate, the demand function is given by:

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon_{p,t}} Y_t, \tag{21}$$

and the domestic price level is defined as follows:

$$P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\epsilon_{p,t}} di\right]^{\frac{1}{1-\epsilon_{p,t}}}, \tag{22}$$

where $P_{H,t}(i)$ is the price for intermediate goods produced by firm i. Note that these variables are also normalized by $1-\nu$.

Intermediate goods firms

The goods sector is characterized by monopolistic competition, and each firm produces a differentiated intermediate good. Firm i's production function is given by the following:

$$Y_t(i) = A_t N_t(i)^{1-\alpha}, \tag{23}$$

[8] Clarida *et al.* (2002) presumed the time-varying wage markup to introduce the exogenous cost-push shock. In this study, instead of introducing the wage markup, a price markup derived from a time-varying elasticity of substitution $\epsilon_{p,t}$ is introduced to examine the effect of a cost-push shock on optimal monetary policy. See Clarida *et al.* (2002) for a detailed discussion of the presence of a wage markup shock.

where α represents the parameter that indicates diminishing returns to scale in the production function. A_t denotes an aggregate productivity disturbance, which follows an AR (1) process given by $\log A_t = \rho_a \log A_{t-1} + \epsilon_t^a$ with $0 \leq \rho_a < 1$, where ϵ_t^a is an independent and identically distributed (i.i.d.) shock with constant variance σ_a^2.

In addition, firm i's employment is characterized by the following CES index:

$$N_t(i) \equiv \left(\int_0^1 N_t(i,j)^{\frac{\epsilon_w - 1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}. \tag{24}$$

The demand for labor is given as follows:

$$N_t(i,j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t(i), \tag{25}$$

and

$$\mathcal{N}_t(j) = \int_0^1 N_t(i,j)\, d_j = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t, \tag{26}$$

where $N_t \equiv \int_0^1 N_t(i)\, di$.

Following Calvo (1983), we assume that price rigidity is present in the goods sector. The following explanation focuses on the home country, whereas we can consider the case for the foreign country. Thus, a fraction $1 - \theta_p$ of all firms adjusts their price, while the remaining fraction of firms θ_p do not. We now consider the firms that can adjust their price. When revising their prices, these firms consider the uncertainty related to when they will next be able to adjust prices. In this case, the firm's optimization problem for the home country is given as follows:

$$E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left(P_{H,t}^o Y_{t+k|t} - \Psi_{t+k}\left(Y_{t+k|t} \right) \right),$$

subject to

$$Y_{t+k|t} = \left(\frac{P_{H,t}^o}{P_{H,t+k}} \right)^{-\epsilon_{p,t+k}} Y_{t+k},$$

where $p_{H,t}^o$ denotes optimal prices.

The first-order condition of this optimization problem yields the following:

$$\sum_{k=0}^{\infty} \theta_p^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left(P_{H,t}^o - \mathcal{M}_p \psi_{t+k|t} \right) \right\} = 0, \tag{27}$$

where $\mathcal{M}_p$ ($\equiv \epsilon_p/(\epsilon_p - 1)$) denotes the price markup. The variable Ψ_t denotes the real marginal cost, which is given as follows:

$$\Psi_{t+k|t} \equiv \frac{W_{t+k}\left(1+\tau_t\right)}{\left(1-\alpha\right) A_{t+k} N_{t+k|t}^{-\alpha}}. \tag{28}$$

We also defined *average* nominal marginal cost as follows:

$$\Psi_t \equiv \frac{W_t\left(1+\tau_t\right)}{\left(1-\alpha\right) A_t N_t^{-\alpha}}. \tag{29}$$

Log-linearizing Equation (27) leads to the following optimal price dynamics:

$$p_{H,t}^o = \mu^p + \left(1-\beta\theta_p\right)\sum_{k=0}^{\infty}\left(\beta\theta_p\right)^k E_t\left\{\psi_{t+k|t}\right\}, \tag{30}$$

where $\mu^p \equiv \log \mathcal{M}^p$ and $\psi_{t+k|t}$ ($\equiv \log \Psi_{t+k|t}$) denote the k-periods-ahead real marginal cost in period t. The log-linearization of the aggregate price index evolves based on the following equation.

$$p_{H,t} = \theta_p p_{H,t-1} + \left(1-\theta_p\right) p_{H,t}^o, \tag{31}$$

where $p_{H,t}^o \equiv \log P_{H,t}^o$. Substituting Equation (31) into Equation (30) and implementing several mathematical manipulations, we obtain the following price new Keynesian Phillips curve (PNKPC) expressed by the real marginal cost.

$$\pi_{H,t}^p = \beta E_t\left\{\pi_{H,t+1}^p\right\} - \lambda_p\left(\mu_t^p - \mu^p\right) + u_t, \tag{32}$$

where $\pi_{H,t}^p$ ($\equiv p_{H,t} - p_{H,t-1}$) denotes the PPI inflation rate, $p_{H,t} = \log P_{H,t}$. u_t is a cost-push shock associated with the time-varying elasticity of substitution for individual goods. μ_t^p ($\equiv p_t - \psi_t$) represents the average price markup, which is related to the real marginal cost. In addition,

$$\lambda_p \equiv \frac{\left(1-\theta_p\right)\left(1-\beta\theta_p\right)}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}.$$

3.5 Equilibrium

We now describe the equilibrium conditions in an open economy. The equilibrium conditions for the goods market are given as follows:

$$\left(1-\upsilon\right) Y_t = \left(1-\upsilon\right) C_{H,t} + \upsilon C_{H,t}^*, \tag{33}$$

$$\upsilon Y_t^* = \left(1-\upsilon\right) C_{F,t} + \upsilon C_{F,t}^*. \tag{34}$$

Using these equations and Equations (2) and (3), we show that the trade balance within each country is zero.

$$P_{H,t}Y_t = P_t C_t,$$

$$P^*_{F,t}Y^*_t = P^*_t C^*_t.$$

In this case, as shown in Clarida *et al.* (2002), home output can be rewritten as follows:

$$Y_t = C_t \mathcal{S}_t^{\upsilon},$$

From the zero trade balance in each country, we obtain the following:

$$C^*_t = Y_t^{1-\upsilon} \left(Y^*_t\right)^{\upsilon} = C_t. \tag{35}$$

Equilibrium with sticky prices and wages

As noted earlier, we express log-deviations from the steady state with lower case letters. In this section, we focus on the equations expressed by a gap term, which is denoted by lower case letters with a tilde. The gap term implies the log-deviation from the equilibrium in which both prices and wages are flexible. While we focus on the equilibrium with sticky prices and wages in the main text, Appendix A provides the structural equations with flexible prices and wages.

First, consider the effect of wage flexibility on an open economy PNKPC. The open economy PNKPC is again given by the following:

$$\pi^p_{H,t} = \beta E_t \left\{ \pi^p_{H,t+1} \right\} - \lambda_p \left(\mu^p_t - \mu^p\right) + u_t,$$

where the average price markup associated with the real marginal cost is given by the following:

$$\mu^p_t - \mu_t = -\frac{\alpha}{1-\alpha}\tilde{y}_t - \tilde{\omega}_t - \upsilon \tilde{s}_t, \tag{36}$$

$\pi^p_{H,t}$ ($\equiv p_{H,t} - p_{H,t-1}$) denotes the PPI inflation rate. $\tilde{y}_t$ is the output gap, $\tilde{\omega}_t$ is the real wage gap, and $\tilde{s}_t$ is the terms of trade gap. Both the real wage gap and the terms of trade gap negatively affect the real marginal cost, resulting in them being positively related to price inflation. This mechanism is similar to one argued in Clarida *et al.* (2002). To do this, we rewrite the PNKPC in the form of the gap variable. Using the definition of consumption $\tilde{c}_t = (1-\upsilon)\tilde{y}_t + \upsilon \tilde{y}^*_t$, the PNKPC expressed by the gap term is derived as follows:

$$\pi^p_{H,t} = \beta E_t \left\{ \pi^p_{H,t+1} \right\} + \frac{\lambda_p \alpha}{1-\alpha}\tilde{y}_t + \lambda_p \tilde{\omega}_t + \lambda_p \upsilon \tilde{s}_t + u_t, \tag{37}$$

Except for the presence of the real wage, this new Keynesian Phillips curve (NKPC) is based on the derivation of the standard two-country NK model (Clarida *et al.*, 2002). Similarly, we can obtain the same relationship for the foreign country as follows:

$$\pi_{F,t}^{p,*} = \beta E_t \left\{ \pi_{F,t+1}^{p,*} \right\} + \frac{\lambda_p^* \alpha}{1-\alpha} \tilde{y}_t^* + \lambda_p^* \tilde{\omega}_t^* - \lambda_p^* \upsilon \tilde{s}_t + u_t^*, \tag{38}$$

where $\pi_{F,t}^{p,*}$ ($\equiv p_{F,t}^* - p_{F,t-1}^*$) denotes PPI inflation in the foreign country, and u_t^* is the exogenous cost-push shock in the foreign country.

In addition, we rewrite the WNKPC in terms of the gap variable as follows:

$$\pi_t^w = \beta E_t \left\{ \pi_{t+1}^w \right\} + \frac{\lambda_w \varphi}{1-\alpha} \tilde{y}_t + \lambda_w \tilde{c}_t - \lambda_w \tilde{\omega}_t. \tag{39}$$

As in the closed economy model, wage inflation is affected by changes in the consumption gap. However, our model addresses the impact of the terms of trade on wage inflation in a two-country model through international risk-sharing. Specifically, as shown in Equation (35), changes in the foreign output gap positively impact home consumption through international consumption risk-sharing. Accordingly, in contrast to Campolmi (2014) and Rhee and Turdaliev (2013), home wage inflation increases in response to the increase in the foreign output gap. However, the real wage gap has a negative impact on wage inflation. Therefore, whether wage inflation rises or not depends on the change in the foreign output gap relative to the change in the real wage gap. The above mechanism is important for understanding the analysis in this paper. Similarly, for the foreign country, we obtain the following:

$$\pi_t^{w,*} = \beta E_t \left\{ \pi_{t+1}^{w,*} \right\} + \frac{\lambda_w^* \varphi}{1-\alpha} \tilde{y}_t^* + \lambda_w^* \tilde{c}_t^* - \lambda_w^* \tilde{\omega}_t^*. \tag{40}$$

The real wage plays a significant role in examining optimal monetary policy in our model. The dynamic behavior of the real wage in terms of the home country is characterized by the following law of motion:

$$\tilde{\omega}_t = \tilde{\omega}_{t-1} + \pi_t^w - \pi_t - \nu \Delta \tilde{s}_t - \Delta \omega_t^n. \tag{41}$$

In this paper, we emphasize the role of nominal wage stickiness in the two-country model. Equation (41) shows that real wages are characterized by inertial changes in the terms of trade. In contrast to a small-open economy, the foreign output gap endogenously affects the dynamics of the terms of trade, leading to inertial changes in the real wage. Since changes in the real wage have a direct impact on the NKPCs for both prices and wages, an inertial change in the terms of trade also has the significant impact of an inertial change in real wages on both the price and wage NKPCs. Thus, our model focuses on the interaction between the effects of real wages and the NKPCs of prices and wages through changes in the terms of trade. This channel is in contrast to the cases in Campolmi (2014), Galí and Monacelli (2016), and Rhee and Turdaliev (2013).

4 Optimal monetary policy in a two-country NK model with nominal wage rigidity

This section examines how the home and foreign central banks implement optimal monetary policy in a two-country economy with nominal price and wage rigidity in both countries. In Section 4.1, we derive the central bank's loss function by implementing the second-order approximation of the household utility function. Section 4.2 explains the optimal monetary policy problem under policy coordination.

4.1 The central bank's loss function

We must derive a well-defined loss function with a micro-foundation to investigate optimal monetary policy in an NK model. Rotemberg and Woodford (1997) and Woodford (2003) showed that the second-order approximation of the household utility function corresponded to the central bank's loss function. In a two-country framework, Clarida *et al.* (2002) and Engel (2011) derived the central bank's loss function by using a second-order approximation of the utility function.

To derive a well-defined loss function, we must perform a Taylor expansion around an efficient steady state. In this paper, an efficient steady state that removes the following distortions is necessary for the derivation of the central bank's loss function.

In our model, there is a distortion of the markups from monopolistic competition in intermediate goods firms and a distortion of the wage markups in the labor unions of each country. As shown in the online appendix, the optimal subsidies that attain an efficient steady state are given as follows:

$$\tau = \tau^* = 1 - \frac{1}{\kappa \mu_p \mu_w}.$$

In deriving these subsidies, it is assumed that the government of each country will calculate the optimal subsidy. These conditions are the same as in Clarida *et al.* (2002), who constructed a two-country model with a monopolistically competitive labor market under flexible nominal wages.

We derive the central bank's loss function by implementing the second-order approximation of the following household utility function:

$$\begin{aligned}
&\sum_{t=0}^{\infty}\beta^t[W_t-\bar{W}] \\
&\approx -\frac{u_c C}{2}\sum_{t=0}^{\infty}\beta^t\Bigg\{(1-\nu)\left[\left(\sigma+\frac{\alpha+\varphi}{1-\alpha}-\nu(\sigma-1)\right)x_t^2+\frac{\epsilon_p}{\lambda_p\Theta}\pi_t^2+\frac{\epsilon_w(1+\epsilon_w\varphi)}{\lambda_w}(\pi_t^w)^2\right] \\
&+\nu\left[\left(\sigma+\frac{\alpha+\varphi}{1-\alpha}-(1-\nu)(\sigma-1)\right)(x_t^*)^2+\frac{\epsilon_p}{\lambda_p^*\Theta}(\pi_t^*)^2+\frac{\epsilon_w(1+\epsilon_w\varphi)}{\lambda_w^*}(\pi_t^{w,*})^2\right] \\
&-2\nu(1-\nu)(1-\sigma)x_t x_t^*\Bigg\}+t.i.p.+O(||\xi||^3), \qquad (42)
\end{aligned}$$

where

$$\Theta=\frac{\epsilon_p}{1-\alpha+\alpha\epsilon_p},\ \lambda_p=\frac{(1-\theta_p)(1-\theta_p\beta)}{\theta_p},\ \lambda_p^*=\frac{(1-\theta_p^*)(1-\theta_p^*\beta)}{\theta_p^*},$$
$$\lambda_w=\frac{(1-\theta_w)(1-\theta_w\beta)}{\theta_w(1+\epsilon_w\varphi)},\ \lambda_w^*=\frac{(1-\theta_w^*)(1-\theta_w^*\beta)}{\theta_w^*(1+\epsilon_w\varphi)}.$$

This loss function is expressed in terms of a consumption equivalence unit. The term $t.i.p$ includes terms that are independent of monetary policy, and $O(||\xi||^3)$ indicates terms of third or higher orders. The appendix B provides a detailed derivation of Equation (42). As in the standard two-country NK model, our loss function contains several standard policy objectives. Inflation and output gap objectives are assigned to both home and the foreign country. The first objective implies the presence of price dispersion caused by nominal price rigidity. In addition, the loss function for each country contains the wage inflation stabilization associated with wage stickiness. This objective is associated with the presence of the wage dispersion caused by staggered wage contracts (Erceg *et al.*, 2000).

Not surprisingly, one may think that this shape of the loss function is easily derived. However, in a small-open economy model, the derivation of the central bank's loss function is quite restrictive in that we require the assumption that both the intertemporal substitution of consumption and the elasticity of substitution between home and foreign goods must be unity(Galí and Monacelli, 2005). [9] In fact, as noted earlier, Campolmi (2014) and Rhee and Turdaliev

[9] De Paoli (2009a) derived the central bank's loss function under more generalized parameterizations. In this

(2013) obtained the central bank's loss function under the assumption of both $\sigma = 1$ and a Cobb-Douglas consumption basket. In contrast to their studies, we can derive the central bank's loss function in a two-country NK model with nominal wage rigidity even in the case of $\sigma \neq 1$.[10]

In the following sections, we use this loss function to derive the optimal monetary policy and evaluate the performance of a commitment policy.

4.2 Optimal monetary policy

This paper first considers optimal commitment policy under policy coordination. Since the purpose of this paper is to simply evaluate the impact of nominal wage rigidity on optimal monetary policy in a two-country NK model, we focus on the commitment solution under policy coordination. Therefore, the issue of policy coordination (i.e., whether the home and foreign central banks can jointly maximize global welfare without reneging on their commitment to policy coordination) is not the main scope of this paper. To examine this issue, it is necessary to explicitly derive the loss function of each central bank under a non-cooperative solution when calculating the optimal monetary policy in a linear-quadratic (LQ) framework. However, in this case, the LQ problem is generally more complicated because the derivation of the loss function requires a second-order approximation of the NKPC and the terms of trade.[11]

Under policy coordination, the benevolent central banks choose the interest rate path by intertemporally solving the minimization problem of the central bank's loss function. We focus on the commitment solution as a benchmark case and, in the next subsection, we explore a comparison of the commitment and discretionary solutions. Under a commitment policy, central banks can commit to future monetary policy stances in the current period. This pa-

derivation of the central bank's loss function, we require the second-order approximation of the NKPC considered in Benigno and Woodford (2005).

[10] In this study, we focus only on the case of policy coordination. In this case, the loss function of the central bank can be derived analytically by computing the second-order approximation of the household utility function. However, when we consider both the cooperative and non-cooperative policy cases, it becomes more difficult to derive a well-defined loss function in our model. See Fujiwara and Wang (2017) for a detailed discussion of the central bank's loss function in a two-country NK model.

[11] If we depart from the problem based on the LQ problem, our model may be able to handle this problem. That is, in the case of a non-linear, two-country NK model, we can use the method developed in Bodenstein, Guerrieri and LaBriola (2019) to solve for both cooperative and non-cooperative policies.

per regards the commitment policy as optimal from a timeless perspective, as suggested by Woodford (2003).[12] Since central banks that commit to an optimal policy can introduce policy inertia into the economy, this policy inertia enables central banks to influence the private sector's expectations. As pointed out in Pappa (2004), a commitment policy can eliminate the deflationary bias induced by a discretionary policy, which is the source of the additional gains from commitment that are associated with an open economy.

Combining the first-order conditions of the central bank's optimization problem with the structural equations, we obtain the following:

$$A_0 X_t = A_1 X_{t-1} + B_1 R_t + \mathcal{U}_t, \tag{43}$$

with

$$\begin{aligned}
X_t &= [X_{1t+1} \; E_t X_{2t+1}]', \\
X_{1t} &= [A_t \; A_t^* \; u_t \; u_t^* \; \phi_{1t} \; \phi_{2t} \; \phi_{3t} \; \phi_{4t}]', \\
X_{2t} &= [\tilde{w}_t \; \tilde{w}_t^* \; \pi_t^w \; \pi_t^{w,*} \; \pi_t \; \pi_t^* \; x_t \; x_t^* \; \phi_{5t} \; \phi_{6t}]', \\
R_t &= [r_t \; r_t^*]'.
\end{aligned}$$

A_0, A_1, and B_1 are coefficient matrices constructed by deep parameters. X_{1t} denotes the vector for the predetermined and state variables, X_{2t} denotes the same for the jump variables, R_t is the vector for the central bank's instrumental variables, and $\mathcal{U}_t$ denotes the vector for structural shocks. ϕ_{1t} and ϕ_{2t} denote the Lagrange multipliers for the PNKPCs, ϕ_{3t} and ϕ_{4t} represent those for the WNKPCs. Finally, ϕ_{5t} and ϕ_{6t} are the Lagrange multipliers for the identities of the real wage. The properties of optimal monetary policy are simulated using the Dynare software package.[13]

5 Quantitative results

In this section, we report the main results of the paper. Section 5.1 briefly explains the deep parameters used in the paper. Section 5.2 explores optimal monetary policy under commitment. In Section 5.2, we demonstrate the effect of wage flexibility in the foreign country on the

[12] See Chapter 7 in Woodford (2003) for a detailed discussion of optimal monetary policy from a timeless perspective.

[13] Dynare is available at http://www.dynare.org/.

impulse responses of the macroeconomic variables. Then, we calculate the effect of foreign wage flexibility on the welfare losses under several key parameter restrictions.

5.1 Calibration

This section describes the deep parameters used in this paper. The parameters calibrated in this paper are the standard ones used in previous studies. The discount factor is set to 0.99. In addition, this paper assumes that parameter φ equals 1.0. The degree of nominal price rigidity for each country is set to 0.75. Following Rotemberg and Woodford (1997), the elasticity of substitution between goods ϵ_p is set to 7.88. The elasticity of substitution for individual labor supply ϵ_w is set to 5.0. The elasticity of substitution among the labor varieties is set to 1.0 on the basis of the calibration used in the standard NK model. Parameter υ, which denotes the degree of openness, is set to 0.4 as a baseline parameter value.

The relative risk aversion coefficient for consumption, σ is 2.0, is a benchmark calibration. This calibrated value is based on Pappa (2004). Thus, we assume a positive international spillover effect. As pointed out in Clarida *et al.* (2002), international spillover effects disappear in the case of $\sigma = 1$. The negative international spillover effect is also considered when $\sigma < 1$. While we focus on the benchmark case $\sigma > 1$, robustness checks are also performed for $\sigma = 1$ and $\sigma < 1$.[14]

The degree of nominal wage rigidity for each country is set to 0.6 as a benchmark parameter value. This parameter value is based on the standard NK model with nominal wage rigidity (Christiano *et al.*, 2005, Erceg *et al.*, 2000, Galí, 2011). The value of the Calvo parameter for nominal wage rigidity is assumed to be fixed at 0.6 in the home country, but is set to three different values in the foreign country when conducting impulse response analysis in Section 5.2. The first case considers a fully flexible nominal wage, $\theta_w^* = 0.01$.[15] The second case corresponds to our benchmark calibration, $\theta_w^* = 0.6$. In the third case, we set θ_w^* to 0.9, assuming a fairly high degree of nominal wage rigidity.

Finally, the standard deviations of productivity, σ_a, and price markup shocks, σ_u, are both set to 0.02. The analogous values are assumed for the foreign country. Regarding autoregressive

[14] See Pappa (2004) and Corsetti, Dedola and Leduc (2010) for a case of departure from the Cobb-Douglas consumption aggregate.

[15] Without loss of generality, we do not set it precisely to zero due to restrictions in the numerical computations.

coefficients for the structural shocks, we set ρ_a and ρ_u to 0.8 and 0, respectively. These calibrated values for economic shocks are presumed to be the same in the foreign country. In the subsequent sections, we focus on the case for the productivity and price markup shocks generated in the foreign country.

5.2 Optimal commitment policy

Impulse response analysis

First, we present an impulse response analysis under a commitment policy as a benchmark calibration. Figure 1 shows the impulse response function when a productivity shock occurs in the foreign country. In this shock, the inflation rate decreases and output increases. An increase in output due to a productivity shock will lead to an increase in the real wage and, consequently, an increase in wage inflation. The results of these impulse responses would be consistent with Erceg *et al.* (2000).

[Figure 1 around here]

The following dynamics are specific to our two-country NK model. An increase in foreign output lowers the relative price of foreign goods to home goods and improves the terms of trade in terms of the home country. This change in the terms of trade leads to higher CPI inflation in the foreign country and lower CPI inflation in the home country. A productivity shock in a foreign country improves the terms of trade in terms of the home country, causing an appreciation of the nominal exchange rate. Shocks in the foreign economy affect the home economy through changes in the terms of trade, as long as σ is not unity.[16] As a result, a productivity shock in the foreign country leads to lower inflation and output in the home country. As the terms of trade improve in the home country, the consumption of foreign goods increases and the output of home goods decreases due to the labor burden. This can also impact real wages through the substitution of labor for consumption, but it appears to be negligible, at least in this simulation.[17]

[16]See Clarida *et al.* (2002) and Pappa (2004) for a detailed explanation of the role of the coefficient of relative risk aversion (CRRA) coefficient in a two-country NK model.

[17]In the baseline calibration, we assume sticky wage settings of $\theta_w = \theta_w^* = 0.6$.

In the two-country model, how does foreign wage flexibility affect the characteristics of a commitment policy? First, let us examine the foreign macroeconomic variables. If the nominal wage is fully flexible in the foreign country (i.e., $\theta_w^* = 0.01$), then productivity shocks lead to a much greater increases in the real wage and wage inflation than when nominal wages are sufficiently sticky in the foreign country. When the nominal wage is sticky in the foreign country, a productivity shock leads to even lower price inflation and higher output. Compared to the case of $\theta_w^* = 0.01$, a foreign productivity shock induces an attenuated response of the nominal exchange rate when $\theta_w^* = 0.6$. The response of home real wages and home wage inflation is dampened when nominal wage rigidity is moderate (i.e., $\theta_w^* = 0.6$). In addition, the response of home inflation and output is weakened by the presence of the sticky nominal wage. However, when nominal wage rigidity is extremely high (i.e., $\theta_w^* = 0.9$), the initial drop in output is greater when $\theta_w^* = 0.9$ than when $\theta_w^* = 0.01$. This transmission mechanism is specific to a two-country NK model with nominal wage rigidity.

Next, we demonstrate the properties of a commitment policy in response to a foreign cost-push shock. Figure 2 shows the home and foreign responses to a foreign cost-push shock. When the shock occurs, the foreign country faces a trade-off between output and inflation; a cost-push shock increases price inflation and decreases output. In a commitment policy, the economy's dynamic response is mitigated by further reducing output and generating a negative output gap (Woodford, 2003). In the case of nominal wage rigidity, a cost-push shock generally causes a reduction in the real wage and wage inflation. On the one hand, even in the case of $\theta_w^* = 0.01$, this reduction is clearly observed in Figure 2. On the other hand, higher nominal wage stickiness considerably mitigates the reduction in both the real wage and wage inflation. Moreover, higher nominal wage stickiness appears to improve the policy trade-off between price inflation and output.

[Figure 2 around here]

This observation is consistent with the NK model of a closed economy with nominal wage rigidity. This study emphasizes how changes in nominal wage rigidity in the foreign country can affect macroeconomic variables in the home country. A decline in foreign output worsens the terms of trade in terms of the home country. More precisely, this change affects the home economy through the depreciation of the nominal exchange rate and an increase in CPI inflation. This case corresponds to that under complete wage flexibility in the foreign country.

In the home country, output rises due to increased consumption in the foreign country, but the appropriate policy reaction attenuates the response to inflation.

Figure 2 shows that the higher the nominal wage stickiness, the stronger this tendency becomes. In this case, the nominal exchange rate in terms of the home currency does not depreciate, but appreciate, in response to a foreign cost-push shock. The higher the nominal wage stickiness, the slower the appreciation in the nominal exchange rate to a cost-push shock in the foreign country, and thus the more attenuated the change in the terms of trade. As pointed out in Monacelli (2003), the responses of the nominal exchange rate and the terms of trade retain the stationary properties of the optimal commitment policy. As we will demonstrate, the response of the nominal exchange rate is non-stationary in the case of a discretionary policy. Thus, even if there is moderate nominal wage stickiness in the home country, fluctuations in both the home country's real wage and the home country's wage inflation rate are alleviated if nominal wage stickiness in the foreign country is sufficiently high. Accordingly, the impact on wage inflation in the home country appears to be negligible.

Welfare analysis

Given nominal wage rigidity in the home country, we find that changes in nominal wage rigidity in the foreign country have a significant impact on the international transmission mechanism of monetary policy and therefore on the performance of commitment policy. Therefore, we assess how changes in nominal wage rigidity in the foreign country affect global welfare under a commitment policy. Under several parameter restrictions, we calculate the welfare loss under a commitment policy. Specifically, we focus on the parameter restrictions of θ_w^*, σ, and υ. As these parameters play an important role in our model, we examine whether they significantly affect the performance of the optimal monetary policy.

Figure 3 shows the welfare loss in the world economy defined by Equation (42) as a function of foreign wage stickiness θ_w^*, expressed as the percentage deviation from the value in the baseline calibration. Consider first the case in which wages in both home and the foreign country are flexible, in the top panel of Figure 3. Each line corresponds to a case in which the home country's wages are flexible ($\theta_w = 0.01$), somewhat sticky ($\theta_w = 0.4$), the baseline ($\theta_w = 0.6$), and highly sticky ($\theta_w = 0.8$).

In the top panel, both central banks can achieve the least loss when both home and foreign wages are fully flexible. The loss is 18.1% smaller than the value in the baseline case. Galí (2011) argued that wage flexibility improves social welfare in a closed economy, and Galí and Monacelli (2016) argued that this holds true in a currency union. We address the fact, however, that this result is obtained in our two-country model.

[Figure 3 around here]

For worldwide welfare, it is important whether wages in *both* home and the foreign country are flexible or not. If wages in either country become somewhat stickier (e.g., from $\theta_w = 0.01$ to $\theta_w = 0.4$), the loss increases by as much as 10 percentage points. In other words, a further increase in wage stickiness (from $\theta_w = 0.4$ to $\theta_w = 0.6$ or 0.8) does not cause such a sharp change in welfare loss.

With home country wage stickiness as a given, the change in welfare loss is not monotonically increasing in accordance with the change in foreign wage stickiness. In fact, in this simulation, it appears that the loss peaks at $\theta_w^* = 0.8$ for all of the lines, and the loss becomes slightly smaller as wage stickiness increases from there.

The non-monotonic relationship between wage stickiness and welfare loss has already been pointed out by Galí (2013), Galí and Monacelli (2016), and Groll and Monacelli (2020).[18] These previous studies have largely discussed the role of wage flexibility within one country or a monetary union, but they did not focus on that mechanism in a two-country model. Compared to previous studies, our results address the fact that the non-monotonicity of wage stickiness and losses is not outstanding. This weak non-monotonicity implies that changes in one country's parameters alone do not have a significant impact on the global economy. The welfare loss is noticeably smaller when wages in *both* countries are flexible. This result cannot directly compare to the welfare loss under the closed economy model. However, we highlight that when nominal wage stickiness differs between home and the foreign country, it is important to take this difference into account to maximize social welfare in the global economy. In particular, if the home country's wage stickiness does not change, changes in the

[18] Galí (2013) showed, in a standard NK model framework, that macroeconomic policies that increase wage stickiness under initial high wage stickiness can lead to welfare losses and argued that "the desirability of more wage flexibility are propositions that one should not take for granted (p.1002)". Galí and Monacelli (2016) showed similar results in a monetary union model. Groll and Monacelli (2020) analyzed how welfare losses varied whether both prices and wages were flexible or sticky and found similar non-monotonicity.

foreign country's wage stickiness will not affect welfare losses to the same extent as in a closed economy. This means that in a closed economy or a small-open economy, changes in the real wage gap in the home country are not affected by foreign endogenous variables. However, as pointed out in Section 3, changes in the foreign country's shocks affect the home real wage gap through changes in the terms of trade in the two-country framework. Importantly, the minimum global welfare loss is attained when the terms of trade externality is fully exploited in the case of complete wage flexibility in both countries. However, this gain is reduced in the case of the presence of nominal wage stickiness in each country because central banks cannot fully use the terms of trade externality. These results indicate the key mechanism underlying the results of our non-monotonic welfare loss in our two-country model.

The middle panel of Figure 3 similarly plots varying intertemporal substitutions of consumption (σ, common between home and the foreign country). The baseline calibration, σ, is set to 2.0. Given that home wages are sticky, a change in σ maintains the shape of the "loss curve" seen in the top panel. If foreign wages are sticky (i.e., $\theta_w^* \geq 0.1$), an increase in σ shifts the loss curve upward, causing welfare losses to increase continuously. As mentioned above and also pointed out in Clarida *et al.* (2002), whether the impact of the foreign economy positively or negatively affects the home country depends on σ. In our model, it has a positive effect when $\sigma > 1$. Why is the relationship between foreign wage stickiness and losses robust to the positive or negative sign of the spillover effect? The reason is that the central bank, with a good understanding of the spillover effect, successfully induces fluctuations in the terms of trade to bring about the optimal spillover.

Interestingly, only when the foreign wage is fully flexible, is the welfare loss minimized when $\sigma = 0.5$. In contrast to this case, when the foreign wages are sticky, the welfare loss curve increases as σ increases. Why does this inversion occur? As seen in the wage version of the NKPC, when wages are sticky, the market distortions induced by slow wage changes become greater as σ increases. Conversely, when the foreign wage is fully flexible, the market distortions from the foreign wage disappear. Here, even if the home country's wages are still sticky, we can control the terms of trade to exploit the spillover effect. Since the spillover effect becomes greater as σ increases, the reversal phenomenon described above occurs only when foreign wages are fully flexible.

We confirm, therefore, that the relationship between foreign wage stickiness and welfare losses is robust, regardless of the intertemporal elasticity of consumption, with only a few exceptions.

The bottom panel of Figure 3 shows the effect of the degree of economic openness υ on worldwide welfare loss. Our baseline calibration is $\upsilon = 0.4$. When the economy is closed ($\upsilon = 0.01$, blue line), foreign wage stickiness changes make little difference to global losses. However, as the economy becomes open, the central bank can effectively manipulate foreign wages to substantially improve economic welfare.

It is noteworthy that when the closed economy is considered, foreign wage stickiness, and the welfare loss become unrelated. This result is not trivial, because flexible wages in the foreign country would improve the foreign county's welfare, which, in turn, would improve global welfare. This mutual indifference can be explained as follows. As the degree of openness decreases, spillovers from the foreign country to the home country are muted, making it impossible to improve welfare using terms of trade externalities, as previously discussed.

In other words, the non-monotonic relationship between foreign wage stickiness and the loss curve becomes more distinct the more the degree of openness increases. This feature indicates that the more the degree of openness increases, the easier it is for the effects of fluctuations in the foreign wage and the terms of trade to spill over into the home country, thus stabilizing the global economy as a whole. This finding suggests that the degree of openness plays an important role in significantly improving the world economy when foreign wages are sufficiently flexible or highly sticky. This result is a notable result for the two-country model in which both countries are large, and it is not found in previous studies such as Galí and Monacelli (2016). Naturally, this is because the role of economic openness cannot be discussed in a closed economy, and, therefore, global welfare is also given in a small-open economy in which the foreign economy is given.

In summary, we find the following results obtained in Sections 5.2 and 5.3. First, the world economy's welfare loss is small when both home and foreign wages are flexible. Second, when either the wages in home or the foreign country are flexible, the welfare loss is also small, although the welfare improvement will be half compared to the case of flexible wages in both countries. Third, the non-monotonicity between wage stickiness and welfare loss is not as apparent as observed in a closed economy and a monetary union. Fourth, regarding the second finding, which states that global welfare improves when foreign wages are flexible, is robust to changes in the intertemporal elasticity in consumption. However, there is also a parameter that would cancel the welfare-improving effect of flexible wages (e.g., the degree of openness, υ).

6 Welfare gains from commitment in a two-country model with sticky prices and wages

We assessed the performance of a commitment policy in a two-country model with nominal wage rigidity in the previous section. This section explores the comparison of the performance of commitment and discretionary policy. In Section 6.1, we calculate the gain from commitment under several key parameter values. Section 6.2 confirms the performance of a commitment policy using impulse response analysis.

6.1 Welfare analysis

In this section, we evaluate the gains from commitment policy in our model. While there are several studies on the effectiveness of commitment policy in economies in which nominal prices and wages are sticky, the role of wage flexibility in a two-country model is an open question. As noted earlier, in the standard NK model, a commitment policy generally leads to smaller welfare losses than a discretionary policy (McCallum and Nelson, 2004).

The result that the welfare gains from a commitment policy are greater than those from a discretionary policy is carried over to the open economy model. Monacelli (2003) showed that welfare losses due to discretionary policy are substantially greater than those due to a commitment policy in a small-open economy. As mentioned earlier, Pappa (2004) pointed out that commitment policy can eliminate the deflationary bias caused by discretionary policy, and thus create welfare gains from commitment in a two-country economy. In addition, the welfare gain from a commitment policy still holds in the presence of sticky wages (Erceg *et al.*, 2000, Galí, 2011). Galí and Monacelli (2016) also pointed out the role of wage flexibility in a monetary union, but did not evaluate the performance of discretionary policy. While Groll and Monacelli (2020) examined the benefits of commitment policy in the cases of both flexible exchange rates and a monetary union, they did not focus on the role of wage flexibility.

It remains unclear whether the welfare losses under a discretionary policy are substantially greater than those under a commitment policy in a two-country NK model in which nominal wages are sticky in both countries. In this paper, we show that the degree of wage flexibility in both countries significantly impacts the performance of optimal commitment policy in the two-country model with sticky prices and wages. However, it is not obvious whether commitment policy are always better than discretionary policy in a two-country model with nominal price and wage rigidity.

The purpose of this section is to quantitatively investigate the extent to which the losses from discretionary policy are greater than those from commitment policy in our model. Figures 4 through 6 compare the results of Figure 3, the performance of commitment, with that of discretion. The commitment case (solid blue line) is the same as each line in Figure 3. Each value is expressed as a percentage deviation from the result at $\theta_w = \theta_w^* = 0.6$, which is the baseline parameter under the commitment policy.

First, Figure 4 shows the performance of both the commitment and discretionary policies on changes in foreign wage stickiness, given the stickiness of the home country wage. We find that when wages are flexible in the home country, welfare losses are much lower in the commitment case than in the baseline calibration. However, this is not the case for the discretionary policy. Under the discretionary policy, although the welfare loss is reduced as the wage becomes more flexible in the home country, the reduction is much smaller than in the commitment policy. In other words, if nominal wages in both countries are completely flexible, the welfare loss of the discretionary policy will be greater than that of the commitment policy. Specifically, if nominal wages are fully flexible in both countries, the gain from commitment policy (i.e., the percentage difference from discretion) is roughly 20% points. Thus, this result indicates that the gain from a commitment policy is greater than the discretionary policy, as long as nominal wages are completely flexible in both countries. We address that this result is not obtained when we consider the two-country NK model excluding the role of nominal wage rigidity.

[Figure 4 around here]

However, when the home country's wage becomes stickier ($\theta_w \geq 0.4$), the welfare loss does not change much under discretionary policy, while the reduction in welfare loss disappears under commitment. Thus, when the degree of foreign wage stickiness increases, the performance of commitment policy deteriorates slightly. In fact, under realistic assumptions ($\theta_w = \theta_w^* = 0.6$), the gain from commitment is only 1.7 percentage points. Importantly, when the nominal wage in both countries become highly sticky, the difference between commitment, and discretionary policies becomes negligible. In particular, the cost of discretionary policy is not necessarily greater than that of commitment policy when considering the role of wage flexibility in a two-country NK model. To the best of our knowledge, none of the previous studies has pointed out this result.

Second, given a value of θ_w, we examine whether a change in the CRRA parameter, σ, affects the gain from commitment policy as θ_w^* changes. Figure 5 computes the welfare losses of both commitment and discretion, relative to the commitment case at the benchmark case, as the degree of foreign nominal wage rigidity changes under several parameter values of σ. As shown in this figure, there appears to be no major differences between the four panels. Thus, welfare losses under a discretionary policy are much greater than those under a commitment policy, as long as nominal wages are completely flexible in the foreign country. In that sense, the results indicate that the welfare loss is robust to σ, regardless of commitment, or discretion. We observe that even in the case of $\sigma < 1$, the gain from commitment policy is not negligible in the case of wage flexibility. This result does not appear to be obtained in the standard two-country NK model considered by Clarida *et al.* (2002).

[Figure 5 around here]

Third, we consider whether the degree of openness affects the performance of the commitment and discretion regimes. We do not need to consider the role of wage flexibility in a two-country economy if the degree of openness never affects the performance of both regimes. Figure 6 computes the welfare losses of both commitment and discretion, relative to the commitment case as the benchmark case, as the degree of foreign nominal wage rigidity changes under several parameter values of υ. In this exercise, we focus on how the degree of openness affects the performance of both the commitment and discretion regimes. The figure shows that the gains from a commitment policy remain around 2% points, in most cases, when the degree of foreign nominal wage rigidity is somewhat sticky (e.g., $\theta_w^* \geq 0.4$). On the other hand, if θ_w^* falls below 0.3, foreign wage flexibility increases the gains from a commitment policy from 2% to 7% points, as the degree of openness υ increases. We find that the degree of openness ensures that flexible wages in both countries improves global welfare. In other words, if wages in both countries are sticky, making wages more flexible will not significantly improve welfare. Therefore, this result implies that wage flexibility in the foreign country increases the gains from a commitment policy when the degree of openness is approximately equal in the two countries.

[Figure 6 around here]

In summary, when nominal wages are perfectly flexible in both countries, the welfare loss under commitment policy is smaller than that under sticky wages, while the welfare loss under discretionary policy is greater. However, when nominal wages are quite sticky in the foreign country, we find that there is no significant difference in the performance of the commitment and discretionary policies. We can say that these results are found to be unaffected by changes in several parameters that are important in a two-country model.

6.2 Impulse response analysis

The results in Section 6.1 show that there is a significant difference in the performance of commitment and discretion with respect to changes in wage flexibility in each country. Why did we focus on the difference between commitment and discretion in a two-country NK model with nominal wage rigidity? To understand this mechanism, we provide an impulse response analysis of both regimes. Figures 7 and 8 show the impulse responses of the macroeconomic variables in both countries.

Figure 7 illustrates the impact of foreign productivity shocks on macroeconomic variables in both countries under commitment and discretion. From this figure, it appears that there is no difference between commitment and discretion, except for the response of wage inflation in both countries. In addition, we observe that the response of the nominal exchange rate to these shocks under both regimes seems to be stationary. This seems to be in stark contrast with the cases presented by Clarida *et al.* (2002), Galí (2013), and Galí and Monacelli (2016). An intuitive mechanism for such a stationary response of the nominal exchange rate may be explained by the dynamics of the terms of trade. Sluggish changes in the terms of trade are driven by gradual movements in the nominal wage in both countries. Thus, as long as the nominal wage is sticky in each country, the nominal exchange rate is characterized by inertial movements in the terms of trade. This is why the nominal exchange rate is stationary. Groll and Monacelli (2020) focused on the role of the terms of trade in explaining the performance of optimal monetary policy in a two-country NK model,[19], but it differed from Groll and Monacelli (2020) in that it addressed the impact of wage flexibility on the terms of trade in a two-country model.

[Figure 7 around here]

In contrast to the case in Figure 7, Figure 8 illustrates that there is a difference between the commitment and discretionary policies when a foreign cost-push shock occurs, but the difference appears to be quantitatively small. Thus, as pointed out in Woodford (2003), our model indicates that the commitment policy succeeds in stabilizing the macroeconomic variables in both countries by introducing policy inertia into the economy. This is in stark contrast with the result in the case of a foreign productivity shock. For each response to a foreign cost-push shock, the response under the commitment policy is smaller than under a discretionary policy regime. The response of the nominal exchange rate is also stationary under both regimes, but the response under discretion is much more depressed and persistent than that under commitment.

[Figure 8 around here]

Figures 9 and 10 show the impulse responses under the both regimes when the foreign country's nominal wages are flexible.[20] First, Figure 9 plots the impulse response to a foreign productivity shock. At first glance, the results appear similar to those in Figure 7, but the response of the home and foreign inflation rates differ in the commitment and discretionary cases. In addition, the response of the nominal exchange rate is stationary, even when there is full wage flexibility in the foreign country. Thus, the nominal exchange rate remains stationary as long as the source of the shock is related to a foreign productivity shock. In other words, in contrast to previous studies, our model shows that wage flexibility in the foreign country does not affect the stationarity issue with respect to the nominal exchange rate.

[Figure 9 around here]

[19] Rhee and Turdaliev (2013) and Campolmi (2014) analyzed the role of wage flexibility in considering optimal monetary policy, but these studies focused only on small-open economies.

[20] In Figures 9 and 10, nominal wages are fixed to the benchmark value in the home country.

Next, Figure 10 plots the impulse responses to foreign cost-push shocks under commitment and discretionary policies when foreign nominal wages are flexible. The results in Figure 10 differ from those in Figure 8. The first thing to note is that the response of the nominal exchange rate under discretionary policy is non-stationary. Monacelli (2003) argued that the stationarity of the nominal exchange rate is related to that of the terms of trade. While the nominal exchange rate is non-stationary under a discretionary policy, the terms of trade retain their stationary property. This result is consistent with the discussion by Monacelli (2003). In addition, the response of foreign CPI inflation is asymmetric between the discretionary and commitment policies. Furthermore, when compared to the case in Figure 8, discretionary policy provides the same policy inertia to the economy as commitment policy, but the latter provides stronger policy inertia than the former. This difference is reflected by the costs from discretion, as shown in the top panel of Figure 3 and the top panel of Figure 4. However, while these figures show the costs from discretion shrink, this observation seems to be consistent with the impulse response obtained in Figure 8.

[Figure 10 around here]

7 Concluding remarks

This paper examined wage flexibility and optimal monetary policy in a two-country NK model. In contrast to a two-country model with no nominal wage rigidity, we addressed the fact that in a two-country model with nominal wage rigidity between the two countries, the dynamics of the terms of trade significantly characterize the key mechanism of wage flexibility.

We showed that in this situation, given wage flexibility in the home country, the degree of foreign wage flexibility substantially impacts worldwide welfare losses and the gain from commitment policy. Specifically, when the nominal wages in both countries are perfectly flexible, the welfare gains from the commitment policy are the greatest. However, when nominal wages in the foreign country are stickier, the gains from commitment are predominantly reduced. This result is robust to any change in several key parameters that play an important role in the two-country NK model.

Finally, we mention the limitations of this study. In this paper, we focus only on the case of international policy coordination. However, if either central bank has an incentive to renege on international policy coordination, the degree of nominal wage rigidity in each country may have a significant impact on the cost of an uncoordinated resolution. This is a very important topic, but it is beyond the scope of this paper. We would like to explore this topic as a future work.

A Appendix A: Equilibrium with flexible prices and wages

In this appendix, we provide the expression of the natural level of the macroeconomic variables in terms of the case in which prices and wages are completely flexible in both countries. These expressions are used to define the gap variables shown in main manuscript. Again, unless otherwise noted, analogous equations hold for the foreign country.

First, the price markup is the surplus of the marginal product relative to the wage cost of hiring a worker, and the two are equal under perfect competition. Under price and wage flexibility, the price markup is given by the following:

$$\begin{aligned}\mu_t^p &= mpn_t^n - \left(w_t^n - p_{H,t}^n\right) \\ &= \log\left(1-\alpha\right) + a_t - \alpha n_t^n - \omega_t^n - \upsilon s_t^n.\end{aligned}$$

Next, the wage markup is the gap between the real wage and the marginal rate of substitution (again, both are equal under perfect competition).

$$\mu_t^w = w_t^n - mrs_t^n = w_t^n - (\sigma c_t^n + \varphi n_t^n),$$

Defining total markup, μ, as the sum of μ_p and μ_w, we obtain the following equation:

$$\begin{aligned}\mu_t^p + \mu_t^w =& \log\left(1-\alpha\right) + \left(\left(1-\sigma\right)\left(1-\upsilon\right)\right)a_t + \left(\upsilon\left(1-\sigma\right) - \sigma\left(1+\alpha\right) - \varphi\right)n_t^n \\ &- \tau_t + \upsilon\left(1-\sigma\right)y_t^*.\end{aligned}$$

Now, we provide the variables under flexible wage and price equilibrium. First, the natural level of employment is given as follows:

$$n_t^n = n - \psi_a a_t + \psi_{y*} y_t^{*,n},$$

where

$$\psi_n \equiv (1+\alpha) + \varphi - \upsilon\left(1-\sigma\right), \; n \equiv \frac{\log\left(1-\alpha\right) - \mu}{\psi_n}, \; \psi_a \equiv \left(\left(1-\sigma\right)\left(1-\upsilon\right)\right), \; \psi_{y*} \equiv \frac{\upsilon(1-\sigma)}{\psi_n}.$$

Output, terms of trade, and consumption, at their natural levels, are given as follows:

$$y_t^n = a_t + (1-\alpha)\, n_t^n,$$
$$s_t^n = y_t^n - y_t^*,$$
$$c_t^n = y_t^n - \upsilon s_t^n.$$

Finally, the natural level of real wages is given by the following:

$$\begin{aligned} w_t^n &= \mu^w + \sigma c_t^n + \varphi n_t^n \\ &= \log(1-\alpha) - \mu^p + a_t - \alpha n_t^n - \upsilon s_t^n. \end{aligned}$$

These equations are used to define the gap variables in the main text.

B Appendix B: Derivation of the central bank's loss function

B.1 Preliminaries

We derive the loss function approximated around a steady state. Before doing so, I define relevant notation. First, $\bar{H}$ denotes the value of the steady state, and H_t^n is the value of the efficient level. Second, as in the main text, we define $h_t = \log(H_t/\bar{H})$ as the deviation of H_t from the steady state. To implement the second-order approximation of the planner's objective function, we introduce the following equation:

$$H_t - \bar{H} = \bar{H}\left(\frac{H_t}{\bar{H}} - 1\right) \approx h_t + \frac{1}{2}h_t^2. \tag{A.1}$$

B.2 Optimal subsidies

The planner's objective function under policy coordination is given by

$$W_t = u(C_t) - (1-\nu)v(N_t) - \nu v(N_t^*), \tag{A.2}$$

The planner maximizes this objective function, subject to the following resource constraint:

$$C = Y^{1-\nu}(Y^*)^{\nu} = (N^{1-\alpha})^{1-\nu}[(N^*)^{1-\alpha}]^{\nu}$$

From the first-order condition, we obtain

$$u_c C = (1-\alpha)v_n N \tag{A.3}$$

We now seek for the optimal subsidies that can eliminate distortions arose from the presence of price and wage mark-ups. From the optimal condition for labor supply,

$$\frac{\upsilon_n N}{u_c C} = \frac{\epsilon_w}{\epsilon_w - 1}\frac{W}{P}$$

Then this equation can be rewritten as follows:

$$\frac{W}{P} = \mu_w \frac{\upsilon_n N}{u_c C} = \mu_w MRS$$

It follows from the real marginal cost in the steady state that

$$\frac{(1-\tau)}{\kappa MPL}\mu_p \mu_w MRS = 1,$$

where MPL denotes the marginal product of labor. In the efficient steady state, the following condition must hold:

$$\frac{MRS}{MPL} = 1$$

Therefore, using these conditions, we find that the optimal subsidies are given by

$$\tau = 1 - \frac{1}{\kappa \mu_p \mu_w} \tag{A.4}$$

Similarly, the optimal subsidies for foreign country are given by

$$\tau^* = 1 - \frac{1}{\kappa \mu_p \mu_w} \tag{A.5}$$

B.3 Derivation of the central bank's loss function

We now provide the second-order approximation of the following household utility function. The derivation of the loss function in the main text is based on the idea of Clarida *et al.* (2002). We implement the second-order approximation of the following planner's objective function around their efficient price equilibrium. The following step of the derivation of the loss function is mainly based on that of Clarida *et al.* (2002) and Engel (2011).

The second-order approximation of first-term of the right-hand side of the planner's objective function are follows:

$$\begin{aligned}
u(C_t) &\approx u(C) + u_c(C_t - C) + \frac{u_{cc}}{2}(C_t - C)^2 + O(||\xi||^3),\\
&= u(C) + u_c C\frac{(C_t - C)}{C} + \frac{u_{cc}C^2}{2}\left(\frac{C_t - C}{C}\right)^2 + O(||\xi||^3),\\
&= u(C) + u_c C\left(c_t + \frac{1}{2}c_t^2\right) + \frac{u_{cc}C^2}{2}\left(c_t + \frac{1}{2}c_t^2\right)^2 + O(||\xi||^3),\\
&= u(C) + u_c C c_t + \frac{u_c C}{2}c_t^2 + \frac{u_{cc}C^2}{2}c_t^2 + O(||\xi||^3),
\end{aligned}$$

Defining $\sigma = -u_{cc}C/u_c$, we obtain

$$U(C_t) \approx u(C) + u_c C\left[c_t + \frac{(1-\sigma)}{2}c_t^2\right] + t.i.p. + O(||\xi||^3), \tag{A.6}$$

The second-order approximation of second-term of the right-hand side of the planner's objective function are follows:

$$\begin{aligned}
v(C_t) &\approx v(N) + v_n(N_t - N) + \frac{v_{nn}}{2}(N_t - N)^2 + O(||\xi||^3),\\
&= v(N) + v_n N\frac{(N_t - N)}{N} + \frac{v_{nn}N^2}{2}\left(\frac{N_t - N}{N}\right)^2 + O(||\xi||^3),\\
&= v(N) + v_n N\left(n_t + \frac{1}{2}n_t^2\right) + \frac{v_{nn}N^2}{2}\left(n_t + \frac{1}{2}n_t^2\right)^2 + O(||\xi||^3),\\
&= v(N) + v_n N n_t + \frac{v_n N}{2}n_t^2 + \frac{v_{nn}N^2}{2}n_t^2 + O(||\xi||^3),
\end{aligned}$$

Defining $\varphi = -v_{nn}N/v_n$, we obtain

$$v(N_t) \approx v(N) + v_n N\left[n_t + \frac{(1+\varphi)}{2}n_t^2\right] + t.i.p. + O(||\xi||^3). \tag{A.7}$$

Similarly, the approximation of the third-term of the right-hand side is given by

$$v(N_t^*) \approx v(N^*) + v_n N\left[n_t^* + \frac{(1+\varphi)}{2}(n_t^*)^2\right] + t.i.p. + O(||\xi||^3). \tag{A.8}$$

Here, price dispersion terms are derived as follows:

$$\begin{aligned}
N_t &= \int_0^1 N_t(i)di\\
&= \int_0^1 \int_0^1 N_t(j)djdi\\
&= \int_0^1 N_t(i)\int_0^1 \left(\frac{N_t(j)}{N_t(i)}\right)dj\ di\\
&= \int_0^1 N_t(i)\int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} djdi\\
&= \int_0^1 N_t(i)di\Delta_{w,t},
\end{aligned} \tag{A.9}$$

where $\Delta_{w,t}$ denotes the wage dispersion, which is defined as follows:

$$\Delta_{w,t} \equiv \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w}. \tag{A.10}$$

Moreover, using production function, we can rewrite the above equation as follows:

$$\begin{aligned}
\int_0^1 N_t(i)di &= \int_0^1 \left(\frac{Y_t(i)}{Y_t}\frac{Y_t}{A_t}\right)^{-1/(1-\alpha)} di \\
&= \left(\frac{Y_t}{A_t}\right)^{-1/(1-\alpha)} \int_0^1 \left(\frac{Y_t(i)}{Y_t}\right)^{-1/(1-\alpha)} di, \\
&= \left(\frac{Y_t}{A_t}\right)^{-1/(1-\alpha)} \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon_p/(1-\alpha)} di \\
&= \left(\frac{Y_t}{A_t}\right)^{-1/(1-\alpha)} \Delta_{p,t} \qquad (A.11)
\end{aligned}$$

where price dispersion, which is defined as follows:

$$\Delta_{p,t} \equiv \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon_p/(1-\alpha)} di \tag{A.12}$$

Hence,

$$N_t = \left(\frac{Y_t}{A_t}\right)^{-1/(1-\alpha)} \Delta_{p,t}\ \Delta_{w,t}, \tag{A.13}$$

Lemma 1 *(Gali, 2015) The following properties hold:*

$$\log \Delta_{p,t} \approx \frac{\epsilon_p}{2\Theta} var_i(P_{H,t}) \tag{A.14}$$

$$\log \Delta_{w,t} \approx \frac{\epsilon_w}{2} var_j(W_t) \tag{A.15}$$

Proof. Following Gali (2015), we derive the above two conditions. Let $\hat{p}_{H,t}(i) = p_{H,t}(i) - p_{H,t}$. Here, $p_{H,t}(i) = \log(P_{H,t}(i))$ and $p_{H,t} = \log(P_{H,t})$. Then,

$$\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\epsilon_p} di = \exp\Big((1-\epsilon_p)\hat{p}_{H,t}(i)\Big)$$

Implementing the McLaughlin expansion of the right-hand side leads to

$$\exp\Big((1-\epsilon_p)\hat{p}_{H,t}(i)\Big) \approx 1 + (1-\epsilon_p)\hat{p}_{H,t}(i) + \frac{(1-\epsilon_p)^2}{2}\hat{p}_{H,t}(i)^2 + O(||\xi||^3). \tag{A.16}$$

Here, using the definition of the price index for $P_{H,t}$, we know

$$1 = \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\epsilon_p} di$$

Then, integrating Equation (A.16) in the interval [0,1], we obtain

$$\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\epsilon_p} di \approx 1 + (1-\epsilon_p)\int_0^1 \hat{p}_{H,t}(i)\ di + \frac{(1-\epsilon_p)^2}{2}\int_0^1 \hat{p}_{H,t}(i)^2\ di + O(||\xi||^3). \tag{A.17}$$

Define

$$E_i\hat{p}_{H,t}(i) = \int_0^1 \hat{p}_{H,t}(i)di.$$

Then we can rewrite Equation (A.17) as follows:

$$E_i\hat{p}_{H,t}(i) = \frac{(\epsilon_p - 1)}{2}E_i\hat{p}_{H,t}(i)^2 \tag{A.18}$$

The objective of this calculation is to derive the second-order approximation of

$$\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon_p/(1-\alpha)} di.$$

The second-order approximation of the above equation becomes

$$\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon_p/(1-\alpha)} \approx 1 - \left(\frac{\epsilon_p}{1-\alpha}\right)\hat{p}_{H,t}(i) + \frac{1}{2}\left(\frac{\epsilon_p}{1-\alpha}\right)^2 \hat{p}_{H,t}(i)^2$$

Integrating in the interval [0,1], we obtain

$$\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon_p/(1-\alpha)} di \approx 1 - \left(\frac{\epsilon_p}{1-\alpha}\right)\hat{p}_{H,t}(i) + \frac{1}{2}\left(\frac{\epsilon_p}{1-\alpha}\right)^2 \hat{p}_{H,t}(i)^2 \tag{A.19}$$

Substituting Equation (A.17) into Equation (A.19), we obtain

$$\begin{aligned}
\Delta_{p,t} &\equiv \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon_p/(1-\alpha)} di \\
&= 1 - \left(\frac{\epsilon_p}{1-\alpha}\right)\left(\frac{\epsilon_p - 1}{2}\right)E_i\hat{p}_{H,t}(i)^2 + \frac{1}{2}\left(\frac{\epsilon_p}{1-\alpha}\right)^2 E_i\hat{p}_{H,t}(i)^2 \\
&= 1 + \frac{1}{2}\left(\frac{1-\alpha-(1-\alpha)\epsilon_p+\epsilon_p}{1-\alpha}\right)E_i\hat{p}_{H,t}(i)^2 \\
&= 1 + \frac{1}{2}\left(\frac{1-\alpha+\alpha\epsilon_p}{1-\alpha}\right)E_i\hat{p}_{H,t}(i)^2 \\
&= 1 + \left(\frac{\epsilon_p}{1-\alpha}\right)\frac{1}{2\Theta}E_i\hat{p}_{H,t}(i)^2
\end{aligned}$$

Here

$$E_i\hat{p}_{H,t}(i)^2 = \int_0^1 (p_{H,t}(i) - p_{H,t})^2 di \equiv var_i(P_{H,t})$$

Then we finally obtain

$$\Delta_{p,t} = 1 + \left(\frac{\epsilon_p}{1-\alpha}\right)\frac{1}{2\Theta} var_i(P_{H,t})$$

Taking the logarithm of this equation, we can also obtain

$$\delta_{p,t} \equiv \log \Delta_{p,t} \approx \frac{\epsilon_p}{2\Theta} var_i(P_{H,t})$$

Similarly, we also the second-order approximation of the wage dispersion as follows:

$$\log \Delta_{w,t} \approx \frac{\epsilon_w}{2} var_j(W_t)$$

■

Using $c_t = (1-\nu)y_t + \nu y_t^*$ in Equation (A.6), using $n_t = (1/(1-\alpha)(y_t - a_t) + \delta_{p,t} + \delta_{w,t}$ and $n_t = (1/(1-\alpha)(y_t - a_t) + \delta_{p,t} + \delta_{w,t}$ in Equations (A.7) and (A.8), respectively, we obtain

$$\begin{aligned}
W_t \approx{}& u(C) + u_c C\left\{\left((1-\nu)y_t + \nu y_t^*\right) + \frac{1-\sigma}{2}\left((1-\nu)y_t + \nu y_t^*\right)^2\right\} \\
&- (1-\nu)\left\{v(N) + v_n N\left[\left(\frac{1}{1-\alpha}(y_t - a_t) + \delta_{p,t} + \delta_{w,t}\right)\right.\right. \\
&\left.\left.+ \frac{1+\varphi}{2}\left(\frac{1}{1-\alpha}(y_t - a_t) + \delta_{p,t} + \delta_{w,t}\right)^2\right]\right\} \\
&-\nu\left\{v(N^*) + v_n N\left[\left(\frac{1}{1-\alpha}(y_t^* - a_t^*) + \delta_{p,t}^* + \delta_{w,t}^*\right)\right.\right. \\
&\left.\left.+ \frac{1+\varphi}{2}\left(\frac{1}{1-\alpha}(y_t^* - a_t^*) + \delta_{p,t}^* + \delta_{w,t}^*\right)^2\right]\right\} + O(||\xi||^3).
\end{aligned}$$

Since we obtain $u_c C = v_n N/(1-\alpha)$ in the efficient steady state, the above equation can be rearranged as follows:

$$\begin{aligned}
W_t - \bar{W} \approx{}& \frac{u_c C}{2}\left[(1-\sigma)(1-\nu)^2 y_t^2 + (1-\sigma)\nu^2 (y_t^*)^2 + 2\nu(1-\nu)(1-\sigma)y_t y_t^*\right. \\
&- \frac{(1-\nu)(1+\varphi)}{(1-\alpha)} y_t^2 - \frac{\nu(1+\varphi)}{(1-\alpha)}(y_t^*)^2 + \frac{(1-\nu)(1+\varphi)}{(1-\alpha)} a_t y_t + \frac{\nu(1+\varphi)}{(1-\alpha)} a_t^* y_t^* \\
&\left.- (1-\nu)(\delta_{p,t} + \delta_{w,t}) - \nu(\delta_{p,t}^* + \delta_{w,t}^*)\right] + t.i.p. + O(||\xi||^3).
\end{aligned}$$

where $\bar{W} = u(C) - (1-\nu)v(N) - \nu v(N^*)$. Further calculation leads to

$$\begin{aligned}
W_t - \bar{W} \approx{}& \frac{u_c C}{2}\left[(1-\nu)[1-\sigma-\nu(\sigma-1)]y_t^2 + (1-\nu)[1-\sigma-(1-\nu)(\sigma-1)](y_t^*)^2\right. \\
&- 2\nu(1-\nu)(1-\sigma)y_t y_t^* - \frac{(1-\nu)(1+\varphi)}{(1-\alpha)} y_t^2 - \frac{\nu(1+\varphi)}{(1-\alpha)}(y_t^*)^2 \\
&\left.+ \frac{(1-\nu)(1+\varphi)}{(1-\alpha)} a_t y_t + \frac{\nu(1+\varphi)}{(1-\alpha)} a_t^* y_t^* - (1-\nu)(\delta_{p,t} + \delta_{w,t}) - \nu(\delta_{p,t}^* + \delta_{w,t}^*)\right] \\
&+ t.i.p. + O(||\xi||^3). \qquad \text{(A.20)}
\end{aligned}$$

The natural rate of output for both countries is given by

$$\left(\frac{1+\varphi}{1-\alpha}\right)\left[(1-\nu)a_t y_t + \nu a_t^* y_t^*\right] = (1-\nu)\left(\sigma + \frac{\alpha+\varphi}{1-\alpha} - \nu(\sigma-1)\right)y_t^n y_t$$
$$+ \nu\left(\sigma + \frac{\alpha+\varphi}{1-\alpha} - (1-\nu)(\sigma-1)\right)y_t^{n,*} y_t^*$$
$$+ \nu(1-\nu)y_t^n y_t + \nu(1-\nu)y_t^{n,*} y_t^*$$

Substituting this equation into Equation (A.20), we obtain

$$W_t - \bar{W} \approx -\frac{u_c C}{2}\Bigg\{(1-\nu)\left[\left(\sigma + \frac{\alpha+\varphi}{1-\alpha} - \nu(\sigma-1)\right)x_t^2 + \frac{\epsilon_p}{\Theta} var_i(P_{H,t}) + \epsilon_w var_j(W_t)\right]$$
$$+ \nu\left[\left(\sigma + \frac{\alpha+\varphi}{1-\alpha} - (1-\nu)(\sigma-1)\right)(x_t^*)^2 + \frac{\epsilon_p}{\Theta} var_i(P_{F,t}^*) + \epsilon_w var_j(W_t^*)\right]$$
$$- 2\nu(1-\nu)(1-\sigma)x_t x_t^*\Bigg\} + t.i.p. + O(||\xi||^3). \tag{A.21}$$

Here, following the proposition of Woodford (2003, Chap 6), we can obtain the following conditions for price dispersion and wage dispersion of both countries. We only derive the second-order approximation for price dispersion for home country.

$$\delta_{p,t} = var_i[\log(p_t(j)) - \bar{P}_{t-1}]$$
$$= E_i[\log(p_{H,t}(i)) - \bar{P}_{H,t-1}]^2 - [E_i \log p_{H,t}(i) - \bar{P}_{H,t-1}]^2$$
$$= \theta_p E_i[\log(p_{H,t-1}(i)) - \bar{P}_{H,t-1}]^2 + (1-\theta_p)(\log p_{H,t}^o - \bar{P}_{H,t-1})^2 - (\bar{P}_{H,t} - \bar{P}_{H,t-1})^2 \tag{A.22}$$

Here

$$P_{H,t} = (1-\theta_p)\log P_{H,t}^o + \theta_p \bar{P}_{H,t-1}$$
$$\bar{P}_{H,t} - \bar{P}_{H,t-1} = (1-\theta_p)\log P_{H,t}^o - (1-\theta_p)\bar{P}_{H,t-1}$$
$$\log P_{H,t}^o - \bar{P}_{H,t-1} = \frac{1}{1-\theta_p}(\bar{P}_{H,t} - \bar{P}_{H,t-1})$$

Then from Equation (A.22), we obtain

$$\delta_{p,t} = \theta_p \delta_{p,t-1} + \frac{1}{1-\theta_p}(\bar{P}_{H,t} - \bar{P}_{H,t-1})^2 - (\bar{P}_{H,t} - \bar{P}_{H,t-1})^2$$
$$= \theta_p \delta_{p,t-1} + \frac{\theta_p}{1-\theta}(\bar{P}_{H,t} - \bar{P}_{H,t-1})^2$$

Since $\bar{P}_t = \log P_t + +o(\|\xi\|)^2$,

$$\delta_{p,t} = \theta_p \delta_{p,t-1} + \frac{\theta_p}{1-\theta_p}\pi_t^2 + O(||\xi||^3)$$

Then $\sum_{t=0}^{\infty} \beta^t \delta_{p,t}$ becomes

$$\begin{aligned}\sum_{t=0}^{\infty} \beta^t \delta_{p,t} &= \theta_p \delta_{p,-1} + \frac{\theta_p}{1-\theta_p}\pi_0^2 + \beta\left(\theta_p \delta_{p,0} + \frac{\theta_p}{1-\theta_p}\pi_1^2\right) + \beta^2\left(\theta_p \delta_{p,1} + \frac{\theta_p}{1-\theta_p}\pi_2^2\right) + \cdots \\ &= \theta_p \delta_{p,-1} + \theta_p \beta(\delta_{p,0} + \beta \delta_{p,1} + \beta^2 \delta_{p,2} + \cdots) + \frac{\theta_p}{1-\theta_p}\left(\pi_0^2 + \beta \pi_1^2 + \beta^2 \pi_2^2 + \cdots\right) \\ &= \theta_p \delta_{p,-1} + \theta_p \beta \sum_{t=0}^{\infty} \beta^t \delta_{p,t} + \frac{\theta_p}{1-\theta_p}\sum_{t=0}^{\infty} \beta^t \pi_t^2 + O(||\xi||^3)\end{aligned}$$

Then

$$\begin{aligned}(1-\theta_p\beta)\sum_{t=0}^{\infty} \beta^t \delta_{p,t} &= \theta_p \delta_{p,-1} + \frac{\theta_p}{1-\theta_p}\sum_{t=0}^{\infty} \beta^t \pi_t^2 + O(||\xi||^3) \\ \Rightarrow \sum_{t=0}^{\infty} \beta^t \delta_{p,t} &= \frac{\theta_p}{(1-\theta_p)(1-\theta_p\beta)}\sum_{t=0}^{\infty} \beta^t \pi_t^2 + t.i.p. + O(||\xi||^3) \qquad \text{(A.23)}\end{aligned}$$

The analogous equations hold for π_t^*, π_t^w, and $\pi_t^{w,*}$.

$$\sum_{t=0}^{\infty} \beta^t \delta_{p,t}^* = \frac{\theta_p^*}{(1-\theta_p^*)(1-\theta_p^*\beta)}\sum_{t=0}^{\infty} \beta^t (\pi_t^*)^2 + t.i.p. + O(||\xi||^3). \qquad \text{(A.24)}$$

$$\sum_{t=0}^{\infty} \beta^t \delta_{w,t} = \frac{\theta_w}{(1-\theta_w)(1-\theta_w\beta)}\sum_{t=0}^{\infty} \beta^t (\pi_t^w)^2 + t.i.p. + O(||\xi||^3), \qquad \text{(A.25)}$$

$$\sum_{t=0}^{\infty} \beta^t \delta_{w,t}^* = \frac{\theta_w^*}{(1-\theta_w^*)(1-\theta_w^*\beta)}\sum_{t=0}^{\infty} \beta^t (\pi_t^{w,*})^2 + t.i.p. + O(||\xi||^3). \qquad \text{(A.26)}$$

Finally, taking the discounted sum of Equation (A.26), we obtain the following the central bank's loss function.

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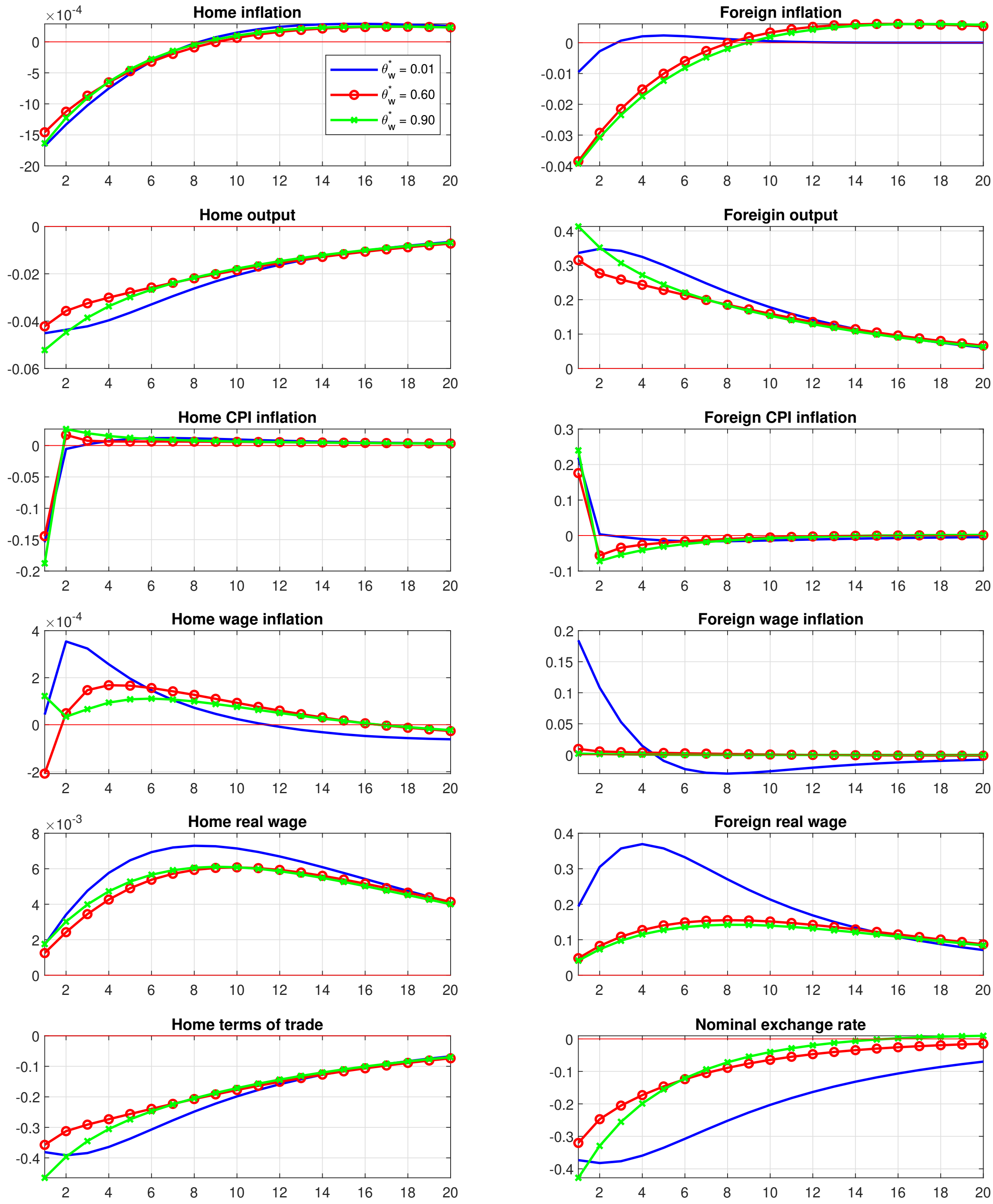

Figure 1: Impulse responses of a foreign productivity shock

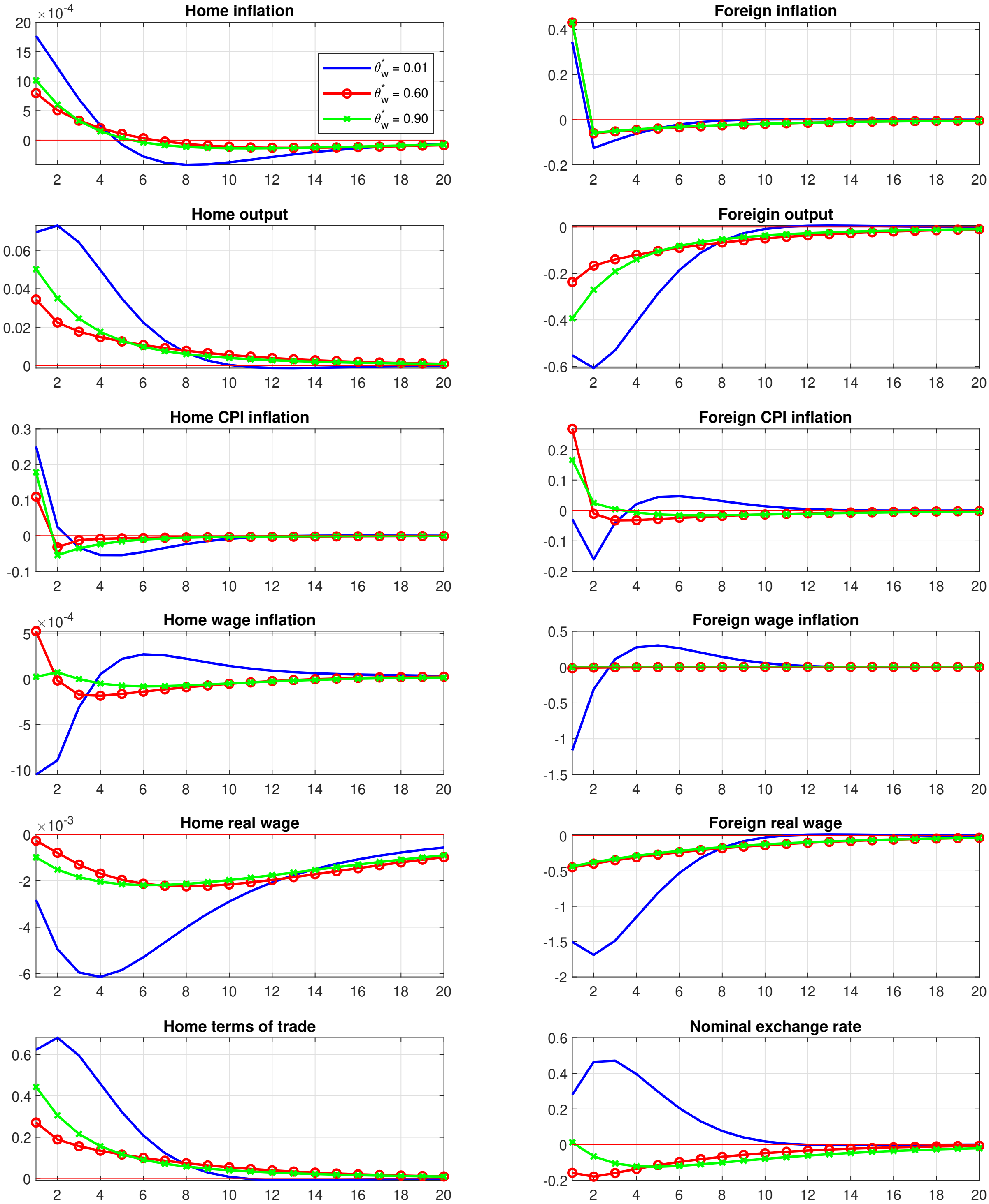

Figure 2: Impulse responses of a foreign cost-push shock

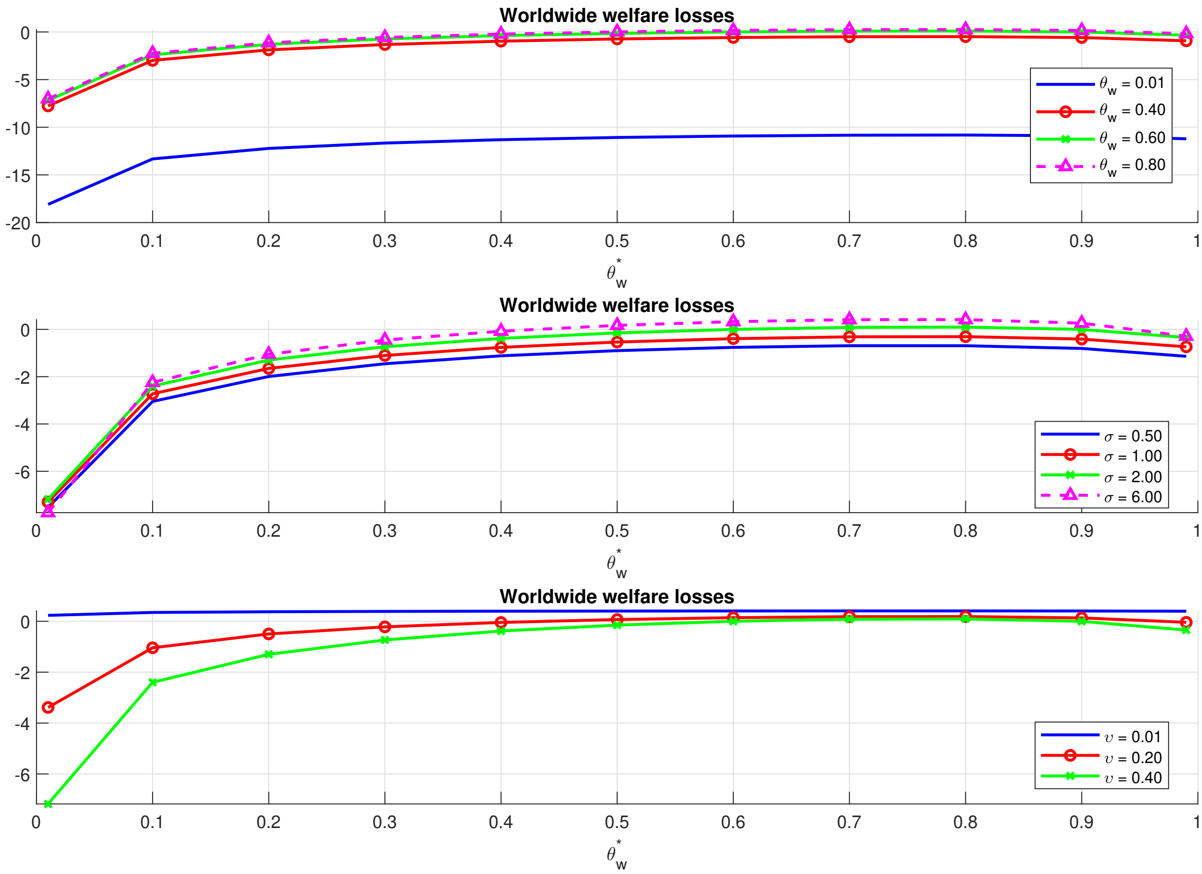

Figure 3: Relative welfare loss (1): Various parameter settings

Note: Each value is expressed as a percentage deviation from the result at the baseline parameter setting of $\theta_w = \theta_w^* = 0.6$.

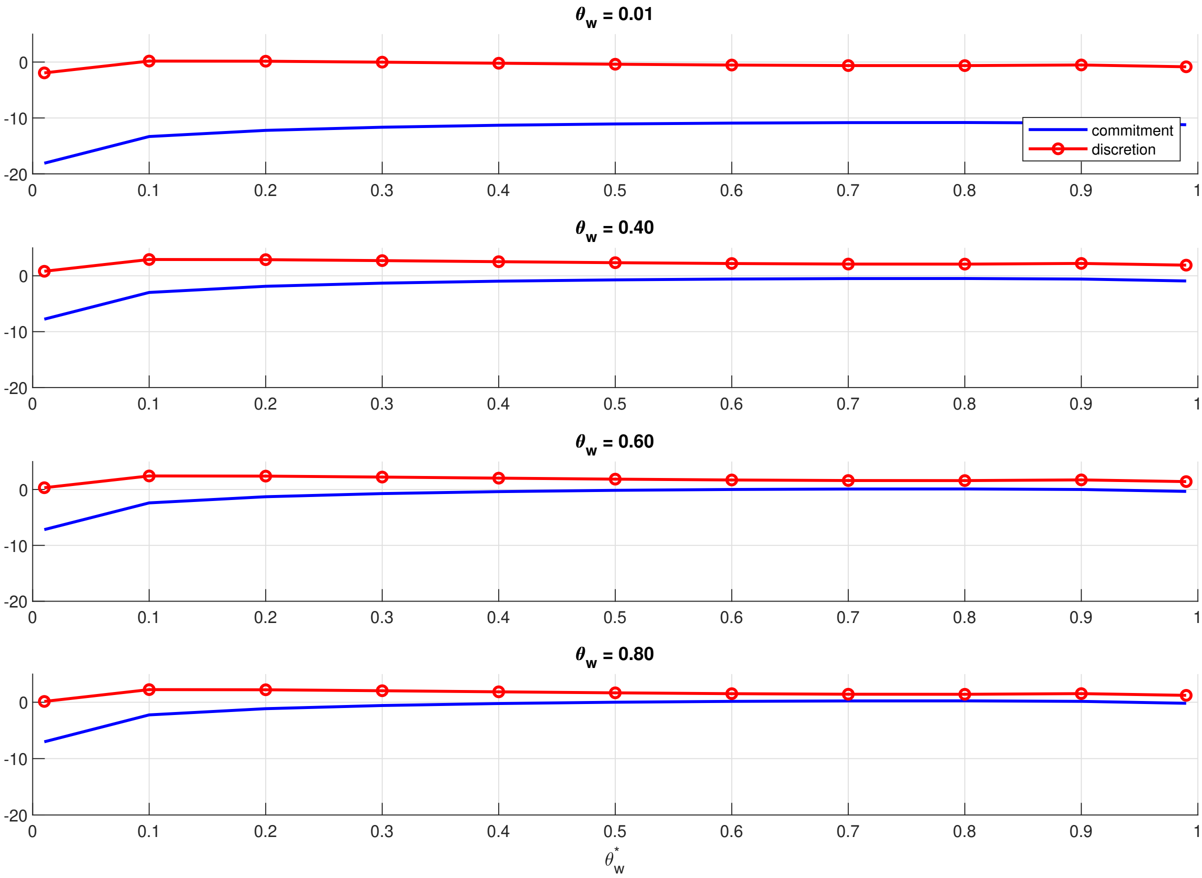

Figure 4: Commitment vs. Discretion (1): Domestic wage stickiness

Note: Each value is expressed as a percentage deviation from the result at $\theta_w = \theta_w^* = 0.6$, which is the setting of the baseline parameter under the commitment policy.

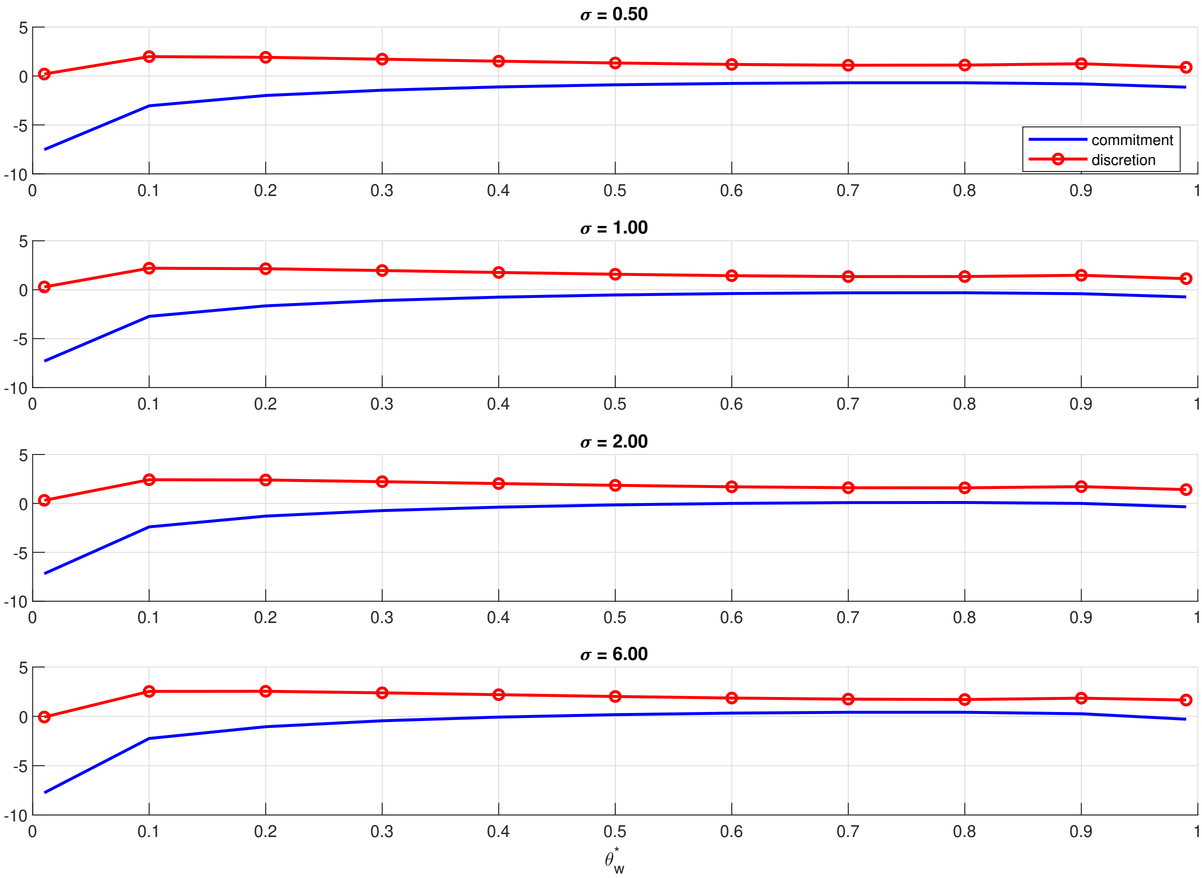

Figure 5: Commitment vs. Discretion (2): Intertemporal substitution of consumption

Note: Each value is expressed as a percentage deviation from the result at $\theta_w = \theta_w^* = 0.6$, which is the setting of the baseline parameters under the commitment policy.

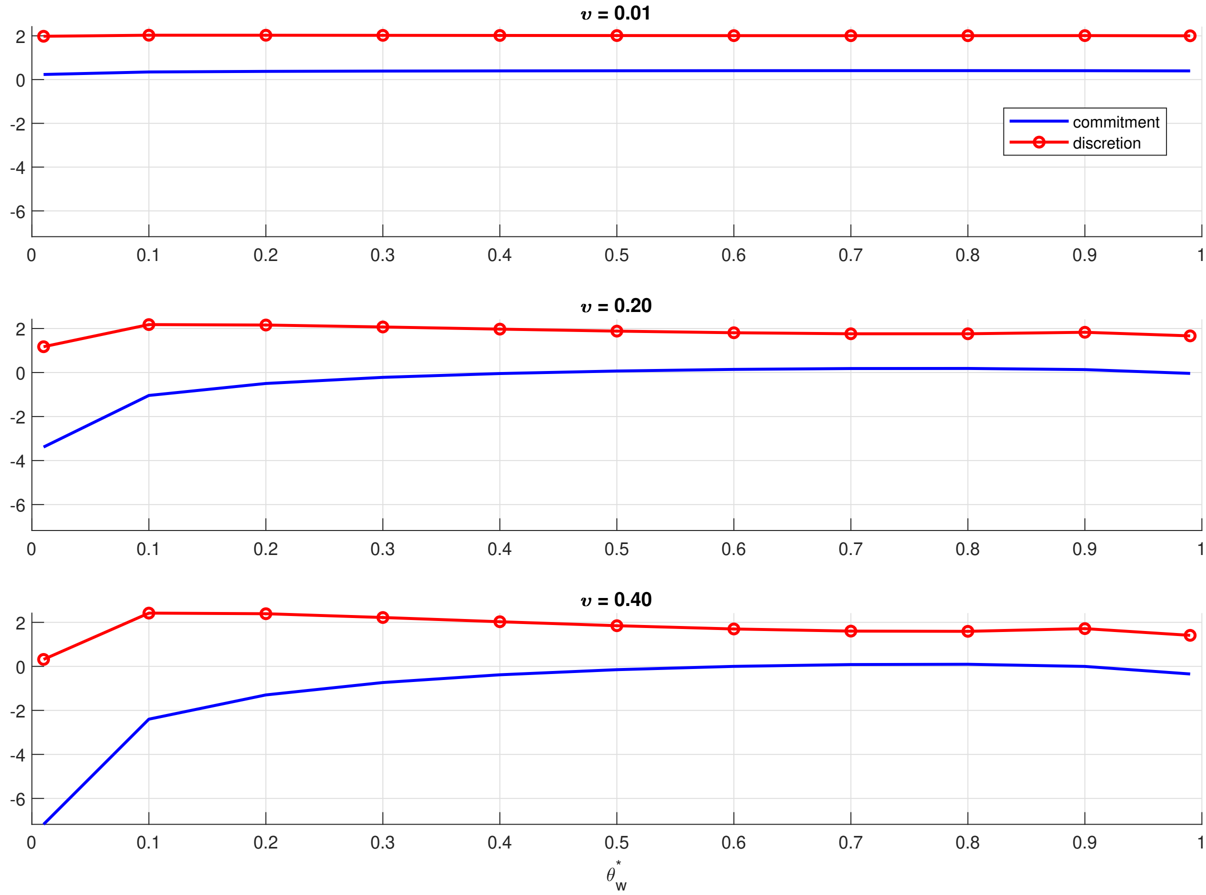

Figure 6: Commitment vs. Discretion (3): Degree of openness

Note: Each value is expressed as a percentage deviation from the result at $\theta_w = \theta_w^* = 0.6$, which is the setting of the baseline parameters under the commitment policy.

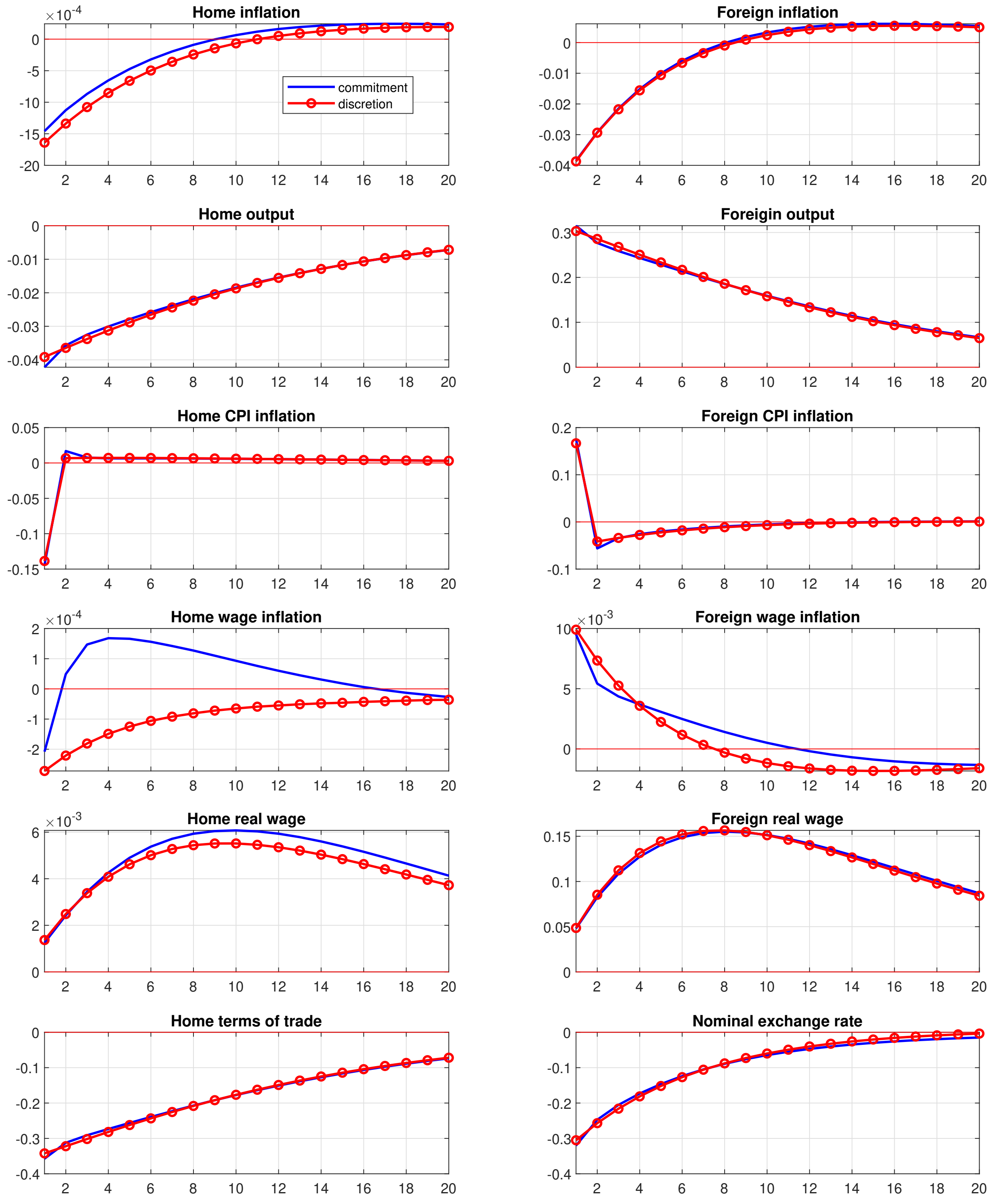

Figure 7: Impulse responses of a foreign productivity shock: Commitment vs. Discretion

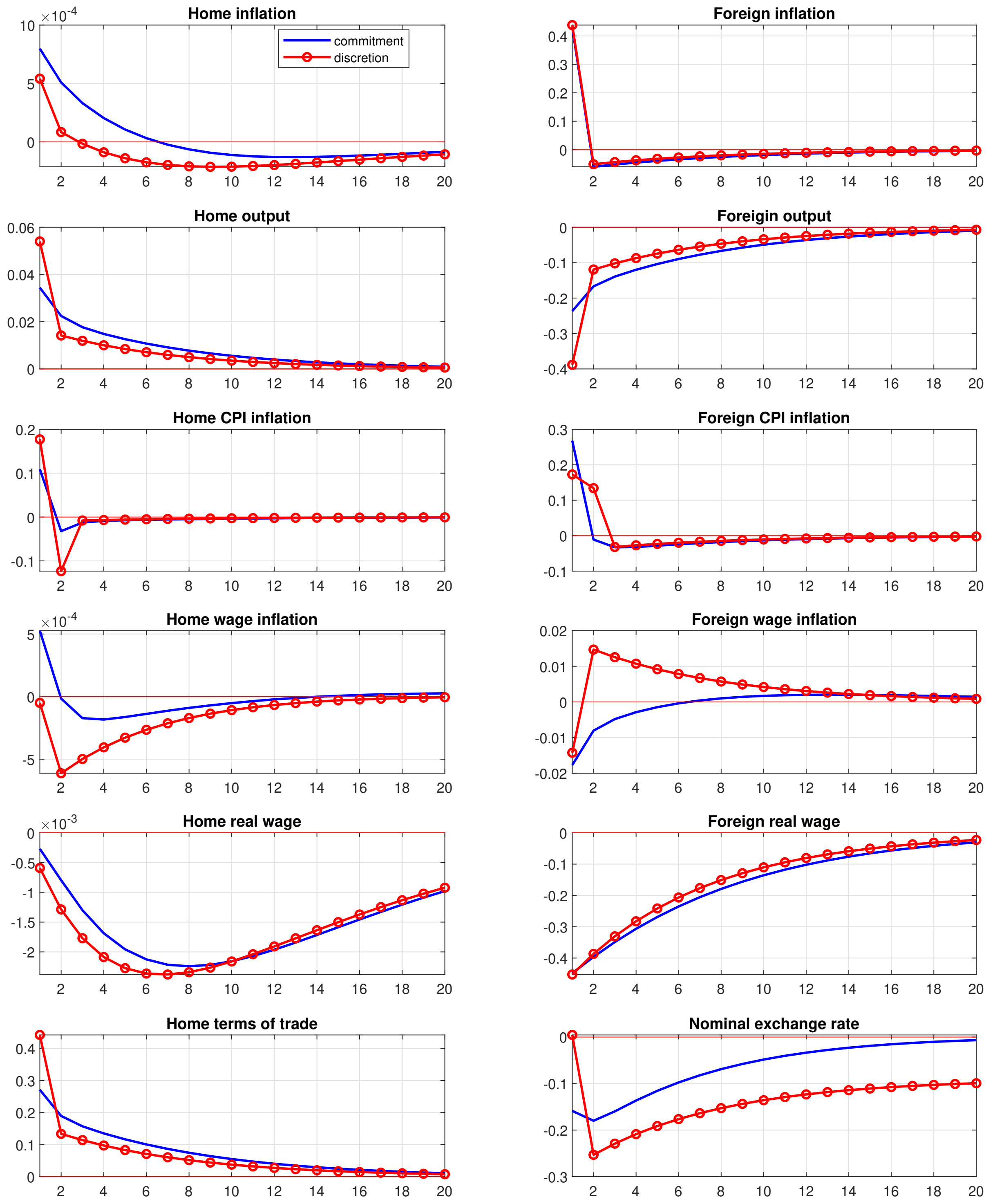

Figure 8: Impulse responses of a foreign cost-push shock: Commitment vs. Discretion

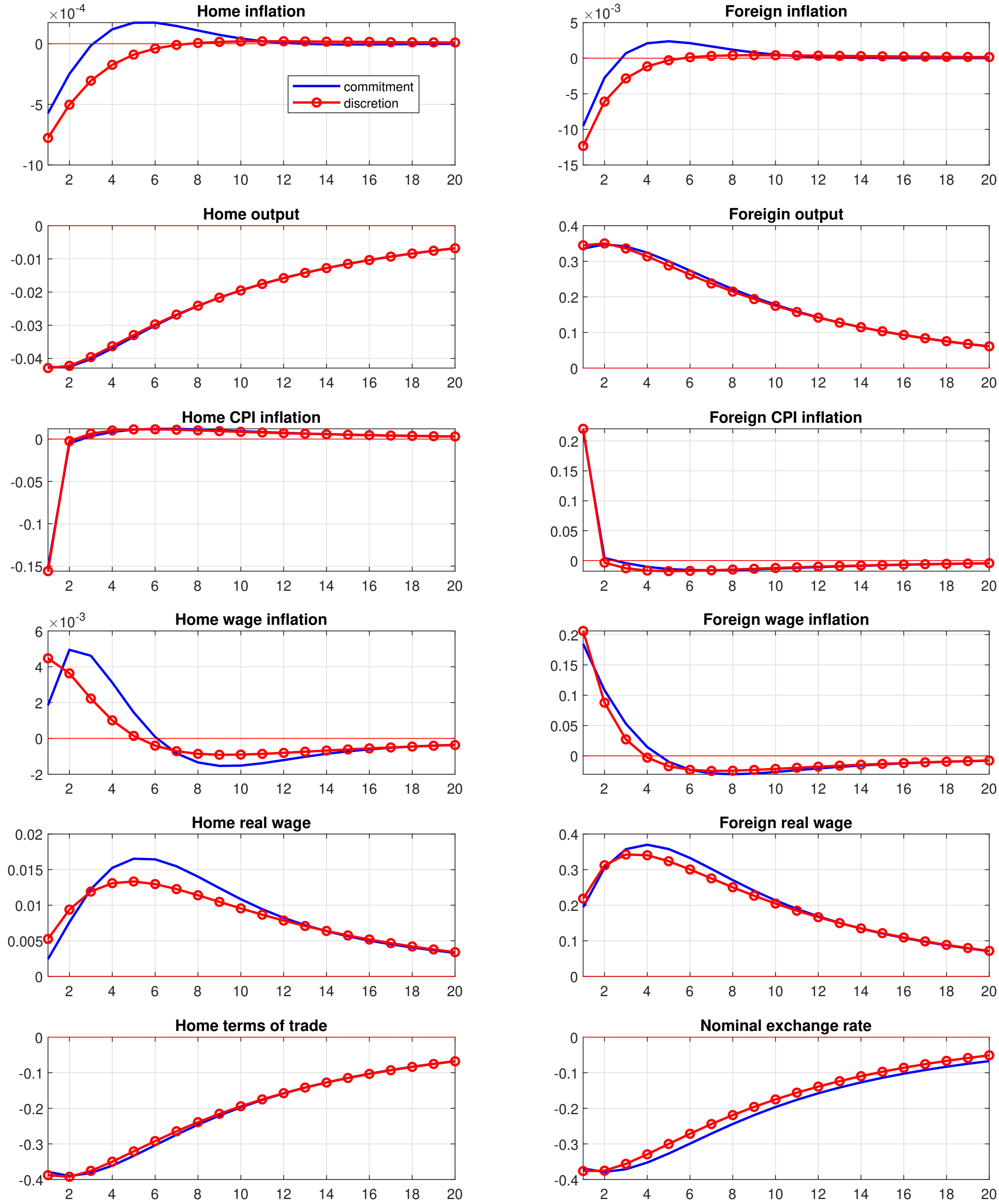

Figure 9: Impulse responses of a foreign productivity shock: In the case of flexible wages

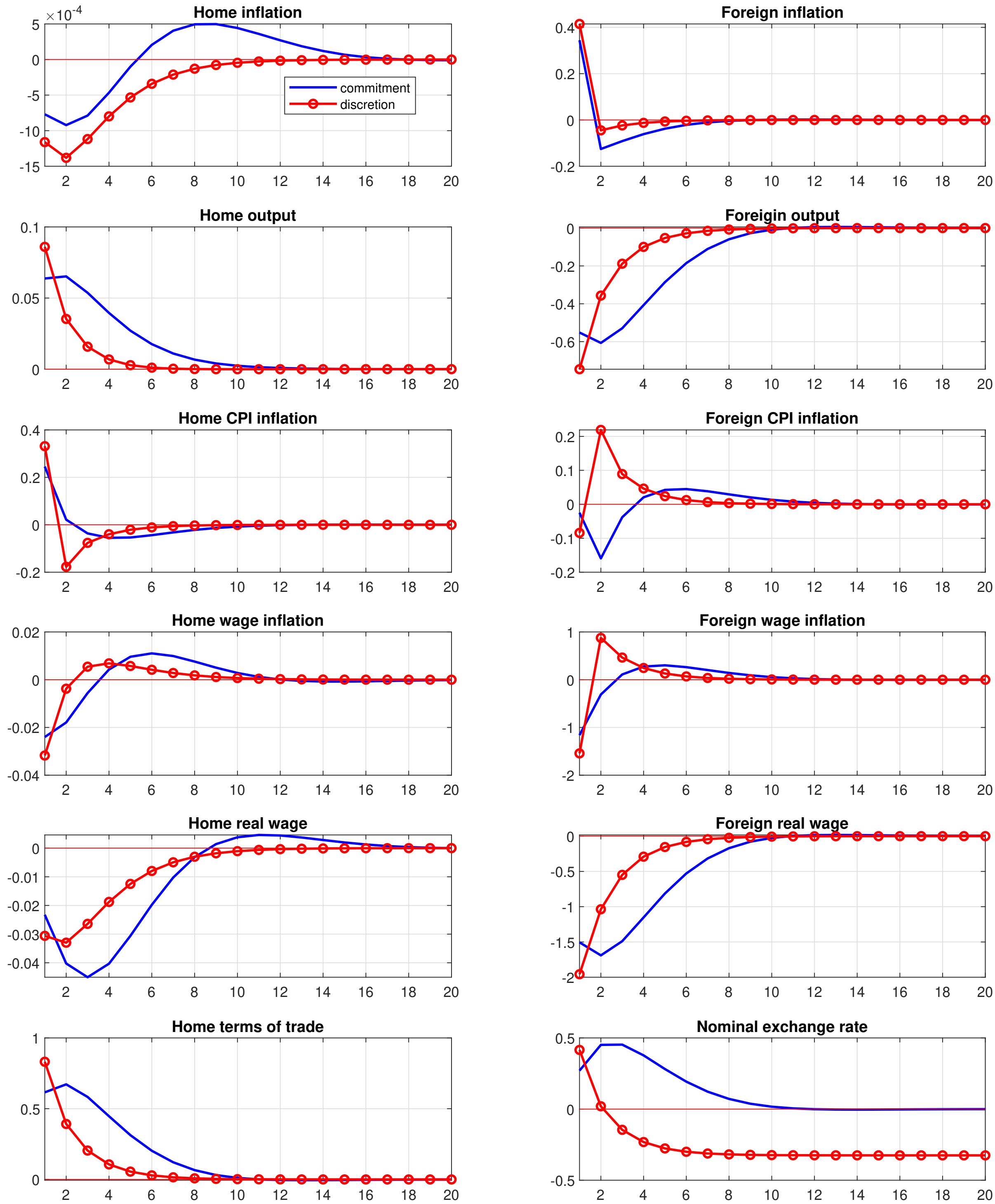

Figure 10: Impulse responses of a foreign cost-push shock: In the case of flexible wages