Research Institute

Discussion Paper Series

No. 19

Title：

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2022 年 4 月

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Robust control and delegating optimal monetary policy inertia in a small open new Keynesian model*

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April 6, 2022

Abstract

This paper explores the delegation of targeting regimes under robust control in a small open new Keynesian model. Under worst-case equilibrium, delegating nominal income growth to the central bank outperforms alternative policy regimes considered in this study, as long as the central bank is concerned about the degree of robustness. We address the fact that, in worst-case equilibrium, nominal income growth targeting outperforms speed limit policy. Even in the approximating equilibrium case, delegating monetary policy inertia can produce better outcomes than inflation targeting. These results are unaffected by the introduction of endogenous inflation persistence.

JEL codes: E52; E58; F41

Keywords : Optimal monetary policy; Targeting regime; Robust control; Small open economy

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*Ida was supported by JSPS KAKENHI Grant Number JP20K01784. Okano was supported by JSPS KAKENHI Grant Number JP20K13531. All remaining errors are our own.

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1 Introduction

This paper attempts to focus on the role of delegating optimal monetary policy inertia in a small open new Keynesian (NK) model with model uncertainty. How should central banks conduct monetary policy in an economy confronted with several uncertainties related to international trade and finance? Needless to say, this concern raises important research questions because policymakers should have considered the uncertain impact of recent globalization on monetary policy conduct (Galí and Gertler, 2010; Rogoff *et al.*, 2003).

Several studies have addressed the role of robust control in the NK model with model uncertainty (Giordani and Söderlind, 2004; Leitemo and Söderström, 2008a; Tillmann, 2009b,c).[1] Leitemo and Söderström (2008b) examined the properties of a discretionary policy under robust control in a small open NK model, but they did not study the case for commitment. Thus, they did not compare the performance of commitment with that of discretion. Although Clarida *et al.* (2001) and Monacelli (2003) addressed the role of commitment policy in a small open NK model, these studies did not consider such role in the presence of model uncertainty.

To the best of our knowledge, no studies have been conducted to determine whether a commitment policy consistently outperforms a discretionary one in a small open NK model with robust control. Consequently, whether a central bank implementing a discretionary policy can achieve performance comparable with that under commitment remains unclear. In other words, as far as we know, this is the first study investigating how the central bank overcomes the problem of stabilization bias in an open economy under robust control. This paper aims to discuss this unresolved issue in the existing literature.

Indeed, a large number of studies have discussed the superiority of commitment over discretion (Woodford, 2003), in the context of optimal monetary policy. This difference is carried over in the case of optimal monetary policy under robust control (Giordani and Söderlind, 2004). However, even if the central bank cannot commit to its future monetary policy, discretionary policies under optimal delegation problem lead to preferable outcomes to a pure discretionary policy (Walsh, 2003). Thus, Walsh (2003) provided the prescription of how the central bank overcomes the problem of stabilization bias caused by a discretionary policy. Recently, Bilbiie (2014) analytically showed the condition that the optimal delegation policy coincides with the commitment solution. Moreover, Ida and Okano (2021) showed the condition of how the

[1] See Hansen and Sargent (2011) for a detailed discussion of the role of robust control in macroeconomics.

optimal delegation policy achieves the commitment solution in a small open NK model.

Although these studies have addressed the role of considering optimal delegation scheme under rational expectations (RE) equilibrium, Walsh (2004) showed that the output gap is related to model misspecification in the worst-case equilibrium under standard inflation targeting (IT). We address the importance of investigating the properties of optimal monetary policy in an open economy model with robust control because this argument should be carried over into open economy models.[2] Does delegating optimal monetary policy inertia play a significant role in a small open NK model with robust control? In other words, how does a central bank overcome the problem of stabilization bias caused by discretionary policies in an open economy where robust controls are important?

This study aims to answer these important questions. To do so, we consider the simple small open NK model developed by Galí and Monacelli (2005). The main findings of this paper are summarized as follows. First and importantly, the role of delegating optimal monetary policy inertia is carried over in a small open NK model with robust control. Under worst-case equilibrium, delegating nominal income growth suggested by Jensen (2002) to the central bank outperforms the considered alternative policy regimes, as long as the central bank worries about the degree of robustness. We specifically address the fact that, in worst-case equilibrium, nominal income growth targeting (NIGT) outperforms commitment policy performance across a range of robustness parameter values. However, in approximating equilibrium, no policy regime can outperform a commitment policy, whereas any policy regime can produce preferable outcomes to IT.

Second, in contrast to the findings of Walsh (2003) and Ida and Okano (2021), the performance of nominal income growth is superior to that of a speed limit policy (SLP) in this model.[3] As shown by Ida and Okano (2021), NIGT consisting of consumer price index (CPI) inflation produces worse outcomes to SLP. However, Ida and Okano (2017) specified the possibility of a commitment solution in a small open NK model due to NIGT based on producer price index (PPI) inflation. We stress the effectiveness of PPI-based NIGT in a small open NK model with robust control.

[2] Walsh (2004) also showed that when the central bank's loss function includes interest rate stabilization in addition to the standard policy objectives, the targeting rule is unaffected by any model misspecification.

[3] Ida (2022) showed the effectiveness of NIGT under robust control in a model with incomplete exchange rate path-through developed by Monacelli (2005).

Third, the optimal response of the nominal exchange rate depends on the targeting regimes that the central bank employs. More concretely, as shown by Monacelli (2003), the optimal response of the nominal exchange rate is stationary under commitment, but nonstationary under discretion. Interestingly, the response of the exchange rate under the SLP is nonstationary, despite the fact that the policy can impart policy inertia into the economy via a change in the output gap. However, NIGT can lead to the exchange rate's stationary response. This is the source of the welfare gain from using NIGT in a small open economy under robust control.

Fourth, the performance of a real exchange rate targeting is very close to that of speed limit targeting suggested by Walsh (2003) under both worst and approximating equilibria. This result is not surprising because the real exchange rate is proportionally related to a change in the output gap. Accordingly, the finding of Ida and Okano (2021) is carried over in the case for robust control because a change in the real exchange rate is proportional to one in the output gap.

Finally, we extend the small open NK model by introducing endogenous inflation persistence. More concretely, following Amato and Labach (2003), we incorporate the rule-of-thumb price setters to investigate the role of backward-looking inflation. The presence of endogenous inflation persistence does not affect our main message. In the case of a worst-case equilibrium, NIGT results in the smallest welfare losses of any policy regime. Even in the approximating equilibrium, discretionary policies with policy inertia produce better results than IT.

This paper is related to the following studies. As noted earlier, we consider the role of delegating optimal monetary policy inertia when robust monetary policy matters in a small open economy. Monacelli (2003) investigated optimal monetary policy in a small open NK model and showed the properties of discretion, commitment, and exchange rate policies. Meanwhile, Ida and Okano (2021) examined the role of optimal delegation problem in a small open NK model by extending the framework suggested by Bilbiie (2014). These studies concentrated on the characteristics of optimal monetary policy in a small open economy. However, we emphasize the importance of considering how the central bank implements optimal monetary policy when it cannot commit to its future monetary policy under model uncertainty. Leitemo and Söderström (2008b) analytically examined the optimal discretionary policy under robust control in a small open economy. However, they did not consider the case of commitment. In contrast to Leitemo and Söderström (2008b), we compare the performance of several alternative policy regimes with pure discretion in that model. Finally, Walsh (2003) found that SLP outperforms NIGT; thus, we showed that a policy regime with nominal income growth leads to preferable outcomes to SLP in a small open NK model with robust control. Hasui (2021) considered the effectiveness of targeting regimes under robust control in a closed economy, whereas we investigate the performance of targeting regimes in an open economy aspect.

The remainder of this paper is structured as follows. Section 2 briefly describes the small open NK model and the central bank's loss function. Section 3 provides a brief overview of robust optimal monetary policy and its properties under robust control. Section 4 provides the main result of the paper. Section 5 discusses the findings in the presence of endogenous inflation persistence. Lastly, Section 6 concludes the study briefly. The appendix contains a detailed derivation of the optimal targeting rule under robust control, which will be discussed in Section 3.2.

2 Model description

This section briefly provides the model description. Section 2.1 explains the small open NK model developed by Galí and Monacelli (2005). Section 2.2 explains the central bank's loss function in an open economy. The model we adopt is based on the standard small open NK model developed by Galí and Monacelli (2005); thus, readers familiar with this model can skip to Section 3.

2.1 A small open new Keynesian model

This section briefly describes the small open NK model based on the work of Galí and Monacelli (2005) and Galí (2015).[4] The home country is infinitesimally small relative to the rest of the world. Representative households in the home country buy domestic and foreign goods. Moreover, households in the home country can access to a complete set of state-contingent securities traded both domestically and internationally. Firms face both monopolistically competitive environments and nominal staggered-price rigidities, as specified by Calvo (1983). We assume producer currency pricing.[5] In this paper, the lower-case variable means the log-linearized

[4]The following model explanation is heavily based on Ida and Okano (2021) in Sections 2.1 and 2.2.

[5]See Monacelli (2005) for a detailed discussion of the NK model based on local currency pricing.

ones. The log-linearized system is given as follows:[6]

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_\nu x_t + u_t, \tag{1}$$

$$x_t = E_t x_{t+1} - \sigma_\nu^{-1}(i_t - E_t \pi_{t+1} - \bar{rr}_t), \tag{2}$$

$$\pi_t^c = \pi_t + \nu(s_t - s_{t-1}), \tag{3}$$

$$q_t = (1-\nu)s_t, \tag{4}$$

$$s_t = \sigma_\nu x_t, \tag{5}$$

where π_t is producer currency inflation, x_t is the output gap, i_t is the nominal interest rate, π_t^c is CPI inflation, s_t is the terms of trade (hereinafter ToT) gap, and q_t is the real exchange rate gap. Gap variables are expressed by the log deviation of the endogenous variables from the efficient level of their variables. In addition, $\bar{rr}_t$ denotes the natural rate of interest, which holds the real interest rate under a flexible price equilibrium, and u_t is the exogenous cost-push shock, which follows an AR(1) process.[7] Finally, the coefficients for each structural equation are defined as follows:

$$\sigma_\nu = \frac{\sigma}{1+\nu[\sigma\eta + (1-\nu)(\sigma\eta - 1) - 1]},$$
$$\kappa_\nu = \delta\left(\sigma_\nu + \frac{1+\varphi}{1-\chi}\right),$$
$$\delta = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\Theta,$$
$$\Theta = \frac{1-\chi}{1-\chi+\chi\epsilon}.$$

The parameters β, σ, η, and φ represent the discount factor, the relative risk aversion coefficient, the substitutability between domestic and foreign goods, and the inverse labor supply elasticity, respectively. Moreover, ν is the degree of openness, and χ is the degree of diminishing returns to scale for labor supply in the production function. α characterizes the degree of nominal price rigidities (i.e., Calvo's lottery), and ϵ denotes the elasticity of substitution for individual goods.

Equation (1) represents the NK Phillips curve (NKPC) for a small open economy, which is derived from the firm's profit maximization problem subject to Calvo pricing. Equation (2) is a

[6]See Galí (2015) for a detailed derivation of a small open NK model.

[7]The presence of a cost-push shock is related to a time-varying price and wage markup owing to the monopolistic competition in the labor market. See Clarida *et al.* (2002) and Steinsson (2003) for a detailed discussion of this issue.

dynamic investment-saving (IS) equation, which results from solving the household's intertemporal optimization problem. Meanwhile, Equation (3) specifies the relationship between CPI and PPI inflation. Equation (4) states that the real exchange rate changes proportionally in response to a change in the terms of trade. Moreover, Equation (5) represents the relationship between the ToT and output gap. To consider the role of the nominal exchange rate, we also derive the nominal exchange rate's log-linearized dynamics. Using the definition of the ToT yields the following nominal exchange rate's law of motion:

$$e_t = e_{t-1} + s_t - s_{t-1} + \pi_t^* - \pi_t, \tag{6}$$

where e_t and π_t^* denote the nominal exchange rate and foreign inflation, respectively.

The effect of using an open economy model is characterized by changes in both ν and $\sigma\eta$. These parameters affect the coefficient of σ_ν, which is included in κ_ν and σ_ν. When $\sigma\eta > 1$, an increase in ν induces a decrease in σ_ν. Thus, changes in ν and $\sigma\eta$ alter the slopes of the IS curve and the NKPC. In this case, the domestic output gap changes according to changes in the ToT, implying a fluctuating real exchange rate through the risk-sharing condition. The IS equation can capture this mechanism.

Noteworthily, changes in the ToT affect the sensitivity of inflation to the real marginal cost in the NKPC through two channels. First, domestic inflation responds positively to changes in ToT caused by changes in the real exchange rate via international consumption risk sharing. Second, changes in ToT cause fluctuations in domestic inflation by inducing changes in real marginal costs. Whether domestic inflation increases depends on the movement of $\sigma\eta$. In the case of $\sigma\eta = 1$, both κ_ν and σ_ν decrease to $\delta(1 + \frac{1+\varphi}{1-\chi}) \equiv \kappa$ and 1, respectively. Thus, the open-economy effect disappears in this case.

Finally, we assume that the home country is infinitesimally small relative to the rest of the world; thus, the other variables for the rest of the world (π_t^*, i_t^*, x_t^*) are exogenous for the home country. Variables with $(*)$ denote affiliation with a foreign country.

2.2 Central bank objectives

The central bank conducts its monetary policy following a targeting rule derived from minimizing the central bank's loss function. As shown by Woodford (2003), the loss function of the central bank is derived from a second-order approximation of a household utility function. In the standard NK model, the central bank's loss function includes both inflation stabilization and the output gap.

However, in an open economy model, we frequently have difficulty deriving a well-defined loss function. In particular, we face this problem in the case of $\sigma\eta \neq 1$. Fortunately, De Paoli (2009) derived the central bank's loss function in the small open NK model, allowing the case of $\sigma\eta \neq 1$. De Paoli (2009) showed that the central bank's loss function includes not only home inflation and the home output gap but also the stabilization term of the real exchange rate. Our model also assumes that the central bank's loss function follows the derivation of De Paoli (2009). [8]

Specifically, the central bank minimizes the following social loss function subject to structural equations:

$$E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda x_t^2 + \tau q_t^2). \tag{7}$$

where λ denotes the stabilization weight on the output gap relative to inflation stabilization, and τ is the stabilization term for the real exchange rate.[9] Using Conditions (4) and (5), we can rewrite the true loss function as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \tilde{\lambda} x_t^2), \tag{8}$$

where $\tilde{\lambda} = \lambda + \tau(1-\nu)^2\sigma_\nu^2$. The loss function can be reduced to the standard loss function that contains both the inflation stabilization and the output gap. However, as long as $\nu \in [0, 1)$, the stabilization weight on the output gap is larger than it is in the case of no stabilization of the real exchange rate (i.e., $\tau = 0$). In addition to the output gap stabilization as the traditional policy objective, the second-term of the right-hand side in $\tilde{\lambda}$ includes the stabilization term of the output gap through a change in the real exchange rate. This term is specific to the open-economy case.

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[8] See De Paoli (2009) for a detailed derivation of the central bank's loss function in an open-economy model with nominal rigidities.

[9] Strictly speaking, the parameters in the loss function in our model might not offer a one-to-one correspondence with those derived by De Paoli (2009). Therefore, although the structural parameters in Equation (7) are constructed according to the deep parameters in the model, we simply calibrate the parameter values of both λ and τ.

3 Robust optimal policy in a small open economy

The role of robust optimal monetary policy in a small open NK model is discussed in this section. In particular, Section 3.1 explains the robust control, and Section 3.2 briefly reviews the properties of optimal monetary policy in a small open NK model. We also discuss the optimal delegation problem in this subsection.

3.1 Model uncertainty: Robust control approach

We consider the effect of model uncertainty on optimal monetary policy in a small open NK model. First, this section considers a Hansen-Sargent type of robust control problem to address the model uncertainty.[10] In the Hansen–Sargent's robust control approach, a robust planner cannot ensure whether the *reference model* for the laws of motion of the economy (i.e., the RE equilibrium) is consistent with the true model. To solve such a problem, or model uncertainty, we adopt the min–max approach by following Hansen and Sargent (2001) and Giordani and Söderlind (2004). In this approach, a hypothetical evil agent—a metaphor for model uncertainty—determines the maximum expected loss under a given budget constraint, and the social planner then minimizes the maximum expected loss. Specifically, the robust optimal monetary policy can be defined as follows:[11]

$$\min_{\{i\}_0^\infty} \max_{\{v\}_1^\infty} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \pi_t^2 + \tilde{\lambda} x_t^2 \right\} \tag{9}$$

subject to the laws of motion of the (distorted) economy:

$$E_t x_{t+1} = A x_t + B i_t + C(v_{t+1} + \varepsilon_{t+1}), \tag{10}$$

and the evil agent's budget constraint

$$E_0 \sum_{t=0}^{\infty} \beta^t v'_{t+1} v_{t+1} \leq \eta \tag{11}$$

where $x_t = [x_{1,t}; x_{2,t}]$ and $C = [C_1; \mathbf{0}_{n2\times n1}]$. In addition, A and B are coefficient matrices constructed by deep parameters. $x_{1,t}$ is an $n_1 \times 1$ vector of predetermined variables, $x_{2,t}$ is an

[10] The following explanation heavily relied on Giordani and Söderlind (2004).

[11] We need to assume that private expectations are either standard or robust in a forward-looking model. Following Hansen and Sargent (2001) and Giordani and Söderlind (2004), we assume that the private sector and the planner share the same loss function, reference model, and degree of robustness. See Giordani and Söderlind (2004) for more details and the numerical solution algorithm.

$n_2 \times 1$ vector of forward-looking variables, and ε_{t+1} is an $n_1 \times 1$ vector of white noise shocks. v_{t+1} is an $n_1 \times 1$ vector of model specification errors, which represents evil agent's control vector. n_1 and n_2 denote the number of predetermined variable and the number of control variable, respectively. Note that this term cannot be identified by the central bank because it is always masked by being multiplied by C, combined with the shock term ε_{t+1}. The parameter η denotes the measure of the central bank's preference for robustness.

Following Giordani and Söderlind (2004), we can rewrite the stated optimization problem as follows:

$$\min_{\{i\}_0^\infty} \max_{\{v\}_1^\infty} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \pi_t^2 + \tilde{\lambda} x_t^2 - \theta v_{t+1}' v_{t+1} \right\}, \tag{12}$$

subject to Equation (10).

The researcher needs to specify only one parameter, the Lagrange multiplier θ, with $0 < \theta < \infty$. The parameter implies the degree of robustness or the evil agent's budget. The evil agent can work well if θ is close to zero and $\theta = \infty$ corresponds to RE.

The solutions of this problem can be described as follows:

$$\begin{bmatrix} x_{1,t+1} \\ \rho_{2,t+1} \end{bmatrix} = M \begin{bmatrix} x_{1,t} \\ \rho_{2,t} \end{bmatrix} + C\varepsilon_{t+1}, \quad \begin{bmatrix} x_{2,t} \\ i_t \\ v_{t+1} \\ \rho_{1,t} \end{bmatrix} = \begin{bmatrix} N_1 \\ -F_i \\ -F_v \\ N_\rho \end{bmatrix} \begin{bmatrix} x_{1,t} \\ \rho_{2,t} \end{bmatrix}, \tag{13}$$

where $M = P^{-1}(A - BF_u - BF_v)P$ and $P = [\mathbf{I}_{n1}\ \mathbf{0}_{n1 \times n1}; N_1]$.[12] In addition, $x_{1,t} = [rr_t, u_t, x_{t-1}]'$, $x_{2,t} = [x_t, \pi_t]'$, and $\varepsilon_{t+1} = [\varepsilon_{t+1}^{rr}, \varepsilon_{t+1}^{u}]'$. We abstract from a natural rate of interest shock in the following discussion because the central bank can achieve optimal policy when this shock only matters in an economy.[13] Note that when the presence of interest rate smoothing is allowed in the loss function, a fluctuation in the natural rate of interest significantly affects optimal monetary policy (Giannoni and Woodford, 2003).[14]

In discussing robust monetary policy, the following two scenarios are often considered: First, Equation (13) is referred as *worst-case model*, which describes the behavior of the economy when

[12]See Giordani and Söderlind (2004) for a detailed calculation of N, F_i, F_v, and N_ρ.

[13]See Woodford (2003) for a detailed discussion.

[14]Walsh (2004) considered the robustly optimal monetary policy rules when the central bank's loss function includes interest rate stabilization.

the planner's pessimism is fully realized. Second, by putting $F_v = 0$ in (13), we can define the *approximate model*.[15] Approximating model assumes the case where the central bank fully implements robust policies under the worst-case scenario, despite the absence of specification errors.

Under a robust pure discretion, the objective function (9) and constraints (10) are shared with a commitment case. By adopting the concept of robust discretionary policy proposed by Giordani and Söderlind (2004), the difference between a robust discretion and a robust commitment in a forward-looking model can be simplified as differences in numerical algorithms.[16] Technically, putting $\rho_{2,t} = \mathbf{0}$ for all t in (13) results in a discretionary robust policy solution.

3.2 Optimal policy under robust control

This section examines the role of an optimal targeting regime in robust control. To emphasize the significance of considering the optimal delegation problem in a small open economy with robust control, we briefly discuss the central bank's optimization problem in a small open economy for commitment and pure discretionary policy cases. More precisely, to arrive at the best monetary policy rule, the central bank minimizes the loss function (8) subject to the NKPC (1) in this model.

First, consider the case for commitment. As discussed by Woodford (2003), a commitment policy allows the central bank to manage the expectations of the private sector because it commits the private sector to future monetary policy at present. Solving the central bank's optimization problem under commitment leads to the *targeting rule*, which is given by[17]

$$\pi_t = -\frac{\tilde{\lambda}}{\kappa_\nu}(x_t - x_{t-1}). \tag{14}$$

The targeting rule (14) is characterized by the presence of a lagged output gap (Woodford, 2003). One can easily conjecture that this property is carried over into the open-economy

[15] Note that under the approximating equilibrium, matrix M becomes $M = P^{-1}(A - BF_u)P$, as shown by Giordani and Söderlind (2004).

[16] Following Giordani and Söderlind (2004), we extend robustness, a metaphor for the concern about model misspecification so that the evil agent optimizes only when the planner optimizes. Applying this principle to the commitment case coincides with the Hansen–Sargent solution.

[17] Commitment policy in this paper is referred to as the optimal monetary policy from timeless perspective proposed by Woodford (2003).

model. Furthermore, as shown in the Appendix, the targeting rule of the form (14) still holds for the case under robust control. By changing the policy variables with a lag, the central bank can manipulate the expectations of the private sector. The differences between closed and open economy models can be captured in the coefficients of the structural equations. The optimal targeting rule (14) in the open economy model reflects the effects of both ToT and risk-sharing effects: the central bank stabilizes the domestic economy by considering the spillover effects from foreign economies.[18]

Next, consider the optimal monetary policy under discretion. Given future expected variables, the central bank under discretion implements optimal monetary policy. Consequently, unlike a commitment policy, a discretionary policy cannot manipulate private sector expectations. Thus, the targeting rule under discretion is changed to

$$\pi_t = -\frac{\tilde{\lambda}}{\kappa_\nu} x_t. \tag{15}$$

In contrast to the targeting rule under commitment, Equation (15) does not depend on the lagged output gap.

In addition to the aforementioned difference, the targeting rule is affected by model misspecification (Walsh, 2004). As shown by Walsh (2004), the relationship between model misspecification and optimal policy under commitment is given by

$$w_{t+1} = -\left(\frac{\beta\tilde{\lambda}}{\kappa_\nu \theta}\right) \sum_{i=0}^{\infty} (\beta\rho_u)^i E_t x_{t+i+1}, \tag{16}$$

where w_{t+1} denotes the Lagrange multiplier associated with the presence of evil agent.[19] As Walsh (2004) specified, the Lagrange multiplier w_{t+1}, which is related to model misspecification, is negatively related to the present value of the expected output gap. When the output gap is negative, and a trade-off exists between stabilizing the output gap and stabilizing inflation, the worst-case scenario for a central bank becomes a reality in that cost-push shocks hit the economy. A robust central bank would prepare for the worst-case scenario by tightening monetary policy further to suppress inflation. As a result, the commitment policy smoothens the effect of a cost-push shock on the output gap over time. Meanwhile, the model misspecification

[18]See Ida and Okano (2021) for more details on the targeting rule (14) in a small open economy.

[19]We briefly provide the derivation of this equation in the Appendix.

under discretion is now given by

$$w_{t+1} = -\left(\frac{\tilde{\lambda}}{\kappa_\nu \theta}\right) x_t. \tag{17}$$

This implies that in contrast to a commitment policy, the central bank implementing optimal policy under discretion has to immediately overcome the policy trade-off between model misspecification and the output gap fluctuation. It must also overcome the standard trade-off between inflation and the output gap associated with a cost-push shock.

To address our motivation in this paper, we focus on the following two international aspects in terms of the relationship between model misspecification and the output gap. First, in our small open economy model, the output gap is related to the real exchange rate and ToT. This means that through (16) or (17), these variables are also related to model misspecification and thus would alter the dynamics under the robust optimal monetary policy. More concretely, substituting Equation (5) into the targeting rule (16) leads to

$$w_{t+1} = -\left[\frac{\beta\tilde{\lambda}}{(1-\nu)\sigma_\nu \kappa_\nu \theta}\right] \sum_{i=0}^{\infty} (\beta\rho_u)^i E_t q_{t+i+1}. \tag{18}$$

This equation implies the negative effect of the discounted present value of the real exchange rate on model specification. In particular, a change in the degree of openness affects the sensitivity of model misspecification to a change in the real exchange rate. This channel is specific to the open economy model.

Second, the aforementioned intertemporal relationship between output gap and model misspecification might be attenuated when considering pure discretion policy. For example, we can easily observe from Equations (5) and (17) that, unlike the commitment policy, only the current real exchange rate influences the model specification under the discretionary policy. Our question is how this difference affects the performance of optimal monetary policy. In other words, just like in the RE model, discretionary policy with lagged endogenous variables produces preferable outcomes compared with pure discretion. As a result, investigating the performance of optimal monetary policy in a small open economy with model misspecification is worthwhile.

3.3 Role of optimal delegation policy under robust control

We now explain the role of the optimal delegation problem in a small open NK model with model uncertainty. As discussed in Section 3.2, the targeting rule (15) does not have a lagged variable in pure discretion case, leading to larger fluctuations (losses) in the economy than that in the commitment case. This difference is the source of stabilization bias generated by a discretionary policy in a forward-looking model.[20] This stabilization bias is also related to model specification because it affects the output gap under the robust optimal monetary policy. To address this issue, we consider delegation policies, which can incorporate inertia even when the central bank lacks commitment capabilities. This paper investigates the role of optimal delegation policy in a small open economy model with model uncertainty.

The following discussion explains the optimal delegation scheme in a forward-looking model. As described by Walsh (2003), the optimal delegation policy regime can be characterized by the policy objectives delegated to the central bank and the stabilization weights for each objective in its loss function. Following Walsh (2003), we also define the targeting rule as one that satisfies (a) the variables in the central bank's objective function and (b) the stabilization weights for the policy objectives in the loss function chosen to minimize the expected discounted value of the social loss function. We consider several delegating regimes summarized in Table 2.[21]

[Table 2 around here]

First, following Jensen (2002), we consider the NIGT, as shown in the first regime in Table 2. As Walsh (2003) noted, the central bank's policy choice will be a function of x_{t-1} under NIGT. This introduces the lagged output gap as an endogenous state variable on the system, even under discretion. As mentioned earlier, this paper adopts a PPI-based NIGT. Section 4.4 discusses how the NIGT specification affects the performance of optimal delegation policies in an open economy. Next, we consider the optimal discretion under the SLP proposed by Walsh (2003)(second regime in Table 2). As in the case of NIGT, SLP includes a change in the output gap in the objective function. Therefore, the central bank that employs SLP can impart policy inertia through the lagged output gap. Following Ida and Okano (2021), and third regime in Table 2, we consider real exchange rate targeting (REX), as a regime specific to small open economy models. The role of targeting the real exchange rate is argued by Taylor (2001). As discussed by Ida and Okano (2021), policy inertia is introduced by the effects of the real exchange rate on the output gap under REX.

[20] See Walsh (2010) for a detailed discussion of stabilization bias.

[21] Ida and Okano (2021) provided a brief survey of delegation policies and some candidates for specific policy regimes.

In the context of the discussion here, we consider the problem that the central bank, which implements optimal monetary policy under a delegated policy regime k, seeks for a kind of the solution

$$w_{t+1} = -\left(\frac{\beta\lambda_k}{\kappa_\nu\theta}\right)\sum_{i=0}^{\infty}(\beta\rho_u)^i E_t x_{t+i+1}, \tag{19}$$

where $k \in \{SLP, NIGT, REX\}$. Notice that the parameter λ_k denotes the stabilization weights optimally chosen under a delegated policy objective, which would generally differ in the true social preference $\tilde{\lambda}$. Unfortunately, we cannot analytically derive Equation (19); thus, we numerically seek for the targeting rule under a delegated regime.

For comparison, we also consider traditional IT, listed as fourth regime in Table 2. Note that IT uses the same formulation as the social loss function. The only difference between the social loss function and IT is the weight on the output gap, which the government delegates in the case of IT. As shown in Table, IT cannot generate the required inertia no matter how the parameters are changed.

To solve these delegation policy problems, we can apply the standard solution algorithm for robust discretionary policies to our model, following Giordani and Söderlind (2004). The only difference between a pure discretionary robust policy and robust delegation policies is that the central bank's loss function is changed from a social loss function to each delegated loss function. Under the regime k, a central bank minimizes

$$\min_{\{i\}_0^\infty}\max_{\{v\}_1^\infty}\ E_0\sum_{t=0}^{\infty}\beta^t[\pi_t^2 + \lambda_k(z_t^k)^2 - \theta v_{t+1}' v_{t+1}], \tag{20}$$

subject to structural equations, for $k \in \{IT, SLP, NIGT, REX\}$. Variable z_t^k represents delegated policy objectives of a central bank's objective function under regime k. Again, note that the stabilization weights in the delegated loss function (λ_k) is selected to minimize the expected discounted value of the social loss function. For instance, the delegated loss function under NIGT is given by

$$\min_{\{i\}_0^\infty}\max_{\{v\}_1^\infty}\ E_0\sum_{t=0}^{\infty}\beta^t[\pi_t^2 + \lambda_{NIGT}(\pi_t + x_t - x_{t-1})^2 - \theta v_{t+1}' v_{t+1}], \tag{21}$$

subject to structural equations, such as the NKPC. The stabilization parameter λ_{NIGT} is optimally chosen to minimize social loss under the NIGT regime. The lagged output gap derived from nominal income growth characterizes the delegated loss function; thus, the targeting rule can impart policy inertia into the economy despite solving optimal policy under discretion. The other targeting regimes are specified in the same way. It follows from Table 2 that except for IT, the targeting regimes are characterized by policy inertia associated with the lagged output gap.

4 Quantitative results

In this section, we present this study's main findings. We examine the performance of the targeting regime numerically because we cannot derive the conditions under which the delegated optimal monetary policy inertia overcomes the stabilization bias problem analytically. Section 4.1 provides a brief explanation of the calibrated values, and Section 4.2 details the welfare losses under alternative targeting regimes. To explain the intuition of the result, we provide the impulse response function to a cost-push shock in Section 4.3. Lastly, Section 4.4 discusses the economic implications of the results presented in Sections 4.2 and 4.3.

4.1 Calibration

The calibrated values in this paper are based on those used in previous studies. The discount factor β is set to 0.99. Meanwhile, the relative risk aversion coefficient and the inverse of labor supply elasticity are set to 2.0 and 5.0, respectively. We set the degree of decreasing return to scale in the production function (χ) to 0.25. The elasticity of substitution for individual goods is set to 9.0, whereas the Calvo lottery (α) is set to 0.75. We set the elasticity of substitution between domestic and foreign goods (η) to 1.0. Consequently, we assume the case of $\sigma\eta > 1$. The degree of openness (ν) is set to 0.4. Finally, the weight on output gap stabilization in the true loss function ($\tilde{\lambda}$) is set to 0.25. Table 1 summarizes these values.

We now tackle choosing the parameter θ representing the degree of robustness. A large value of θ implies no robustness, whereas a small value of θ indicates that the central bank is more concerned about robustness. This study assumes that the value of θ ranges from 80 to 1000. Our calibration strategy is based on Tillmann (2009a). As mentioned previously, model uncertainty disappears as the value of θ becomes larger. Following Giordani and Söderlind (2004) and Tillmann (2009a), we consider the values of θ in which the detection error probability ranges from 0 to 0.5. The values of θ that we selected satisfy the suggested range of detection error probability.[22] We emphasize that our calibration strategy allows us to explore how the optimal delegation policy overcomes the stabilization bias problem for several candidate values of θ in this paper.

4.2 Welfare losses under targeting regimes

Table 3 depicts the welfare losses in alternative targeting regimes when the cost-push shocks are not serially correlated. The welfare loss in each targeting regime is expressed as a relative loss to that in the commitment. In the targeting regimes shown in Table 2, welfare losses are calculated using a delegated loss function with optimally chosen policy weights.

[Table 3 around here]

We first consider the result of Table 3(a) that reports the welfare losses under the worst-case equilibrium. Clearly, IT leads to the worst welfare loss of all policy regimes. Also, as Jensen (2002) and Walsh (2003) showed, the policy regimes with policy inertia produce outcomes that are preferable over IT. Importantly, regardless of the value of θ, welfare losses under NIGT are smaller than those under speed limit targeting. In particular, the performance of NIGT is very close to that of commitment policy, but it does not outperform commitment policy. Hasui (2021) showed the effectiveness of SLP in a closed economy. Meanwhile, our findings suggest the efficacy of NIGT in a small open economy. We also discovered that the real exchange rate target performs very similarly to the speed limit target. The performance of real exchange rate targeting, in particular, produces preferable results to that of SLP. This is not surprising given that the real exchange rate varies in proportion to the output gap, as previously stated.

Table 3(b) reports the welfare losses of alternative targeting regimes under the approximating equilibrium. As discussed in Giordani and Söderlind (2004), in the absence of distortions from evil agents, the central bank implements robust policy under the approximating equilibrium. As shown in Table 3(a), in the worst-case scenario, reducing the value of θ increases welfare loss. However, welfare losses under the remaining regimes decrease. Meanwhile, under the approximating equilibrium, all regimes' welfare losses increase as the value of θ decreases. Notably, under the approximating equilibrium, NIGT performance appears to be unaffected by any values of θ.

[22] The values of θ less than 80 cannot lead to stable properties of equilibrium dynamics. Therefore, we consider the range 80–1000 for the value of θ.

We ruled out the presence of serial correlation of a cost-push shock in the previous analysis. Accordingly, we did not consider the impact of cost-push shock persistence on robust policy. We examine this by introducing a serial correlation and reporting the performance of targeting regimes in Table 4. We assume $\rho_u = 0.5$ in this simulation.[23] Table 4(a) shows the welfare losses under the worst-case equilibrium. As with no serial correlation, the welfare losses in IT are the largest of all policy regimes. Additionally, the policy regimes containing the lagged output gap lead to preferable outcomes to IT.

[Table 4 around here]

Surprisingly, NIGT performance dominates the commitment policy performance when $\theta \leq 200$. In other words, NIGT becomes an effective tool for overcoming stabilization bias when the problem of robust control matters. To the best of our knowledge, no study has addressed this finding in a small open NK model. As shown by Hasui (2021), in a closed economy model, the performance of NIGT is comparable with that of speed limit targeting.[24] When we consider the approximating equilibrium, the performance of all targeting regimes under discretion never outperforms that under commitment, as shown in Table 4(b). Regardless, NIGT becomes the most effective of all targeting regimes under discretion.

Summing up, the performance of targeting regimes with the lagged output gap leads to preferable outcomes to IT when the central bank implements a discretionary policy. Thus, as suggested by Ida and Okano (2021), the delegated targeting regimes can become an effective tool for overcoming stabilization bias in a small open economy with robust control. In particular, NIGT outperforms the commitment policy under the worst-case equilibrium in the presence of a serial correlation of a cost-push shock.

4.3 Impulse response analysis

We demonstrated that the performance of NIGT produces preferable outcomes to alternative targeting regimes when robustness is crucial in a small open NK model. In some cases, an NIGT policy outperforms the commitment policy. Why? To aid in understanding this result,

[23] We confirm that the equilibrium dynamics are unstable when $\rho_u > 0.5$.

[24] He demonstrated that when endogenous inflation persistence exists in a closed economy, NIGT performance is nearly identical to that of speed limit targeting.

we provide the impulse response function of inflation, the output gap, and the exchange rate. The parameter values of the stabilizing weights of the production gap are fixed in these impulse responses, and serial correlation of the cost-push shocks is permitted.

Figures 1 and 2 show the impulse responses of the inflation rate and the output gap to a cost-push shock under the worst-case equilibrium, respectively. As shown by Woodford (2003), a cost-push shock generates a policy trade-off between inflation and the output gap. As shown in Figures 1 and 2, among the regimes, a pure discretionary policy has the worst trade-off because it cannot manipulate the private sector's expectations. As opposed to a pure discretionary policy, a precommitment policy introduces policy inertia into the economy. Thus, when compared with pure discretion, through a precommitment policy, the central bank can ease the trade-off between inflation and the output gap by managing private sector expectations. Consequently, a discretionary policy is associated with a stabilization bias.

[Figure 1 around here]

[Figure 2 around here]

Figures 1 and 2 show that a targeting regime with a lag in the output gap creates policy inertia in the economy, even when the central bank conducts discretionary policy. Although the response of the output gap under an NIGT is nearly identical to that under a speed limit target, a central bank with a policy goal of stabilizing nominal income growth is likely to steadily introduce more inertial behavior into the inflation rate than that under speed limit targeting.

However, why does NIGT outperform speed limit targeting in an open economy? Figure 3 depicts the nominal exchange rate's impulse response to a cost-push shock to explain the reason. As shown by Monacelli (2003), after a cost-push shock, the response of the nominal exchange rate returns to a steady state under commitment, but it is nonstationary under discretion. This is because the price level under commitment retains stationarity, whereas that under discretion has a unit root.

[Figure 3 around here]

Interestingly, this study determines that the nominal exchange rate response has unit root properties under the speed limit target, but it returns to stationarity after a cost-push shock under the nominal income growth rate target. Remember the nominal exchange rate law of motion to grasp this mechanism. Under a speed-limiting policy, the central bank can only indirectly impart policy inertia to the nominal exchange rate through ToT. However, this inertia is insufficient to ensure that the SLP maintains exchange rate stationarity. Meanwhile, under a nominal income growth target, the central bank can introduce policy inertia not only through ToT but also through inflation. Therefore, NIGT demonstrates that the degree of policy inertia on the exchange rate is greater than that of speed-limited policy. Thus, the nominal exchange rate's stationarity reflects the difference in policy inertia between the two regimes. This finding supports the effectiveness of NIGT in small open economies by demonstrating its potential to create exchange rate stationarity.

4.4 Discussion

This study obtains the following findings. Unlike speed limit targeting, optimal discretionary policy under nominal income growth can produce performance very close to precommitment by appropriately manipulating the nominal exchange rate's policy inertia. When model uncertainty matters in an open economy, such a policy regime becomes a very effective tool. This finding appears to contradict previous research. For instance, Ida and Okano (2021) demonstrated that NIGT that consists of CPI inflation produces worse outcomes compared with SLP. They contended that when the central bank can manipulate the ToT externality under the NIGT based on CPI inflation, it has an incentive to break an optimal delegation scheme. This result appears to support the case for robust control. Intuitively, in the case of a robust control problem, the policymaker and the evil agent have an incentive to exploit the ToT externalities.

Meanwhile, Ida and Okano (2017) suggested that NIGT based on PPI inflation could lead to a commitment solution in a small open NK model. We can avoid their criticism because our specification constructs nominal income growth in terms of PPI-based inflation.[25] This is because the performance of PPI-based NIGT can be very close to that of a commitment policy in the sense that the central bank that follows this regime no longer has an incentive to manipulate ToT externality. We highlight the effectiveness of PPI-based NIGT in a small

[25] Although we attempt to calculate the performance of CPI-based NIGT, we find that equilibrium dynamics are unstable. Therefore, we cannot report the results of CPI-based NIGT. To put it another way, as claimed by Ida and Okano (2021), CPI-based NIGT is a less effective tool in a small open NK model.

open NK model with robust control in this paper.

5 Extension: Endogenous inflation persistence

The NKPC is depicted by only the forward-looking component in the previous section. However, several studies have argued the importance of the backward-looking inflation in the NKPC (Amato and Labach, 2003; Galı and Gertler, 1999; Steinsson, 2003). For instance, Amato and Labach (2003) and Steinsson (2003) showed the decrease in welfare gain from a commitment policy as the NKPC becomes more backward-looking. Also, Walsh (2003) showed that the performance of delegated targeting regimes is crucially affected by the presence of endogenous inflation persistence. Furthermore, Hasui (2021) showed that the degree of backward-looking inflation in the NKPC affects the performance of targeting regimes under robust control. According to these studies, the delegated optimal policies with lagged output gap create preferable outcomes to the IT in the presence of moderate inflation persistence.

Therefore, this section examines whether the performance of NIGT remains effective in the presence of endogenous inflation persistence in a small open NK economy with robust control. More concretely, following Amato and Labach (2003), we incorporate endogenous inflation persistence into the small open NK model. We also determine that a fraction $1-\alpha$ of all firms adjusts their price, whereas the remaining fraction of firms α do not, following Calvo (1983). In addition to this specification, among firms that can adjust price, a fraction ω sets price based on a rule-of-thumb, whereas a fraction $1-\omega$ sets price optimally. These assumptions allow us to incorporate the lagged inflation rate in the NKPC. More concretely, under these assumptions, the hybrid NKPC is derived:

$$\pi_t = \gamma_1 E_t \pi_{t+1} + \gamma_2 \pi_{t-1} + \tilde{\kappa}_\nu x_t + u_t, \tag{22}$$

where

$$\gamma_1 = \frac{\alpha\beta}{\alpha + \omega(1-\alpha(1-\beta))}, \; \gamma_2 = \frac{\omega}{\alpha + \omega(1-\alpha(1-\beta))},$$
$$\tilde{\kappa}_\nu = \frac{\omega(1-\alpha)(1-\alpha\beta)}{\alpha + \omega(1-\alpha(1-\beta))}\left(\sigma_\nu + \frac{1+\varphi}{1-\chi}\right).$$

When ω approach zero, Equation (22) corresponds to the purely forward-looking NKPC.[26]

[26] See Amato and Labach (2003) and Galı and Gertler (1999) for a detailed derivation of the hybrid NKPC.

Table 5 shows the performance of robustly controlled targeting regimes in the presence of endogenous inflation persistence. Following previous research, we set the calibrated value of ω to 0.5.[27] We have a few thoughts on this outcome. First, when the degree of robustness is reduced in the case of endogenous inflation persistence, NIGT outperforms commitment policy. However, under the approximating equilibrium, among all policy regimes, a commitment policy can lead to the most preferable outcomes. Second, the performance of speed limit targeting is never superior to that of NIGT. Third, real exchange rate targeting clearly outperforms speed limit targeting in terms of performance. Finally, when compared with a model without endogenous inflation persistence, the performance of IT improves in both the worst and approximate cases.

[Table 5 around here]

In summary, NIGT performance can be a very effective tool for overcoming the problem of stabilization bias in small open economy models with robust controls. As a result, despite the introduction of endogenous inflation persistence, the performance of NIGT retains its effectiveness.

6 Concluding remarks

Globalization through international trade and finance has expanded rapidly in recent years, implying the increasing importance for central banks to manage monetary policy in light of the various model uncertainties associated with globalization. When model uncertainty matters in an open economy, how should the central bank implement monetary policy? In particular, in an open economy with model uncertainty, how a central bank that cannot implement commitment policies can improve social welfare is an open question.

This paper investigated the delegation of targeting regimes under robust control in a small open NK model to answer this question. Delegating nominal income growth to the central bank outperforms alternative policy regimes we consider in worst-case equilibrium, as long as the central bank is concerned about the degree of robustness. We address the fact that, in worst-case equilibrium, NIGT outperforms SLP. Even in the approximating-equilibrium case,

[27] We found that when the parameter ω is less than 0.4, the equilibrium dynamics are unstable. In addition to this parameterization of ω, we confirmed that equilibrium dynamics are unstable when ρ_u is greater than 0.3.

delegating monetary policy inertia can produce preferable results than IT. Moreover, introducing endogenous inflation persistence has no effect on these results. Our findings emphasize the significance of considering the optimal delegation problem in a small open NK model where the stabilization bias associated with discretion matters under model uncertainty.

Finally, we would like to mention future works that we were not able to cover in this paper. We chose a simple small open NK model to make the role of the optimal delegation problem under model uncertainty as simple and intuitive as possible for understanding. As a result, investigating the optimal delegation problem under robust control in a medium-scale open economy model may be worthwhile.

Appendix : Detailed derivation in Section 3.2

Following Walsh (2004), we briefly provide a detailed derivation of Equation (16). This appendix examines how to derive optimal conditions under a commitment policy. To solve the optimization problem for both the central bank and evil agent under a commitment policy, we define the following Lagrangian

$$\min_{\{x,\pi\}_0^\infty} \max_{\{w\}_1^\infty} E_0 \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \tilde{\lambda} x_t^2 - \beta\theta w_{t+1}^2 - \phi_{1t}(\pi_t - \beta\pi_{t+1} - \kappa_\nu x_t - u_t)$$
$$- \phi_{2t}(\rho_u u_t + w_{t+1} + \varepsilon_{t+1} - u_{t+1})], \tag{A.1}$$

where ϕ_{1t} and ϕ_{2t} denote the Lagrange multipliers for the NKPC and the cost-push shock, respectively. The first-order conditions are given as follows:

$$\pi_t + \phi_{1t} - \phi_{1t-1} = 0, \tag{A.2}$$
$$\tilde{\lambda} x_t - \kappa_\nu \phi_{1t} = 0, \tag{A.3}$$
$$- \phi_{1t} + \rho_u \phi_{2t} - \frac{1}{\beta}\phi_{2t-1} = 0, \tag{A.4}$$
$$- \theta w_{t+1} + \phi_{2t} = 0. \tag{A.5}$$

From Equations (A.2) and (A.3), we obtain equation (14), leading to the standard optimal targeting rule proposed by Woodford (2003). Next, substituting Equations (A.3) and (A.5) into (A.4) leads to

$$w_t = -\frac{\beta\tilde{\lambda}}{\theta\kappa_\nu} x_t + \rho_u \beta w_{t+1}.$$

Finally, iterating this equation forwardly leads to Equation (16).

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Table 1: Parameterization

	Parameter	Value
β	Discount rate	0.99
σ	Relative risk aversion coefficient for consumption	2.0
φ	Inverse of the elasticity of labor supply	5.0
χ	Degree of decreasing return on labor	0.25
ϵ	Elasticity of substitution for individual intermediate goods	9.0
α	Calvo lottery	0.75
η	Elasticity of substitution between domestic and foreign goods	1.0
ν	Degree of openness	0.4
$\tilde{\lambda}$	Weight on the output gap in the true loss function	0.25

Table 2: Alternative policy regimes

Regime	Loss function
Inflation targeting (IT)	$\pi_t^2 + \lambda_{IT} x_t^2$
Speed limit policy (SLP)	$\pi_t^2 + \lambda_{SLP}(x_t - x_{t-1})^2$
Nominal income growth targeting (NIGT)	$\pi_t^2 + \lambda_{NIGT}(\pi_t + x_t - x_{t-1})^2$
Real exchange rate targeting (REX)	$\pi_t^2 + \lambda_{REX}(q_t - q_{t-1})^2$

Table 3: Welfare losses under alternative delegated policy regimes: $\rho_u = 0$

(a) worst-case equilibrium

Worst case	IT	SLP	NIGT	REX
$\theta = 1000$	130.44	105.00	100.88	104.99
$\theta = 750$	130.45	105.00	100.87	104.99
$\theta = 500$	130.46	104.99	100.87	104.98
$\theta = 400$	130.47	104.98	100.86	104.97
$\theta = 300$	130.48	104.97	100.85	104.96
$\theta = 200$	130.51	104.95	100.83	104.94
$\theta = 100$	130.59	104.89	100.76	104.89
$\theta = 80$	130.63	104.86	100.72	104.86

(b) approximating equilibrium

	IT	SLP	NIGT	REX
$\theta = 1000$	130.46	105.02	100.89	105.01
$\theta = 750$	130.47	105.02	100.89	105.01
$\theta = 500$	130.49	105.02	100.89	105.01
$\theta = 400$	130.51	105.02	100.89	105.01
$\theta = 300$	130.54	105.03	100.89	105.01
$\theta = 200$	130.59	105.03	100.89	105.02
$\theta = 100$	130.76	105.04	100.90	105.03
$\theta = 80$	130.84	105.04	100.90	105.04

(Note) IT, inflation targeting; SLP, speed limit policy; NIGT, nominal income growth targeting; REX, real exchange rate targeting. The welfare loss in each targeting regime is expressed as a relative loss to that in the commitment regime. Each loss is multiplied by 100.

Table 4: Welfare losses under alternative delegated policy regimes: $\rho_u = 0.5$

(a) worst-case equilibrium

Worst case	IT	SLP	NIGT	REX
$\theta = 1000$	135.88	103.96	100.42	103.96
$\theta = 750$	135.86	103.92	100.37	103.91
$\theta = 500$	135.88	103.83	100.28	103.83
$\theta = 400$	135.78	103.76	100.21	103.76
$\theta = 300$	135.72	103.65	100.09	103.65
$\theta = 200$	135.58	103.44	99.86	103.43
$\theta = 100$	135.17	102.78	99.16	102.77
$\theta = 80$	134.96	102.44	98.80	102.44

(b) approximating equilibrium

	IT	SLP	NIGT	REX
$\theta = 1000$	136.00	104.10	100.56	104.10
$\theta = 750$	136.02	104.10	100.56	104.10
$\theta = 500$	136.06	104.11	100.56	104.11
$\theta = 400$	136.09	104.11	100.57	104.11
$\theta = 300$	136.13	104.12	100.57	104.12
$\theta = 200$	136.22	103.14	100.58	104.14
$\theta = 100$	136.44	104.18	100.59	104.18
$\theta = 80$	136.55	104.20	100.60	104.20

(Note) IT, inflation targeting; SLP, speed limit policy; NIGT, nominal income growth targeting; REX, real exchange rate targeting. The welfare loss in each targeting regime is expressed as a relative loss to that in the commitment regime. Each loss is multiplied by 100.

Table 5: Welfare losses under alternative delegated policy regimes under inflation persistence: $\omega = 0.5$ and $\rho_u = 0.3$

(a) worst-case equilibrium

	IT	SLP	NIGT	REX
$\theta = 500$	112.94	100.84	100.11	100.71
$\theta = 400$	112.93	100.81	100.08	100.67
$\theta = 300$	112.91	100.77	100.04	100.65
$\theta = 200$	112.87	100.68	99.95	100.56
$\theta = 100$	112.76	100.41	99.68	100.30
$\theta = 80$	112.70	100.27	99.54	100.17

(b) approximating equilibrium

	IT	SLP	NIGT	REX
$\theta = 500$	113.02	100.95	100.22	100.82
$\theta = 400$	113.03	100.95	100.22	100.82
$\theta = 300$	113.04	100.95	100.22	100.82
$\theta = 200$	113.06	100.95	100.22	100.82
$\theta = 100$	113.11	100.95	100.21	100.82
$\theta = 80$	113.14	100.94	100.20	100.82

(Note) IT, inflation targeting; SLP, speed limit policy; NIGT, nominal income growth targeting; REX, real exchange rate targeting. The welfare loss in each targeting regime is expressed as relative to that in the commitment regime. Each loss is multiplied by 100.

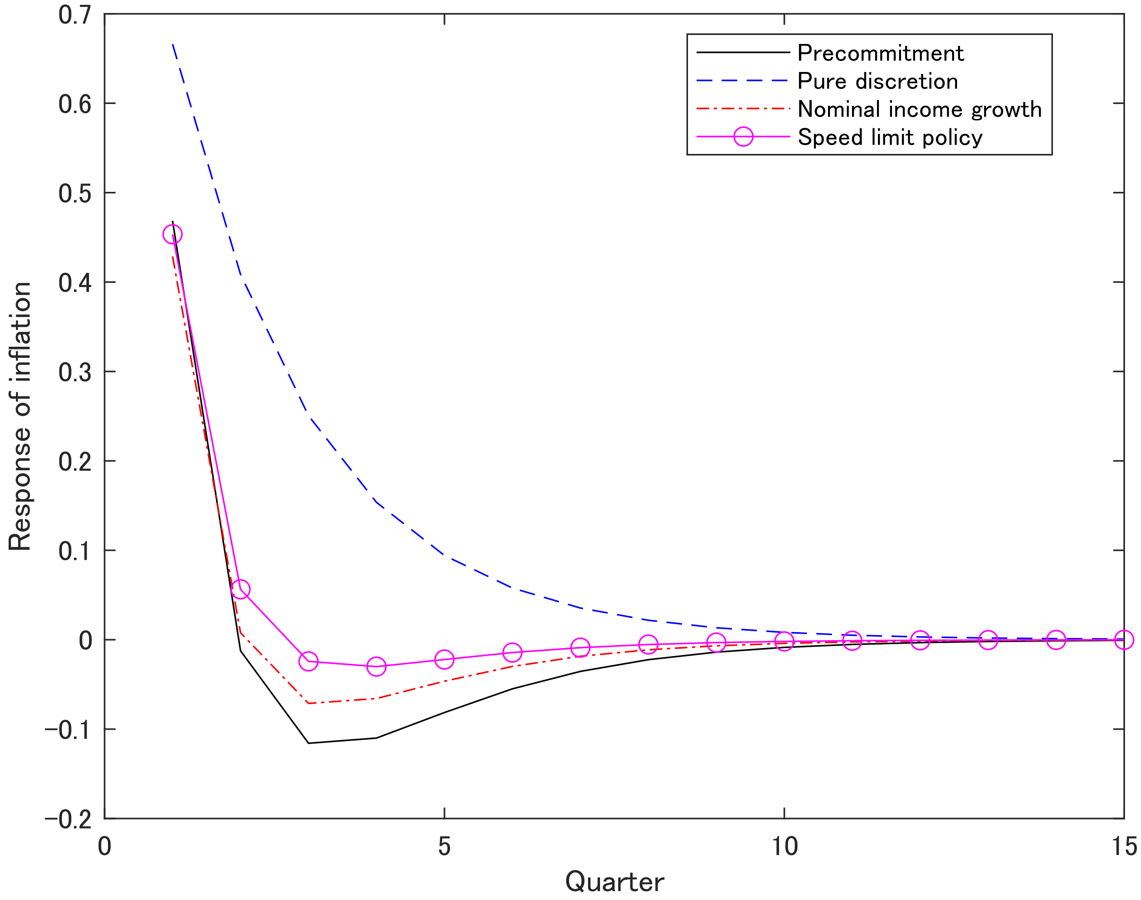

Figure 1: Impulse response of inflation to a cost-push shock under worst-case equilibrium

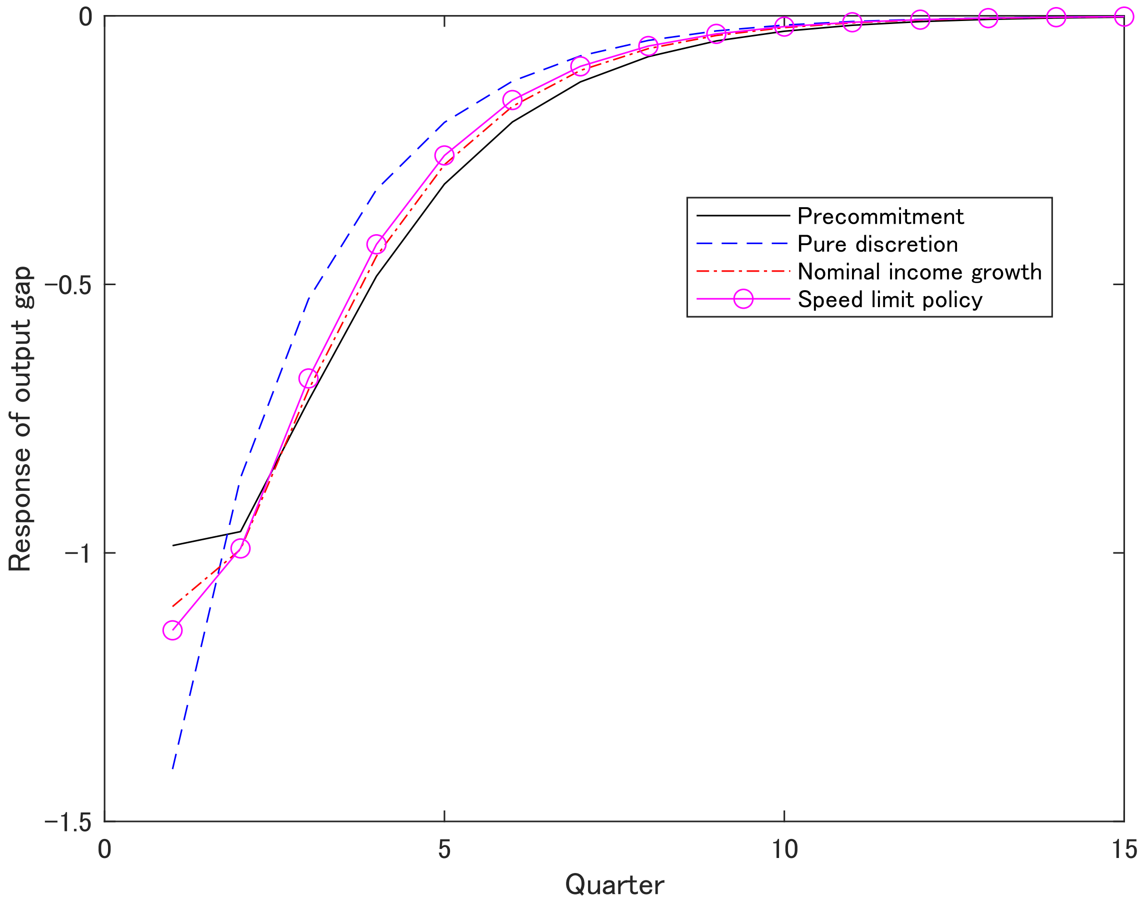

Figure 2: Impulse response of output gap to a cost-push shock under worst-case equilibrium

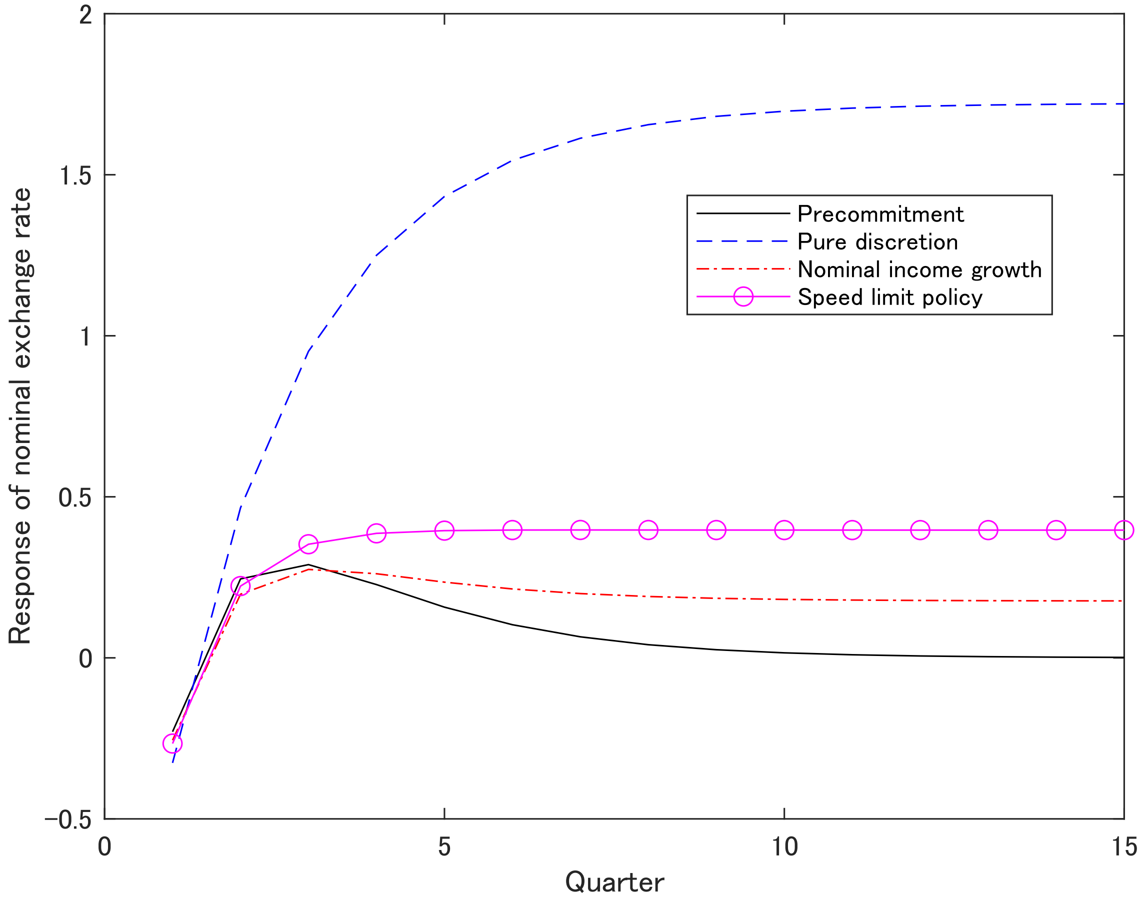

Figure 3: Impulse response of nominal exchange rate to a cost-push shock under worst-case equilibrium