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Progressive taxation and optimal monetary policy in a two-country New Keynesian model*

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Abstract

This paper examines the effect of progressive taxation on optimal monetary policy in a two-country New Keynesian (NK) model. This paper's main results are summarized as follows. First, although both the structural equation and the shape of the central bank's loss function are the same as the standard two-country NK model, we address that the deeper parameters indicated by the structural equation and the loss function are crucially affected by the progressive taxation parameters in both countries. Second, when home and foreign central banks cooperatively implement optimal monetary policy under discretion, an increase in the progressive tax rate generally worsens worldwide social welfare, regardless of which country raises the progressive tax rates. These results are robust to any changes in structural parameters.

JEL codes: E52; E58; F41;

Keywords : Progressive taxation; Optimal monetary policy; two-country New Keynesian model;

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1 Introduction

This paper aims to examine the role of progressive taxation in a two-country New Keynesian (NK) model. The recent rapid progress of globalization and digital transformation (DX) has challenged international taxation systems. For example, standard international macroeconomics indicates that if financial globalization progresses rapidly, output growth of one country may be strongly affected by changes in the output of other countries through international asset markets and trade (Obstfeld and Rogoff, 1996). Therefore, it is natural to conjecture that changes in corporate or income taxes could be similarly affected.

This paper focuses on how progressive international taxation affects the properties of optimal monetary policy in a two-country NK model. If complete asset markets exist internationally, then theoretically, consumption differences among countries should be equalized through international consumption risk sharing (Chari *et al.*, 2002). This may suggest that, if we consider the international aspect of taxation under a complete international asset market, progressive tax rates that each country's government separately sets may be equalized across countries. This mechanism appears to be observed in advanced countries (Facundo *et al.*, 2017). Therefore, national tax systems will play an important role when exploring international macroeconomic stabilization in the near future. We must ask how policymakers should consider the interaction of monetary and fiscal policies in a two-country economy.

However, the standard two-country NK model lacks this global perspective of progressive taxation systems, and therefore may induce the misleading policy prescription for international macroeconomic policy. While Mattesini and Rossi (2012) focused on the role of modern tax systems in a closed economy,[1] they did not consider the role of international taxation systems. In contrast to Mattesini and Rossi (2012), this paper focuses on the effect of progressive taxation on optimal monetary policy in a two-country NK model. To do this, we incorporate progressive taxation into the two-country NK model developed by Clarida *et al.* (2002), based on the framework considered in Mattesini and Rossi (2012). We consider how changes in a progressive tax affect international spillover effects of structural shocks under an optimal cooperative policy. To the best of our knowledge, the extant literature has not examined the role of progressive

[1] See Chen and Guo (2014) and Wenli and Sarte (2004) for the discussion about the role of progressive taxation in a neo-classical growth model. While Galí *et al.* (2007) examined the role of government spending in the NK model, their model does not consider the role of progressive taxation.

taxation from the aspect of the global economy.

The paper's main findings are summarized as follows. First, both the structural equation and the shape of the central bank's loss function are the same as Clarida *et al.* (2002). However, we point out that, in contrast to Clarida *et al.* (2002), the deep parameters indicated by the structural equation and the loss function are now crucially affected by the progressive taxation parameters in both countries. Second, we demonstrate that when home and foreign central banks jointly minimize the worldwide welfare loss function under discretionary policies, an increase in the progressive tax rate generally worsens global social welfare, regardless of which country raises the progressive tax rates. However, in the absence of cost-push shocks in the home country, it is possible that given the tax rates in the foreign country, an increase in the home tax rate can improvement the global welfare. Third, we show that the value of the constant relative risk aversion coefficient (CRRA) in the household utility function is highly relevant to capturing the impact of the degree of progressive taxation on global welfare losses.

This study contributes to the extant literature; more concretely, our study is related to Mattesini and Rossi (2012) who explored the effect of progressive taxation on optimal monetary policy in a closed-economy NK model by employing the approach of Guo (1999) and Guo *et al.* (1998). To address our motivation of considering the role of the international progressive taxation system, we incorporate the approach of Guo (1999) in the progressive tax in a standard two-country NK model. While Mattesini and Rossi (2012) showed that the existence of a progressive tax rate significantly impacts inflation dynamics through the effect of tax rate changes on the marginal cost of firms. We emphasize the importance of the interdependence of home and foreign progressive tax rates in the presence of international complete asset markets in a two-country model, even if each country's government determines its progressive tax rate separately. In particular, our model contributes to policy implications for the role of the international taxation system.

Our study is also related to Clarida *et al.* (2002) who developed a two-country NK model and showed the potential gain from policy coordination.[2] We incorporate the role of progressive

[2]This paper we does not focus on the gain from policy coordination because we attempt to analytically examine the effect of progressive international taxation on international monetary policy. If we use the algorithm developed by Bodenstein *et al.* (2019), we can investigate how a change in the progressive tax rates in one country affects the gain from policy coordination. However, this task is beyond this paper's scope, and we would like to examine this topic in future work.

taxation into a two-country NK model developed by Clarida *et al.* (2002). While we abstract from the international dimension of the macroeconomic policy game between home and foreign central banks, we address the potential role of international taxation under international policy coordination.[3] Additionally, this study is related to Beetsma and Jensen (2005), Cook and Devereux (2011), and Gali and Monacelli (2008). Although Beetsma and Jensen (2005) and Gali and Monacelli (2008) focused on the role of fiscal policy in a monetary union, these studies did not investigate the effect of progressive tax rates on international macroeconomic dynamics. Cook and Devereux (2011) examined the role of monetary and fiscal policies in a two-country NK model under a liquidity trap. However, they also abstract from the role of progressive international taxation in their model.

Our paper is related to Strehl and Engler (2015). They examined the effect of progressive taxation on optimal monetary policy solving the Ramsey problem in monetary union, while we consider its effect on optimal discretionary policy under policy coordination in a two-country NK model with full exchange rate pass-through. In addition, they did not explicitly derive the central bank's loss function, but we do so by computing a second-order approximation of the household utility function. Furthermore, in contrast to Strehl and Engler (2015), we can consider how changes in home and foreign progressive tax rates affect global welfare losses under international monetary policy cooperation.

Finally, our study may be closely related to Chen *et al.* (2021). They considered the role of the international consumption tax system under international monetary policy coordination in a two-country NK model with local currency pricing. Although our model is based on producer currency pricing, to the best of our knowledge, this is the first study investigating the role of the international progressive taxation system in a two-country NK model. While Auray *et al.* (2017) examined the role of competitive tax reform in monetary union, they did not explore the role of progressive taxation systems.

This paper is constructed as follows. Section 2 explains a two-country NK model with progressive taxation, and Section 3 provides the role of optimal monetary policy in an economy with progressive taxation. More concretely, we analytically examine the role of progressive

[3] Our framework is built on a standard two-country model under producer currency prices (PCP), which implies that purchasing power parity holds. Engel (2011) considered the international monetary policy coordination in a two-country NK model under local currency pricing.

tax in an optimal cooperative policy under discretion. Section 4 provides the quantitative results. Following the main results obtained in the previous section, Section 5 provides the policy implication for the international progressive taxation system under optimal monetary policy coordination. Section 6 briefly concludes. Appendix A provides the detailed derivation of the central bank's loss function by calculating the second-order approximation of home and foreign households' utility functions.

2 Model

We examine the role of progressive taxation in a two-country NK model. More concretely, we add progressive tax, as argued in Mattesini and Rossi (2012) into the two-country NK model developed by Clarida *et al.* (2002). Consider an economy with two large symmetric countries: home and foreign. The size of the economy for home and foreign is $1-\gamma$ and γ, respectively. The following explanation focuses on the home country (H), whereas we can consider the case for foreign country (F).

There are two production sectors in each country: a final goods sector, characterized by perfect competition, and an intermediate goods sector, wherein firms face monopolistic competition and Calvo (1983)-type nominal price rigidity. We allow the the degree of price stickiness to differ in each country and assume that both countries have complete markets. We also postulate that only final goods are traded and that the number of final goods producers is equal to the number of households in each country. As noted earlier, in this paper purchasing power parity holds because we consider the PCP model according to Clarida *et al.* (2002).

Finally, unless otherwise noted, analogous equations hold for the foreign country. Note that an asterisk denotes the variables for the foreign country.

As noted earlier, except for the introduction of progressive taxation, this paper follows a standard two-country NK model developed by Clarida *et al.* (2002). Therefore, readers familiar with the two-country NK model can skip to Section 2.4, which provides the log-linearized model.

2.1 Households

The households solve their utility maximization problem through two stages in each country. In the first stage, they solve the intratemporal cost-minimization problem. In the second stage, the households solve the intertemporal utility maximization problem.based on the optimal conditions obtained in the first stage.

2.1.1 Preferences

First, consider the intratemporal cost minimization problem for the consumption basket. Following Clarida *et al.* (2002), preferences for consumption C_t in the home country are given by

$$C_t \equiv C_{H,t}^{1-\gamma} C_{F,t}^{\gamma}, \tag{1}$$

where $C_{H,t}$ is the consumption of domestic goods and $C_{F,t}$ is the consumption of foreign goods. From the cost-minimization problem of home households, we obtain the following equations:

$$C_{H,t} = (1-\gamma)\left(\frac{P_{H,t}}{P_t}\right)^{-1} C_t, \tag{2}$$

$$C_{F,t} = \gamma\left(\frac{P_{F,t}}{P_t}\right)^{-1} C_t. \tag{3}$$

Here the price index in the home country is given by:

$$P_t = k^{-1} P_{H,t}^{1-\gamma} P_{F,t}^{\gamma} = k^{-1} P_{H,t} S_t^{\gamma}, \tag{4}$$

where $k \equiv (1-\gamma)^{(1-\gamma)}\gamma^{\gamma}$, $P_{H,t}$ is the price of domestic goods, and $P_{F,t}$ is the price of foreign goods. Futhermore, S_t represents the terms of trade, given by

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}}. \tag{5}$$

2.1.2 Household optimization problem

In the second stage, households h in each country maximize their utility by solving the intertemporal utility maximization problem. More concretely, they consider the intertemporal utility maximization problem. Here, an infinitely lived representative households intertemporal utility is as follows:

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t(h)) &= E_0 \sum_{t=0}^{\infty} \beta^t \Big[u(C_t) - v(N_t) \Big] \\ &= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t(h)^{1+\eta}}{1+\eta} \right\}, \end{aligned} \tag{6}$$

where $N_t(h)$ denotes the household's h labor hour. The parameters σ and η denote the CRRA parameter and the inverse of the elasticity of the labor supply, respectively.

The representative household maximizes the above utility function subject to the following budget constraint:

$$P_t C_t + E_t[Q_{t,t+1} B_{t+1}] = B_t + (1 - \tau_t) W_t(h) N_t(h) + \Gamma_t, \tag{7}$$

where B_t represents nominal bonds held for one period and Γ_t are dividends earned from a domestic intermediate goods firm. $W_t(h)$ is the corresponding nominal wage. $Q_{t,t+1}$ denotes a stochastic discount factor, and τ_t denotes the taxes on labor income. Following Mattesini and Rossi (2012), we assume that labor income tax τ_t is given as follows:

$$\tau_t = 1 - \delta \left(\frac{Y_n}{Y_{n,t}} \right)^{\phi_n}, \tag{8}$$

where $Y_{n,t} = W_t N_t / P_t$ denotes the household's taxable income and Y_n denotes its steady-state level. $\delta \in (0, 1]$ governs the level of the tax schedule and $\phi_n \in [0, 1)$ denotes the slope of the tax schedule.[4] We assume that this labor income tax is common to each household type h. When $\phi_n > 0$ the tax rate increases as the household's taxable income increases. Also, the marginal tax rate τ_t^m is characterized by the following property:

$$\tau_t^m = \frac{\partial(\tau_t Y_{n,t})}{\partial Y_{n,t}} = 1 - \delta(1 - \phi_n) \left(\frac{Y_n}{Y_{n,t}} \right)^{\phi_n}.$$

As postulated in Mattesini and Rossi (2012), we consider the case where the marginal tax rates are larger than the average tax rate because we assume $\phi_n > 0$. In this case, the tax schedule becomes progressive.[5]

We assume that a complete market is present in both countries, and introduce the following stochastic discount factor:

$$E_t(Q_{t,t+1}) = \frac{1}{1 + r_t}, \tag{9}$$

where r_t is the risk free short-term nominal interest rate. Solving the intertemporal utility maximization problem leads to the following consumption Euler equation:

$$E_t[Q_{t,t+1}] = \frac{1}{1 + r_t} = \beta E_t \left[\frac{u_c(C_{t+1})}{u_c(C_t)} \frac{P_t}{P_{t+1}} \right]. \tag{10}$$

[4]See Chen and Guo (2014), Guo *et al.* (1998), and Wenli and Sarte (2004) for a detailed discussion about specifying the progressive tax rule.

[5]This paper assumes $\phi_n > 0$. Thus, we focus on only the role of progressive taxation system. Notice that we can consider the role of regressive taxation if we assume $\phi_n < 0$.

Regarding the demand for labor following Clarida *et al.* (2002), the household is a supplier of labor under a monopolistically competitive environment. In this case, solving the firm's cost minimization problem leads to the following labor demand:

$$N_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\xi_{w,t}} N_t,$$

where

$$N_t = \left[\frac{1}{1-\gamma}\int_0^{1-\gamma} N_t(h)^{\frac{\xi_{w,t}-1}{\xi_{w,t}}} dh\right]^{\frac{\xi_{w,t}}{\xi_{w,t}-1}}. \tag{11}$$

Substituting this labor demand function into (11), we obtain the following aggregate wage index:

$$W_t = \left[\frac{1}{1-\gamma}\int_0^{1-\gamma} W_t(h)^{1-\xi_{w,t}} dh\right]^{\frac{1}{1-\xi_{w,t}}}. \tag{12}$$

Accordingly, the first order condition concerning labor supply in the household's utility maximization problem is given by

$$\frac{W_t(h)}{P_t} = \mu_t^w \frac{v_n(N_t(h))}{u_c(C_t)}, \tag{13}$$

where μ_t^w denotes the time-varying wage markup associated with monopolistic competition in the labor market. Substituting Equation (8) into Equation (13) leads to

$$\delta\left(\frac{Y_n}{Y_{n,t}}\right)^{\phi_n} \frac{W_t(h)}{P_t} = \mu_t^w \frac{v_n(N_t(h))}{u_c(C_t)}. \tag{14}$$

In symmetric equilibrium, the following relationship holds

$$W_t = W_t(h); \; N_t = N_t(h).$$

2.1.3 International risk-sharing

Next, we consider a risk-sharing condition between countries. The Euler equation for foreign consumption denominated in domestic currency is

$$\frac{1}{1+r_t^*} = \beta E_t\left[\frac{u_c(C_{t+1}^*)}{u_c(C_t^*)} \frac{P_t^* \mathcal{E}_t}{P_{t+1}^* \mathcal{E}_{t+1}}\right]. \tag{15}$$

where $\mathcal{E}_t$ denotes the nominal exchange rate. We assume that the purchasing power parity condition holds for this economy:

$$P_t = \mathcal{E}_t P_t^*, \tag{16}$$

where P_t^* is the price level in the foreign country. Following Clarida *et al.* (2002), we assume that state-contingent bonds exist, allowing both domestic and foreign households to trade internationally. Under international complete market and consumption preferences (1), combining the foreign Euler equation (15) with that in the home country leads to the following result under a suitable normalization on initial conditions:

$$C_t = C_t^*, \tag{17}$$

for all t.[6] In our model, international consumption risk sharing remains unaffected by the presence of international progressive tax rates. As later explained, the effect of international progressive tax rates is mainly derived from a change in the terms of trade.

2.2 Firms

Each country has two production sectors. The first is the final goods sector, which produces final goods using intermediate goods and is characterized by perfect competition. The second is the intermediate goods sector, wherein firms face monopolistic competition and Calvo (1983)'s nominal price rigidity.

Final goods firms

The final goods sector is perfectly competitive, and producers use inputs produced in the intermediate goods sector. Final goods are produced according to the following CES aggregate:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\xi_p - 1}{\xi_p}} di\right]^{\frac{\xi_p}{\xi_p - 1}}, \tag{18}$$

where Y_t is aggregate output, $Y_t(i)$ is demand for intermediate goods produced by firm i, and ξ_p is the elasticity of substitution. Note that both variables are normalized by $1-\gamma$.

Under the CES aggregate, the demand function is given by:

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\xi_p} Y_t, \tag{19}$$

and the domestic price level is defined as follows:

$$P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\xi_p} di\right]^{\frac{1}{1-\xi_p}}, \tag{20}$$

[6] See Chari *et al.* (2002), Corsetti *et al.* (2010), and Pappa (2004) for a detailed discussion of international consumption risk-sharing when we consider the constant elasticity of substitution (CES) consumption basket.

where $P_{H,t}(i)$ is the price for intermediate goods produced by firm i. Note that these variables are also normalized by $1-\gamma$.

Intermediate goods firms

The intermediate goods sector is characterized by monopolistic competition, and each firm produces a differentiated intermediate good. Firm i's production function is given by:

$$Y_t(i) = A_t N_t(i), \tag{21}$$

where A_t denotes an aggregate productivity disturbance, following an autoregressive (AR)(1) process given by $\log A_t = \rho_a \log A_{t-1} + \epsilon_t^a$ with $0 \le \rho_a < 1$, where ϵ_t^a is an independent and identically distributed (i.i.d.) shock with constant variance σ_a^2.

As in Clarida *et al.* (2002), the intermediate firm's real marginal cost is given as follows:

$$\varphi_t = (1-\tau^s)\frac{W_t}{P_{H,t}}\frac{1}{A_t}, \tag{22}$$

where τ^s is a subsidy rate for the wage bill used to eliminate the price and wage markup distortion associated with monopolistic firms and labor unions at the steady state (Woodford, 2003). Using Equation (14), we can rewrite the above real marginal cost as follows:

$$\begin{aligned}
\varphi_t &= (1-\tau^s)\frac{W_t}{P_{H,t}}\frac{1}{A_t} \\
&= (1-\tau^s)\frac{\mu_t^w}{kA_t}\frac{N_t^\eta}{C_t^{-\sigma}}(1-\tau_m)^{-1}S_t^\gamma \\
&= (1-\tau^s)\frac{\mu_t^w}{kA_t}\frac{N_t^\eta}{C_t^{-\sigma}}\delta^{-1}(1-\phi_n)^{-1}\left(\frac{Y}{Y_t}\right)^{-\phi_n}S_t^\gamma.
\end{aligned} \tag{23}$$

In contrast to Clarida *et al.* (2002), the real marginal cost now depends on the progressive taxation for the home country. The progress taxation effects are captured by the parameters ϕ_n and δ. The foreign tax rate also induces a change in the home real marginal cost through changes in the terms of trade in a two-country economy model. Therefore, as we more concretely discuss below, the key to understanding the role of international monetary policy in our model is to consider the effect of progressive taxation on the terms of trade.

2.3 Equilibrium

2.3.1 Equilibrium with flexible prices

First, we explain the equilibrium conditions under flexible price equilibrium. The goods market conditions for both countries are given by

$$(1-\gamma)Y_t = (1-\gamma)C_{H,t} + \gamma C^*_{H,t} + (1-\gamma)G_t, \tag{24}$$

$$\gamma Y^*_t = (1-\gamma)C_{F,t} + \gamma C^*_{F,t} + \gamma G^*_t. \tag{25}$$

Here, using the government budget constraints and the definition of the progressive tax in the home country leads to the following equation:

$$G_t = (1-\tau_t)Y_t. \tag{26}$$

Thus, in contrast to Mattesini and Rossi (2012), in our model the size of government spending is endogenously adjusted by a change in the progressive tax.[7] It follows that the higher the progressive tax rates, the smaller government spending in the home country. Since two countries are symmetric, the analogous equation holds for the foreign country.

Substituting Equations (2), foreign counterparts, Equation (24), and (26) leads to the following relation:

$$(1-\tau_t)P_{H,t}C_t = P_tC_t. \tag{27}$$

In contrast to Clarida *et al.* (2002), the home net exports are affected by their progressive tax rates. This model defines net exports as a fraction of steady-state output after taxes. Therefore, this model considers the trade balance between the two countries to be equal on an after-tax basis. Similarly, we obtain the foreign country's counterpart

$$(1-\tau^*_t)P^*_{F,t}C^*_t = P^*_tC^*_t. \tag{28}$$

Using Equations (27) and (28) and the definition of progressive taxes, we can rewrite the home terms of trade as follows:

$$S_t = \frac{\delta\left(\frac{Y}{Y_t}\right)^{\phi_n} Y_t}{\delta^*\left(\frac{Y^*}{Y^*_t}\right)^{\phi^*_n} Y^*_t}, \tag{29}$$

[7] Therefore, in contrast to the role of an exogenous government spending shock in Mattesini and Rossi (2012), our model abstracts from the role of an exogenous fiscal stimulus effect.

where Y and Y^* denotes home and foreign output at the steady state, respectively. It follows from this equation that both ϕ_n and ϕ_n^* significantly influence the terms of trade dynamics. To obtain this equation's intuition, we rewrite it as follows:

$$S_t = \frac{\delta Y^{\phi_n} Y_t^{(1-\phi_n)}}{\delta^* (Y^*)^{\phi_n^*} (Y_t^*)^{(1-\phi_n^*)}}, \tag{30}$$

For instance, given home output, a rise in foreign output causes a depreciation in the home terms of trade in the case of an increase in the progressive tax rate in the foreign country. In particular, as an extreme case, the home terms of trade react proportionally to the home output in the case of 100% progressive taxation in the foreign country. This mechanism holds for the foreign terms of trade. Without loss of generality, we assume $\delta = \delta^*$ in the following discussion.

We now derive the natural rate of output When prices are flexible worldwide. More concretely, the home natural rate of output Y_t^n is given as follows:

$$Y_t^n = \left(\frac{k^{1-\sigma} A_t^{1+\eta} (1-\phi_n) (Y_t^{n,*})^{-\kappa_0}}{(1-\tau^s) \mu_t^w \mu^p \delta^{(\sigma-1)} Y^{(1-\gamma)(\sigma-1)\phi_n} (Y^*)^{\gamma(\sigma-1)\phi_n^*}} \right)^{\frac{1}{\kappa}}, \tag{31}$$

where

$$\kappa = (\sigma - 1)(1-\gamma)(1-\phi_n) + 1 + \eta; \ \kappa_0 = (\sigma - 1)\gamma(1-\phi_n^*).$$

Also, $Y_t^{n,*}$ denotes the natural rate of foreign output. Notice that unlike Clarida *et al.* (2002), the impact output $Y_t^{n,*}$ on Y_t^n in our model depends on the CRRA coefficient σ and the degree of home progressive taxation ϕ_n. On the one hand, if $\sigma > 1$, then Y_t^n is negatively affected by an increase in $Y_t^{n,*}$. In contrast, the home natural rate of output co-moves following a change in $Y_t^{n,*}$ when $\sigma < 1$. Except for the presence of progressive tax rates in our model, this mechanism is consistent with the result in Clarida *et al.* (2002). In contrast to Clarida *et al.* (2002),the degree of ϕ_n and ϕ_n^* in our model significantly affects the natural rate of output in both countries. Note that the analogous equation holds for the foreign country. Section 2.3.2 discusses how changes in home and foreign progressive tax rates affect the real marginal cost through changes in κ and κ_0 in our two-country model.

2.3.2 Equilibrium with sticky prices

Following Calvo (1983), we assume that price rigidity is present in the intermediate goods sector. More precisely, a fraction $1 - \omega$ of all firms adjusts their price while the remaining fraction of firms ω does not. We consider the intermediate firms that can adjust their price. When revising their prices, these firms contemplate uncertainty concerning when they can adjust prices again. In this case, the intermediate firm's optimization problem for country H is given by

$$E_t \sum_{t=0}^{\infty} \omega^j Q_{t,t+j} Y_{t+j}(i)(P_{H,t}^o - P_{H,t+j}\varphi_{t+j}). \tag{32}$$

where $P_{H,t}^o$ is the firm's optimal price.[8] The first order condition of this maximization problem is as follows:

$$E_t \sum_{t=0}^{\infty} \omega^j Q_{t,t+j} Y_{t+j}(i)(P_{H,t}^o - \mu^p P_{H,t+j}\varphi_{t+j}) = 0. \tag{33}$$

where the variable $\mu = 1/(\theta - 1)$ is the price markup. Log-linearizing this equation around the zero inflation steady state, we obtain the new Keynesian Phillips curve (NKPC) in terms of the real marginal cost:[9]

$$\pi_t = \beta E_t \pi_{t+1} + \zeta \varphi_t, \tag{34}$$

where π_t denotes the producer price index inflation rate and the slope of the NKPC is defined as follows:

$$\zeta = \frac{(1-\omega)(1-\omega\beta)}{\omega}.$$

Here we briefly explain some notations for the log-linearization. Z represents the value of steady-state value, and H_t^n is the value of efficient level. Also, we define $z_t = \log(Z_t/Z)$ as the deviation of Z_t from its steady state.

To obtain the intuitive mechanism of the role of international progressive taxation, we consider log-linearization of several key equations in our paper. First, we consider the role of the real marginal cost in our model. More concretely, the log-linearization of the home real marginal cost is given by

$$\varphi_t = [(\sigma - 1)(1-\gamma)(1-\phi_n) + 1 + \eta]x_t + (\sigma - 1)\gamma(1-\phi_n^*)x_t^* + \mu_t^w. \tag{35}$$

[8] Clarida *et al.* (2002) assume that the degree of nominal price rigidity in the home country is the same as that in the foreign country. Unlike them, we allow for different degrees of price stickiness in the home and foreign countries.

[9] See Walsh (2010) for a detailed derivation of the new Keynesian Phillips curve.

Equation (35) is the same shapes as those derived by Clarida *et al.* (2002). However, in our model, the deep parameters in the real marginal cost are now affected by parameters ϕ_n and ϕ_n^*. On the one hand, the sensitivity of the real marginal cost to the home output gap κ is negatively affected by an increase in ϕ_n. This is in stark contrast with Clarida *et al.* (2002) and Mattesini and Rossi (2012) in that both international risk sharing and the terms of trade channels are affected by the parameter ϕ_n. Interestingly, the higher home progressive tax rate counteracts the open economy effect on the real marginal cost in its country. Clarida *et al.* (2002) and Mattesini and Rossi (2012) do not show this mechanism; they focused on the role of progressive taxation in a closed economy model. In an extreme case, the first term on the right-hand side in κ becomes zero. This implies that the open-economy effect completely shuts down, regardless of $\sigma \neq 1$. In addition, sensitivity to the foreign output gap κ_0 is also negatively affected by a change in ϕ_n^*. In contrast to Clarida *et al.* (2002), the higher the value of ϕ_n^*, the smaller the open economy effect through changes in the foreign output gap regardless of $\sigma \neq 1$. Thus, a higher progressive tax in the *foreign* country attenuates the effect of the foreign output gap on home inflation through international risk sharing and the terms-of trade channels. Therefore, in a two-country model, a change in *foreign* progressive tax rate changes the home inflation through a change in the terms of trade.

Finally, in contrast to the Clarida *et al.* (2002), in a very extreme case where the progressive tax rates of the two countries are very close to unity, the open economy effect disappears regardless of $\gamma > 0$ and $\sigma \neq 1$. To confirm, we consider a case of log-lineraized terms of trade expressed by the gap term, as follows:

$$\tilde{s}_t = (1 - \phi_n)x_t - (1 - \phi_n^*)x_t^*. \tag{36}$$

where $\tilde{s}_t$ $(= s_t - s_t^n)$ denotes the terms of trade gap. $x_t = y_t - y_t^n$ denotes the home output gap and x_t^* denotes the corresponding gap in the foreign economy. The home (and foreign) terms of trade become constant when both ϕ_n and ϕ_n^* are very close to unity. Although this case of simultaneously adopting a 100% progressive tax in both countries is unrealistic, this mechanism is based on theoretical aspects derived from our two-country model.

Similar mechanism holds for the foreign country. The foreign real marginal cost is given by

$$\varphi_t^* = [(\sigma - 1)\gamma(1 - \phi_n^*) + 1 + \eta]x_t^* + (\sigma - 1)(1 - \gamma)(1 - \phi_n)x_t + \mu_t^{w,*}, \tag{37}$$

where its coefficients are given as follows:

$$\kappa^* = (\sigma - 1)\gamma(1 - \phi_n^*) + 1 + \eta; \;\; \kappa_0^* = (\sigma - 1)(1 - \gamma)(1 - \phi_n).$$

Next, we derive the aggregate supply relations in both home and foreign countries. Substituting (35) for (34) leads to the following NKPC:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda x_t + \lambda_0 x_t^* + u_t. \tag{38}$$

We obtain a similar expression for the foreign country as follows:

$$\pi_t^* = \beta E_t \pi_{t+1}^* + \lambda^* x_t^* + \lambda_0^* x_t + u_t^*, \tag{39}$$

where

$$\zeta^* = \frac{(1 - \omega^*)(1 - \omega^* \beta)}{\omega^*}; \;\; \lambda = \zeta\kappa; \;\; \lambda_0 = \zeta\kappa_0; \lambda^* = \zeta^*\kappa^*; \;\; \lambda_0^* = \zeta^*\kappa_0^*.$$

Also, u_t and u_t^* denotes the exogenous home and foreign cost-push shocks associated with the presence of time-varying wage markup, respectively. We assume that the home cost-push shock follows an AR(1) process:

$$u_t = \rho u_{t-1} + \epsilon_t^c, \tag{40}$$

where $\rho < 1$ is a cost-push shock persistence and ϵ_t^c is an independent and identically distributed (i.i.d.) shock with constant variance σ_c^2. A similar expression holds for the foreign country. As noted earlier, the progressive tax rate in each country significantly affects inflation dynamics through a change in the real marginal costs.

We also derive the dynamic aggregate demand (AD) equation, as follows:

$$x_t = E_t x_{t+1} + \vartheta E_t \Delta x_{t+1}^* - \sigma_0^{-1}(r_t - E_t \pi_{t+1} - \bar{rr}_t), \tag{41}$$

$$x_t^* = E_t x_{t+1}^* + \vartheta^* E_t \Delta x_{t+1} - (\sigma_0^*)^{-1}(r_t^* - E_t \pi_{t+1}^* - \bar{rr}_t^*), \tag{42}$$

where

$$\sigma_0 = \sigma - \gamma(\sigma - 1)(1 - \phi_n^*); \;\; \vartheta = (\sigma_0)^{-1}\kappa_0;$$
$$\sigma_0^* = \sigma - (1 - \gamma)(\sigma - 1)(1 - \phi_n); \;\; \vartheta^* = (\sigma_0^*)^{-1}\kappa_0^*.$$

It follows from Equations (41) and (42) that the shape of these equations are similar to those derived by Clarida *et al.* (2002). In our model the slope of the AD curve is now affected by its country's progressive tax rate in its country because an increase in ϕ_n counteracts the sensitivity of the output gap to a change in the real interest rate. The AD curve slope is positive as long as parameter σ_0 takes a negative value. That case corresponds to the inverted AD curve argued by Bilbiie (2008), and the model under a regressive taxation system. While this is interesting, we postulate that the slope of the AD curve in our model never takes a negative value because of $\phi_n \in [0, 1]$. Variables $\bar{rr}_t$ and $\bar{rr}_t^*$ denote the natural rate of interest for each country, defined as follows:

$$\bar{rr}_t = \sigma_0(\Delta\hat{Y}_{t+1}^f + \kappa_0\Delta\hat{Y}_{t+1}^*);\ \bar{rr}_t^* = \sigma_0^*(\Delta\hat{Y}_{t+1}^{f,*} + \kappa_0^*\Delta\hat{Y}_{t+1}).$$

Finally, using the definition of the terms of trade, the nominal exchange rate evolves to

$$e_t = e_{t-1} + s_t - s_{t-1} + \pi_t^* - \pi_t \tag{43}$$

In contrast to the standard two-country NK model, the nominal exchange rate is now affected by ϕ_n and ϕ_n^*. When both ϕ_n and ϕ_n^* approach unity, the nominal exchange rate becomes a one-to-one correspondence with the difference in inflation rates between the two countries. This is because a change in the terms of trade dampens as parameters ϕ_n and ϕ_n^* increase.

3 Optimal monetary policy

This section examines optimal monetary policy in a two-country model with progressive taxation. Section 3.1 provides the central bank's loss function by calculating the second-order approximation of the household's utility function. In Section 3.2 we derive optimal monetary policy rules under a discretionary policy and investigate the effect of international progressive taxation on the properties of optimal monetary policy under policy coordination.

3.1 Central bank's loss function

We derive the central bank's loss function under policy coordination. More precisely, we consider the derivation of the central bank's loss function by implementing the second-order approximation of the household's utility function around the efficient steady state (Woodford, 2003). If the distorted distortion is significant, we cannot use the second-order approximation of the household's utility function. Benigno and Woodford (2005) derived the central bank's loss function in a case where the steady state distortion is not small for us to obtain the exact second-order approximation of the household's utility function. As Woodford (2003) postulated, we assume that the distortion associated with the price markup is sufficiently small so that the method of second-order approximation is still valid in the presence of the price-markup at the steady state. Thus, this paper derives the central bank's loss function without calculating the second-order approximation of the Phillips curves. Following Clarida *et al.* (2002), we obtain the efficient steady state in a two-country NK model by using the optimal subsidy to completely offset the distortions associated with the presence of price and wage markups and progressive taxes.[10]

In the cooperative case, home and foreign policymakers jointly maximize their objective function weighted by $1-\gamma$ and γ, respectively. More concretely, the worldwide utility function is defined as:

$$U_t^W = E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ (1-\gamma) U(C_t, N_t) + \gamma U(C_t^*, N_t^*) \Big\}.$$

Since international consumption risk-sharing is complete, we can rewrite the above worldwide welfare as follows:

$$U_t^W = E_0 \sum_{t=0}^{\infty} \beta^t \Big[u(C_t) - (1-\gamma) v(N_t) - \gamma v(N_t^*) \Big].$$

Consider a social planner's problem of solving for optimal subsidies under policy coordination to obtain an efficient steady state. The social planner selects the optimal subsidy rate to maximize the period utility function at the steady state subject to the equilibrium conditional on steady state consumption:

$$C = \kappa \delta \left(\frac{Y}{N} \right)^{\phi_n} N S^{-\gamma}$$

with

$$S = \frac{N^{1-\phi_n}}{{N^*}^{1-\phi_n^*}}.$$

Solving the above maximization by social planner, we obtain the optimal subsidy rates to eliminate the distortions associated with price and wage markups. More concretely, as shown in

[10] We derive the central bank's loss function by considering the standard second-order approximation of the utility function around the efficient steady state, whereas Mattesini and Rossi (2012) derive the central bank's loss function in a case of a distorted steady-state. Even if we consider the derivation of the loss function around the distorted steady state, without loss of generality, we can say that this study's main masses are unaffected.

Appendix A, we show that the optimal subsidy obtained from the command planner's optimization problem satisfies the following equation:

$$(1-\tau^s)\mu^{w-1}\left(\frac{\xi_p - 1}{\xi_p}\right) = 1.$$

Under the optimal subsidies, we derive a central bank loss function by computing a second-order Taylor expansion of households' home and foreign utility functions weighted by the degree of openness. More concretely, we obtain the following central bank's objective function under policy coordination:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ (1-\tilde{\gamma})L_t^H + \tilde{\gamma} L_t^F - 2\Lambda x_t x_t^* \right\} + t.i.p. + o(\|a\|^3). \quad (44)$$

In the above cooperative loss function of central banks the periodic loss function for each country is derived as follows:

$$L_t^H = \pi_t^2 + \alpha_x x_t^2;$$
$$L_t^F = (\pi_t^*)^2 + \alpha_x^* (x_t^*)^2.$$

The coefficients in this loss function are defined as follows:

$$\varpi = (1-\gamma)(1-\phi_n)\zeta^{-1} + \gamma(1-\phi_n^*)(\zeta^*)^{-1};\ \tilde{\gamma} = \frac{\gamma(1-\phi_n^*)(\zeta^*)^{-1}}{\varpi};$$
$$\alpha_x = \frac{\zeta[(\sigma-1)(1-\gamma)(1-\phi_n)+1+\eta]}{\xi_p};\ \alpha_x^* = \frac{\zeta^*[(\sigma-1)\gamma(1-\phi_n^*)+1+\eta]}{\xi_p};$$
$$\Lambda = \frac{(1-\sigma)(1-\gamma)(1-\phi_n)\gamma(1-\phi_n^*)}{\varpi\xi_p}.$$

Also, $t.i.p$ includes terms independent of monetary policy, and $o(||\xi||^3)$ indicates the terms of third or higher orders. Appendix A provides a detailed derivation of the above loss function by calculating the second-order approximation of the household utility function. The first and second terms on the right side of Equation (44) represent home and foreign welfare losses, respectively. The third term on the right hand side of this equation denotes the cross-term of x_t and x_t^* captures the international spillover effect. According to Clarida *et al.* (2002), this term is the source of the international monetary policy coordination gains.

While this loss function's policy objectives are the same as Clarida *et al.* (2002), the stabilization coefficients in each term are now affected by the parameters ϕ_n and ϕ_n^*. First, the weights on home and foreign policy objectives ($1-\tilde{\gamma}$ and $\tilde{\gamma}$, respectively) are affected by ϕ_n and ϕ_n^*. For example, for a given value of ϕ_n, an increase in the foreign progressive tax rate reduces the foreign share in the global loss function. Second, the output gap stabilization in each country is also affected by the degree of each country's progressive tax rates. For instance, other things being equal, an increase in ϕ_n lowers the coefficient for the output gap stabilization in the home country's loss function.

Third, the cross-term of x_t and x_t^* is also affected by both ϕ_n and ϕ_n^*. As Equation (30) states, the progressive tax rates in both countries significantly affect the terms of trade. For instance, in the case of $\sigma > 1$, an increase in the home and foreign tax progressivity expands the cross-term influence in the loss function. Although the case of $\phi_n = \phi_n^* = 1$ is extreme and unrealistic, the cross-term of x_t and x_t^* in this case disappears in spite of $\gamma > 0$. Unlike this extreme example, increasing progressive tax rates in both countries decreases the stabilization weight on the cross-term that captures the spillover effect of international risk-sharing and the terms of trade effects.

Although our model is a simple extension of Clarida *et al.* (2002), these characteristics are not observed in Clarida *et al.* (2002). To capture the role of progressive taxation in a two-country NK model, we analytically derive the optimal targeting rule under a discretionary policy in the next subsection. Section 4, numerically confirms the effect of a progressive taxation system on the optimal monetary policy under policy coordination.

3.2 Optimal policy under discretion: Analytical results

This section analytically derives the optimal response of several key macro variables to grasp how the international progressive taxation system affects the optimal discretionary policy under international monetary policy coordination. Solving the central bank's optimization problem leads to the following targeting rules for each country:

$$x_t = -\xi_p \pi_t - \frac{\kappa_0}{\kappa}\left(x_t^* + \xi_p \pi_t^*\right), \tag{45}$$

$$x_t^* = -\xi_p \pi_t^* - \frac{\kappa_0^*}{\kappa^*}\left(x_t + \xi_p \pi_t\right). \tag{46}$$

Combining these equations yields

$$x_t = -\xi_p \pi_t, \tag{47}$$

$$x_t^* = -\xi_p \pi_t^*. \tag{48}$$

These conditions state the optimal targeting rule under the discretionary policy. The shape of the targeting rule seems to be the same as that of Clarida *et al.* (2002). However, we emphasize that in our model, the degree of the progressive tax rates significantly affects the rational expectations' equilibrium solutions.

The following analytical discussion, without loss of generality, focuses on a case where cost-push shocks are present in both countries.[11] To explain the role of progressive taxation under optimal monetary policy, we focus on the minimum state variable (MSV) solution in the following discussion (McCallum, 1983).The MSV solutions under the cooperative discretionary policy are derived in the symmetric equilibrium. We use the methods for undetermined coefficients (Uhlig *et al.*, 1995), and the MSV solutions of home and foreign inflation are given by

$$\pi_t = au_t + bu_t^* \tag{49}$$

$$\pi_t^* = a^* u_t + b^* u_t^* \tag{50}$$

where

$$a = \frac{\psi^*}{\psi\psi^* - \lambda_0\lambda_0^*\xi_p^2}; \ a^* = -\frac{\lambda_0^*\xi_p}{\psi\psi^* - \lambda_0\lambda_0^*\xi_p^2}; \quad b = -\frac{\lambda_0\xi_p}{\psi\psi^* - \lambda_0\lambda_0^*\xi_p^2}; \ b^* = \frac{\psi}{\psi\psi^* - \lambda_0\lambda_0^*\xi_p^2}$$

$$\psi = 1 - \beta\rho + \xi_p\lambda; \ \psi^* = 1 - \beta\rho + \xi_p\lambda^*.$$

By substituting Equations (49)-(50) into targeting rules (47)-(48), we also obtain the reduced form of home and foreign output gaps. Except for the presence of progressive tax rates, the shape of the reduced form of home and foreign inflation rates is line with Clarida *et al.* (2002). We next analytically examine how changes in progressive tax rates affect the response of home and foreign inflation rates to country-specific cost-push shocks. To obtain the intuitive solution, we restrict the analysis to a symmetric case where the probabilities home and foreign firms keep their prices fixed (implying $\zeta = \zeta^*$). The effect of a change in ϕ_n on home and foreign inflation is summarized as the following proposition:

Proposition 1 *A change in the home progressive tax rate amplifies the impact of its cost-push shock on home inflation; however it weakens the impact of the foreign cost-push shock on home inflation.*

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[11] Even when we account for the productivity shock, the following discussion remains unaffected. See Clarida *et al.* (2002) for a detailed discussion about the role of productivity shocks under a discretionary policy in a two-country model.

Proof. Differentiating the coefficient a in Equation (49) concerning ϕ_n, we obtain the following results:

$$\frac{\partial a}{\partial \phi^n} = -\frac{\psi^* \xi_p}{(\psi\psi^* - \lambda_0\lambda_0^*\xi_p^2)^2}[(1-\beta\rho) + \xi_p(\lambda^* - \lambda_0)]\frac{\partial \lambda}{\partial \phi^n}.$$

For plausible parameterizations, the sign in parentheses is positive. Since we can easily show that $\partial\lambda/\partial\phi_n < 0$ when $\sigma > 1$, the sign of the above equation becomes positive. Next, differentiating the coefficient b in equation (49) leads to

$$\frac{\partial b}{\partial \phi^n} = \frac{\lambda_0 \xi_p^2}{(\psi\psi^* - \lambda_0\lambda_0^*\xi_p^2)^2}[(1-\beta\rho) + \xi_p(\lambda^* - \lambda_0)]\frac{\partial \lambda}{\partial \phi^n} < 0.$$

This completes the proof. ■

This proposition's intuition is as follows. On the one hand, an increase in ϕ_n amplifies the effect of home cost-push shock on home inflation. Although a rise in ϕ_n alleviates a fluctuation in the output gap caused by the cost-push shock, the policy trade-off increases the inflation rate. However, a rise in ϕ_n dampens the impact of a foreign cost-push shock on home inflation as long as $\sigma > 1$. Thus, the higher the home progressive tax rate, the more it contributes to mitigating the impact of terms of trade on the real marginal cost in the home country. Still, since $\partial b/\partial\phi_n$ becomes negative in the case of $\sigma < 1$, an increase in the home tax rates amplifies the effect of a foreign cost-push shock on the home inflation rate.

Next, we consider the effect of changes in home progressive tax rates on the *foreign* inflation rate. This result is summarized as follows:

Proposition 2 *A change in the home progressive tax rate bolsters the impact of its cost-push shock on foreign inflation; however, it weakens the impact of the foreign cost-push shock on foreign inflation.*

Proof. Differentiating the coefficient a^* in Equation (49) with respect to ϕ_n, we obtain

$$\frac{\partial a^*}{\partial \phi^n} = -\frac{\psi^* \xi_p}{(\psi\psi^* - \lambda_0\lambda_0^*\xi_p^2)^2}[(1-\beta\rho) + \xi_p(\lambda - \lambda_0^*)]\frac{\partial \lambda}{\partial \phi^n}.$$

For plausible parameterizations, the sign in parentheses is positive in this calculation. As noted in proposition 1, we also know that $\partial\lambda/\partial\phi_n < 0$ when $\sigma > 1$. Therefore, we can say that the sign of the above equation becomes positive. Next, differentiating the coefficient b^* in equation (50) leads to

$$\frac{\partial b^*}{\partial \phi^n} = \frac{\lambda_0 \xi_p^2}{(\psi\psi^* - \lambda_0\lambda_0^*\xi_p^2)^2}[(1-\beta\rho) + \xi_p(\lambda - \lambda_0^*)]\frac{\partial \lambda}{\partial \phi^n} < 0.$$

This completes the proof. ■

First, a rise in ϕ_n also induces the positive impact of a home cost-push shock on foreign inflation. This result is not surprising because it simply echoes the effect of ϕ_n on the coefficient b in Proposition 1. Second, consider the intuition on the effect of ϕ_n on b^*. We underline that this channel is also specific to a two-country model with progressive taxation. A parameter change in ϕ_n helps the foreign central bank to reduce the volatility of foreign inflation associated with its country's cost-push shock. Proposition 2 implies that as long as $\sigma > 1$, a rise in home progressive tax rates can ease the volatility of foreign inflation through the terms of trade channel. This also simultaneously indicates an increase in the foreign output gap due to a policy trade-off caused by cost-push shocks.

Propositions 1 and 2 state the effect of cost-push shock for each country on the home inflation following a change in ϕ_n. However, we do not understand whether a change in ϕ_n enhances global social welfare or how does a change in ϕ_n affects the worldwide welfare loss under international monetary policy coordination. We analytically calculate the unconditional expectation of welfare losses to answer these questions. More precisely, substituting equations (49) and (50) into the loss function (44), and taking unconditional expectations, we obtain the following welfare loss function based on unconditional expectations:

$$\begin{aligned} E(\mathcal{L}) = {} & [(1-\tilde{\gamma})(1+\alpha_x {\xi_p}^2)a^2 + \tilde{\gamma}(1+\alpha_x^* {\xi_p}^2)(a^*)^2 - 2\Lambda a a^* {\xi_p}^2]\sigma_c^2 \\ & + [(1-\tilde{\gamma})(1+\alpha_x {\xi_p}^2)b^2 + \tilde{\gamma}(1+\alpha_x^* {\xi_p}^2)(b^*)^2 - 2\Lambda b b^* {\xi_p}^2](\sigma_c^*)^2. \end{aligned}$$

To execute an analytical investigation, we assume that this derivation contains no correlation between the home and foreign cost-push shocks. Furthermore, for simplicity, we set $(\sigma_c^*)^2 = 0$ to obtain an intuitive expression.[12] Differentiating the above equation regarding ϕ_n leads to

$$\begin{aligned} \frac{\partial E(\mathcal{L})}{\partial \phi_n} = {} & (1+\alpha_x {\xi_p}^2)\left[2(1-\tilde{\gamma})\frac{\partial a}{\partial \phi_n} - \frac{\partial \tilde{\gamma}}{\partial \phi_n}a^2\right] + (1-\tilde{\gamma})\frac{\partial \alpha_x}{\partial \phi_n}\xi_p^2 a^2 \\ & + (1+\alpha_x^* {\xi_p}^2)\left[2\tilde{\gamma}\frac{\partial a^*}{\partial \phi_n} + \frac{\partial \tilde{\gamma}}{\partial \phi_n}(a^*)^2\right] - 2{\xi_p}^2\left[a a^* \frac{\partial \Lambda}{\partial \phi_n} + \Lambda\left(a^* \frac{\partial a}{\partial \phi_n} + a\frac{\partial a^*}{\partial \phi_n}\right)\right]. \end{aligned}$$

[12] Section 4 numerically examines the effect of progressive taxation on the worldwide welfare losses in the presence of home and foreign cost-push shocks.

Rearranging this equation, we obtain

$$\frac{\partial E(\mathcal{L})}{\partial \phi_n} = 2(1+\alpha_x {\xi_p}^2)(1-\tilde{\gamma})\frac{\partial a}{\partial \phi_n} + (1-\tilde{\gamma})\frac{\partial \alpha_x}{\partial \phi_n}\xi_p^2 a^2 + (1+\alpha_x^* {\xi_p}^2)\left(2\tilde{\gamma}\frac{\partial a^*}{\partial \phi_n} + \frac{\partial \tilde{\gamma}}{\partial \phi_n}(a^*)^2\right) - 2\xi_p^2\left[aa^*\frac{\partial \Lambda}{\partial \phi_n} + \Lambda\left(a^*\frac{\partial a}{\partial \phi_n} + a\frac{\partial a^*}{\partial \phi_n}\right)\right] - (1+\alpha_x\xi_p^2)a^2\frac{\partial \tilde{\gamma}}{\partial \phi_n}.$$

From Propositions 1 and 2, under plausible calibrated values in the existing literature the partial derivative $\partial E(\mathcal{L})/\partial \phi_n$ should be positive. This implies that when two countries are interdependent, an increase in one country deteriorates worldwide welfare despite the absence of a cost-push shock in the other country. Thus, we have

Proposition 3 *An increase in the home progressive tax increases the worldwide welfare losses in the presence of a home cost-push shock.*

Based on our analytical investigation, we address the role of progressive taxation when considering international monetary policy analysis. Therefore, this result provides an important implication for international optimal monetary policy coordination when progressive tax rates matter internationally. The next section considers these results quantitatively.

4 Quantitative results

This section presents quantitative results on optimal monetary policy under discretion. First, we briefly explain this paper's calibrated values; second, we examine how international progressive taxation affects the transmission mechanism of structural shocks in a two-country economy; third, we show the impact of international progressive taxation on global welfare; fourth, we conduct several sensitivity experiments to check the robustness of the previous sections. Finally, as in Section 3.2, we focus our quantitative analysis on an exogenous cost-push shock because it generates a policy trade-off between the inflation rate and the output gap. In what follows, we use the Dynare software package to simulate the properties of optimal discretionary policy.[13]

4.1 Calibration

This subsection briefly describes this study's parameters based on existing literature on an open-economy NK model. The discount factor, β, is set to 0.99. Regarding the degree of

[13]Dynare is available at http://www.dynare.org/.

openness, the value of γ is set to 0.4. The following explanation of calibrated parameters focuses on the parameters for the home country. Unless otherwise noted, the same parameters hold for the foreign country. The degree of price rigidity, ω, is set to 0.75. A value of 2.0 is used for the risk aversion coefficient σ as a benchmark value. The elasticity of household labor supply, η, is set to 3.0. The elasticity of substitution for individual goods, ξ_p, is set to 5.0. The cost-push shock persistence ρ is set to 0.5 as a benchmark case.

Finally, we explain how to calibrate the values of progressive taxation parameters ϕ_n and ϕ_n^*. We focus on the international taxation system implying that fiscal policymakers in the home and foreign countries choose the same tax rate between the two countries.[14] The parameterization regarding progressive tax rates is based on Mattesini and Rossi (2012). First, we consider the case where home and foreign fiscal authorities do not consider a change in the progressive tax rate: $\phi_n = \phi_n^* = 0$. Then, following the discussion of Mattesini and Rossi (2012), we consider the calibrated values for ϕ_n and ϕ_n^*. We set these values to 0.3 for a small increase in tax rates and 0.6 for a higher increase in ones. Finally, while the case of $\phi_n = \phi_n^* = 0.9$ is an extreme example, we consider this case the complete opposite to $\phi_n = \phi_n^* = 0$ to obtain economic intuitions of the results.

4.2 Impulse response analyses

Figure 1 shows the impulse response to a foreign price markup shock, which leads to a policy trade-off between the inflation rate and the output gap in the foreign country. Interestingly, in the country where the shock occurs, the figure shows that neither the inflation rate nor the output gap is affected by the degree of progressive taxation. According to Propositions 1 and 2, this result represents the observation that simultaneous and identical changes in home and foreign progressive tax rates offset the effects of foreign cost-push shocks on that country's macro variables.

A foreign cost-push shock causes an increase in the home country's terms of trade and increases the home country's output gap, and the home country's inflation rate rises in response to the increase in the home country's output gap. This seems to be inconsistent with the

[14] We confirm that the quantitative results are unaffected by an individual change in the tax rate only in each country. Therefore, we do not report the above results. We can underscore the importance of the international progressive taxation system in that home and foreign countries cooperatively consider their tax rates.

result in Proposition 1. However, we conjecture that in this impulse response, the terms of trade effect dominates the consumption risk-sharing effect, increasing the home inflation rate. Moreover, in contrast to the foreign country case, the macro variables in the home country are crucially affected by changes in ϕ_n and ϕ_n^* because changes in these parameters dampen the response of the home terms of trade to the foreign cost-push shock. In the case of significantly higher progressive taxation in both countries, the macro variables in the home country remain unchanged in response to a foreign cost-push shock.

[Figure 1 around here]

Figure 2 also shows the impulse response to the nominal exchange rate in the home currency to a foreign cost-push shock. As discussed in Monacelli (2003), the response of the nominal exchange rate under discretion is characterized by its non-stationary property. Figure 2 shows that the nominal exchange rate depreciates immediately due to a foreign cost-push shock. However, if the home and foreign fiscal authorities both impose heavy progressive taxes, the initial exchange rate depreciation becomes attenuated. Additionally, increasing ϕ_n and ϕ_n^* dampens the nominal exchange rate depreciation. As Equation (43) states, this is because it counteracts the effect of the terms of trade on the nominal exchange rate.

[Figure 2 around here]

Next, we consider how more persistent cost-push shocks affect the performance of international progressive taxation. Figure 3 illustrates the response of the nominal exchange rate to the foreign cost-push shock in the case of $\rho = 0.8$.[15] Compared to Figure 2, when home and foreign progressive tax rates take a lower value, the impact of the foreign cost-push shock on the exchange rate is significant in the initial period. Also, in the case of ϕ_n and ϕ_n^* taking a more significant value, the exchange rate continues to appreciate in response to the foreign cost-push shock. We confirm that the persistence of the cost-push shock does not qualitatively affect inflation and the output gap for both home and foreign countries. However, Figure 3 shows that the cost-push shock persistence significantly affects the dynamics of the nominal exchange rate.

[15] We do not report the impulse response function of key macro variables in the case of $\rho = 0.8$ because we confirm that the impulse response function, in this case, does not qualitatively change.

[Figure 3 around here]

Therefore, this implies that progressive taxation in both countries might ease the volatility of home macro variables caused by the foreign structural shocks. If so, can an international progressive tax system play a significant role in enhancing social welfare worldwide? In other words, can home and foreign fiscal authorities achieve higher worldwide social welfare by manipulating progressive tax rates? We conduct a welfare analysis in the following subsection to answer this question.

4.3 Welfare analyses

We found that the international progressive taxation systems can reduce the volatility of home macro variables induced by a foreign cost-push shock. This leads to the natural question: does the presence of international progressive taxation systems enhance worldwide social welfare? We calculate the worldwide welfare losses under optimal monetary policy coordination with discretion for any combinations of ϕ_n and ϕ_n^* to answer this question.

Figure 4 illustrates the worldwide welfare loss when cost-push shocks are present in both countries. This figure shows that an increase in ϕ_n and ϕ_n^* produces worse welfare losses, which makes intuitive sense. Under a cost-push shock, central banks generally face a policy trade-off between inflation and output stabilization. Thus, when cost-push shocks are present in both countries, home and foreign central banks also face a trade-off between these two endogenous variables. The role of progressive taxation reduces output volatility, as progressive taxation acts as an effective automatic stabilizer of output. However, the presence of a progressive taxation system increases the inflation rate's volatility due to cost-push shocks. Thus, a progressive taxation system may produce more significant policy trade-offs. Accordingly, the worldwide welfare loss increases when home and foreign fiscal authorities both increase ϕ_n and ϕ_n^*, respectively. Therefore, we address that an international progressive taxation system renders worse worldwide welfare losses when home and foreign economies suffer from cost-push shocks.

[Figure 4 around here]

Next, Figure 5 shows the worldwide welfare loss when the cost-push shock is present only in the foreign country. As shown in Figure 4, an increase in a progressive tax worsens the worldwide welfare. Interestingly, for smaller value of ϕ_n^*, the worldwide welfare loss decreases as the home fiscal authority increases its progressive tax rate even when the home country does not face the cost-push shock. We underline that the presence of an international progressive tax reduces the worldwide welfare loss when asymmetric cost-push shocks exist between countries.

[Figure 5 around here]

The intuition behind this result is as follows. Since there are no cost-push shocks in the home country, the welfare costs associated with foreign cost-push shocks are transferred to the home country. Increasing the home country's progressive tax rate can mitigate such losses without incurring additional welfare costs because the fluctuation in the output gap through the terms of trade is attenuated by a rise in the home tax rate. No welfare costs caused by inflationary pressure arise in the home country because it does not face the cost-push shock. Therefore, a higher progressive tax rate counteracts the fluctuation in the output gap in the home country; this is the gain from considering the role of the international progressive taxation system. While our model is constructed as a natural extension of the two-country NK model, no existing studies have focused on the policy implication for the international progressive taxation system.

Now we consider the role of the CRRA coefficient, which plays an important role in the open economy. As shown in Clarida *et al.* (2002), the open-economy effects disappear when σ takes unity. Therefore, whether σ is above unity affects worldwide welfare losses. In particular, we focus on how increases in progressive tax rates affect the welfare loss under several calibrated values of σ. As mentioned in the Clarida *et al.* (2002), when considering the international spillover effects of monetary policy, values where σ is smaller than unity are also addressed in a two-country NK model.[16] A case of $\sigma < 1$ corresponds to a case where home and foreign goods are complements. The baseline calibration of σ is based on the value used in Pappa (2004). The case where σ is above unity corresponds to where home and foreign goods are substitutes. Indeed, several values of the CRRA coefficient that takes above unity are calibrated in several previous studies (Fujiwara *et al.*, 2013; Pappa, 2004). Corsetti *et al.* (2010) also argued whether the value of σ is above unity affects the gain from international monetary policy coordination.[17]

[16]See Corsetti *et al.* (2010) for a detailed discussion regarding the effect of the value of σ on optimal monetary policy in a two-country NK model.

[17]Corsetti *et al.* (2010) shows the elasticity of substitution between home and foreign goods also affects the

Pappa (2004) also explored the welfare gain from international policy coordination in a two-country model. Unlike Corsetti *et al.* (2010) and Pappa (2004), this paper examines how a change in the progressive tax rates affects the worldwide welfare loss under several values of σ, in a case where home and foreign central banks jointly solve optimal cooperative policy with discretion.

Figure 6 illustrates the worldwide welfare loss when σ changes from 0.3 to 4. A change in σ significantly affects the global welfare losses. When σ takes unity, the welfare losses are affected by any change in the degree of progressive tax rates. However, except for $\sigma = 1$, a change in the tax rates affects the global welfare losses. On the one hand, in the case of $\sigma < 1$, the smaller a change in the tax rates, the more significant global welfare losses. On the contrary, smaller progressive tax rates enhance global welfare when $\sigma > 1$. In the case of $\sigma > 1$, a foreign cost-push shock increases the home terms of trade when home and foreign goods are substitutes. This shock generates a policy trade-off between inflation and output stabilization; the higher the tax rate between the two countries, the worse the trade-offs become. Progressive taxation reduces the output gap volatility but increases inflation volatility. In a two-country model, large σ values exacerbate this trade-off by activating a strong channel through changes in terms of trade when home and foreign fiscal authorities impose strict progressive taxes on labor income.[18]

[Figure 6 around here]

4.4 Sensitivity experiments

This subsection performs some sensitivity checks on the results obtained in sections 4.2 and 4.3. First, we consider how the cost-push shock persistence affects the welfare loss under several combinations of ϕ_n and ϕ_n^*. According to Figure 7, increasing in ρ creates larger welfare losses. This is simply because the policy trade off between inflation and the output gap in

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gain from policy coordination and the parameter value of σ. Our model postulates that this elasticity is set to unity because it is based on Clarida *et al.* (2002). See Corsetti *et al.* (2010) and Pappa (2004) for a detailed discussion about how the value of elasticity of substitution between home and foreign goods affects the gain from policy coordination.

[18] If each fiscal authority uses progressive taxation as a macroeconomic policy instrument in a non-cooperative manner, it may be worthwhile to consider the gains from international policy coordination. This work is beyond the scope of this paper but will be considered in future work.

both countries is amplified by the persistent effect of the cost-push shocks. When cost-push persistence is low, Welfare losses are almost unaffected by combinations of ϕ_n and ϕ_n^* when the cost-push persistence is low. As long as ρ is not excessive, larger values of ϕ_n and ϕ_n^* slightly increase welfare loss. In the case of $\rho = 0.9$, the welfare loss under $\phi_n = \phi_n^* = 0.9$ is the largest of all combination values of ϕ_n and ϕ_n^*. Therefore, while cost-push shock persistence significantly increases the worldwide welfare loss under discretion, the loss remains unaffected by any combinations of home and foreign progressive tax rates unless cost-push persistence takes a higher value.

[Figure 7 around here]

Second, we consider whether the degree of nominal price rigidity affects the worldwide welfare loss under several parameterizations of ϕ_n and ϕ_n^*. If flexible price equilibrium is achieved in both countries, the distortion caused by price dispersion disappears. Also, as Rogoff *et al.* (2003) indicated, it is important to note how price flexibility from globalization affects worldwide welfare losses under international monetary policy coordination. This exercise explores the relationship between the degree of nominal price rigidity and progressive taxes in an open economy. We consider a case where ω and ω^* take ranges from 0.4 to 0.9.[19]

Figure 8 illustrates the worldwide welfare loss under discretion when the Calvo parameter changes. Regardless of the degree of progressive tax rates, the worldwide welfare loss is close to zero under flexible price equilibrium. Since price dispersion from staggered prices disappears, the effect of a cost-push shock is negligible. Put differently, according to our result, since globalization induces more flexible prices (Rogoff *et al.*, 2003), under global price flexibility, fluctuations in markup shocks should be fully absorbed by home and foreign progressive taxation. However, this result requires careful discussion; it does not necessarily mean that international progressive taxation provides an effective means of stabilizing global output under monetary policy coordination. This is because it is challenging to precisely identify whether this production gap stabilization can be fully achieved solely through a system of international progressive taxation.

The stickier nominal prices, the larger the worldwide welfare loss due to the presence of price dispersion. In the case of $\omega = \omega^* = 0.9$, larger values of $\phi_n = \phi_n^* = 0.9$ worsen worldwide

[19] We do not report the result for the parameter range from 0 to 0.4 because it does not affect the result in Figure 8.

social welfare. As home and foreign fiscal authorities raise progressive tax rates, home and foreign central banks can reduce the output volatility. However, the welfare losses associated with price dispersion increase because the cost-push shock generates a policy trade-off between inflation and the output gap. Overall, our results are robust to a change in the Calvo parameter.

[Figure 8 around here]

5 Discussion

This section argues the role of the international taxation system based on our results and provides some policy implications for the practical aspect of international monetary and fiscal policies. Our model indicates that the international progressive taxation system plays a significant role in the conduct of international monetary policy coordination. In particular, when home and foreign central banks jointly minimize the worldwide loss function, the higher the tax rates in both countries, the more significant the welfare losses. Since a change in the tax rates causes a policy trade-off between inflation and the output gap stabilization, higher tax rates create the greater welfare losses associated with inflation volatility under a progressive taxation system. Therefore, our result indicates that lower progressive taxes in both countries are desirable for home and foreign central banks to jointly minimize the worldwide welfare losses associated with the cost-push shocks.

How does our result provide the policy implication for international monetary and tax policies? Since Facundo *et al.* (2017) reported the time-series properties in the progressive tax rates in advanced countries, we use their calculation to explain our study's policy implications. The time-series data in the tax rates developed by Facundo *et al.* (2017) are shown in Figure 9, showing changes in top income tax rates in rich countries. This figure illustrates the time series data of progressive tax rates in five advanced countries. Before the 1980s, the tax rates are significantly different between countries. However, as shown in Figure 9, recent evidence implies that progressive tax rates seem to converge around the ranges from 40% to 60%.

[Figure 9 around here]

Also, as Rogoff *et al.* (2003) indicated, the inflation rate in advanced countries globally decreased since the latter half of the 1980s. Bernanke (2004) argued that the advanced economies, mainly the United States, experienced a period of low inflation and stable economic growth created by the success of the monetary policy. This fact might be called *great moderation*. Furthermore, we can almost certainly say that it has addressed the role of international monetary policy coordination since the 1980s (Canzoneri and Henderson, 1991).

How do these facts relate to this paper's results? This paper's central message is that home and foreign fiscal authorities may be able to enhance global welfare by lowering progressive tax rates under international monetary policy coordination. Our results might suggest that —until at least the global financial crisis of 2009— a global decline in progressive tax rates in advanced economies may also have contributed to low inflation and stable economic growth under international monetary policy coordination.[20] Therefore, we believe that while our model is constructed in a simple two-country model, our study can contribute to previous research because our model framework plays a vital role in examining international progressive taxation under monetary policy coordination.

6 Conclusions

Recent international macroeconomic policy has addressed the role of international taxation systems. We examined the role of international progressive taxation in the standard two-country NK model. This study's primary findings are summarized as follows. First, we showed that although the structural equation and shape of the central bank's loss function are the same as the standard two-country NK model, the deeper parameters indicated by the structural equation and the loss function are crucially affected by the progressive taxation parameters in both countries. Second, we demonstrated that when home and foreign central banks cooperatively implement optimal monetary policy under discretion, an increase in the progressive tax rate generally worsens worldwide social welfare, regardless of which country raises the progressive tax rates.

We also would like to mention several caveats that our paper cannot address. This paper does not focus on the gain from policy coordination because we attempt to analytically examine

[20] Our model does not consider the non-negativity constraints on the nominal interest rates. Therefore, we cannot assess whether a global decline in the progressive tax rates has helped stimulate the real economy under a global liquidity trap since 2009s. This is beyond this paper's scope, but it may be an important research question.

the effect of international progressive taxation on international monetary policy. Whether the gain from policy coordination is affected by international progressive taxation systems is an important topic for future research. We focused on the role of international progressive taxation systems in a two-country NK model by assuming a simple progressive tax rule. Therefore, we may consider how a change in progressive taxes affects the interaction between optimal fiscal and monetary policies. In other words, it may be interesting if we derive the optimal policy coordination between progressive taxation and monetary policies in a two-country framework. While these topics are open questions, our paper contributes to significant policy implications for the role of international taxation systems from the international aspect of monetary policy.

Appendix A: Detailed derivation of the central bank's loss function under policy coordination

This appendix derives the central bank's loss function under policy coordination by calculating the second-order Taylor expansion of the household's utility function. We first derive the optimal subsidy to eliminate the distortion caused by price and wage markups at the steady state. In the cooperative case, the policymaker maximizes the period utility $U(C_t) - (1 - \gamma)V(N_t) - \gamma V(N_t)$. The optimal subsidy rate is chosen to maximize the period utility subject to the equilibrium conditional on consumption:

$$C = \kappa\delta\left(\frac{Y}{N}\right)^{\phi_n} N S^{-\gamma}$$

with

$$S = \frac{N^{1-\phi_n}}{N^{*1-\phi_n^*}}.$$

Solving the command planner's problem, we can get the optimal condition $V'(N)N = (1 - \phi_n)U'(C)C$. Hence, using this condition, we show that the optimal rate of τ in the equilibrium steady state satisfies

$$(1 - \tau^s)\mu^{w-1}\left(\frac{\xi_p - 1}{\xi_p}\right) = 1.$$

Next, we derive the second-order approximation of the household's utility function. Before deriving the loss function, we define some notations. First, as noted earlier, Z represents the steady-state value, and Z_t^n is the value of efficient level. Also, we define $z_t = \log(Z_t/Z)$ as the deviation of Z_t from its steady state. In addition to these notations, we introduce the following equation:

$$Z_t - Z = Z\left(\frac{Z_t}{Z} - 1\right) \simeq z_t + \frac{1}{2}z_t^2.$$

where this equation is used to obtain the well-defined second-order approximation of the utility function.[21]

Now, we consider the second-order approximation of the following periodic utility function:

$$u(C_t) - (1-\gamma)v(N_t) - \gamma v(N_t^*). \tag{A.1}$$

First, calculating the second-order approximation of the right-hand side of Equation (A.1), we obtain

$$U(C_t) \simeq U'(C)C\left[c_t + \frac{(1-\sigma)}{2}c_t^2\right] + t.i.p. + o(\|a\|^3). \tag{A.2}$$

Again note that $t.i.p$ includes terms that are independent of monetary policy, and $o(||\xi||^3)$ indicates the terms of third or higher orders. Substituting the log-linearization of consumption and Equation (30), Equation (A.2) is rewritten as follows:

$$\begin{aligned} U(C_t) \simeq U'(C)C\bigg\{&(1-\gamma)(1-\phi_n)y_t + \sigma(1-\phi_n^*)y_t^* + \frac{(1-\sigma)}{2}[(1-\gamma)^2(1-\phi_n)^2 y_t^2 \\ &+ 2(1-\gamma)\gamma(1-\phi_n)(1-\phi_n^*)y_t y_t^* + \gamma^2(1-\phi_n^*)^2(y_t^*)^2]\bigg\} + t.i.p. + o(\|a\|^3). \end{aligned} \tag{A.3}$$

Next, the second-order approximation of the second and third terms of the right-hand side of Equation (A.1) is calculated by

$$V(N_t) \simeq V'(N)N\left[n_t + \frac{(1+\eta)}{2}n_t^2\right] + t.i.p. + o(\|a\|^3), \tag{A.4}$$

$$V(N_t^*) \simeq V'(N)N\left[n_t^* + \frac{(1+\eta)}{2}(n_t^*)^2\right] + t.i.p. + o(\|a\|^3). \tag{A.5}$$

The log-lineraization of the aggregate production function for both countries is given by

$$n_t = a_t + y_t + \log\Delta_t, \tag{A.6}$$

$$n_t^* = a_t^* + y_t + \log\Delta_t^*, \tag{A.7}$$

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[21]See Walsh (2010) for a detailed explanation of using this equation in the derivation of the central bank's loss function.

where $\log \Delta_t$ and $\log \Delta_t^*$ denote the home and foreign log price dispersion term caused by the distortion of nominal rigidity, respectively. Combining (A.4) with (A.6) and also doing so for the foreign country's counterpart, we obtain

$$V(N_t) \simeq V'(N)N\left[y_t - a_t + \log \Delta_t + \frac{1}{2}(1+\eta)(y_t - a_t + \log \Delta_t)^2\right] + t.i.p. + o(\|a\|^3), \quad \text{(A.8)}$$

$$V(N_t^*) \simeq V'(N)N\left[y_t^* - a_t^* + \log \Delta_t^* + \frac{1}{2}(1+\eta)(y_t^* - a_t^* + \log \Delta_t^*)^2\right] + t.i.p. + o(\|a\|^3). \quad \text{(A.9)}$$

Substituting (A.3), (A.8) and (A.9) into periodic worldwide social welfare (A.1) leads to

$$\begin{aligned} U_t^W &= U(C_t) - (1-\gamma)V(N_t) - \gamma V^*(N_t^*) \\ &\simeq U'(C)C\Bigg\{\frac{(1-\sigma)}{2}\Big[(1-\gamma)(1-\phi_n)y_t + \gamma(1-\phi_n^*)y_t^*\Big]^2 \\ &\quad - (1-\gamma)(1-\phi_n)\left[y_t - a_t + \log \Delta_t + \frac{1}{2}(1+\eta)(y_t - a_t + \log \Delta_t)^2\right] \\ &\quad - \gamma(1-\phi_n^*)\left[y_t^* - a_t^* + \log \Delta_t^* + \frac{1}{2}(1+\eta)(y_t^* - a_t^* + \log \Delta_t^*)^2\right]\Bigg\} \\ &\quad + t.i.p. + o(\|a\|^3), \end{aligned} \quad \text{(A.10)}$$

where we used the fact that the second equality follows the optimal condition $V'(N)N = (1-\phi_n)U'(C)C$ due to the presence of the optimal subsidies.

We consider the second-order approximation of price dispersion term using the following lemma, which relates price dispersion to variance in prices:

Lemma 1 *(Gali, 2015) The following properties hold:*

$$\log \Delta_t \approx \frac{\xi_p}{2} var_i(P_{H,t}), \quad \text{(A.11)}$$

$$\log \Delta_t^* \approx \frac{\xi_p}{2} var_i(P_{F,t}^*). \quad \text{(A.12)}$$

Proof. See Galí (2015). ■

Now, we rewrite the logarithm of the variance of prices as $\delta_{p,t} = \log var_i(P_{H,t})$. Then, we obtain the following variance of prices:

$$\begin{aligned} \delta_{p,t} &= var_i(P_{H,t}) = var_i[\log(P_{H,t}(i)) - \bar{P}_{H,t-1}] \\ &= E_i[\log(P_{H,t}(i)) - \bar{P}_{H,t-1}]^2 - [E_i \log P_{H,t}(i) - \bar{P}_{H,t-1}]^2 \\ &= \omega E_i[\log(P_{H,t-1}(i)) - \bar{P}_{H,t-1}]^2 + (1-\omega)(\log P_{H,t}^o - \bar{P}_{H,t-1})^2 - (\bar{P}_{H,t} - \bar{P}_{H,t-1})^2, \end{aligned} \quad \text{(A.13)}$$

where $\bar{P}_{H,t}$ denotes the mean of prices. Also,

$$P_{H,t} = (1-\omega)\log P^o_{H,t} + \omega \bar{P}_{H,t-1}$$
$$\bar{P}_{H,t} - \bar{P}_{H,t-1} = (1-\omega)\log P^o_{H,t} - (1-\omega)\bar{P}_{H,t-1}$$
$$\log P^o_{H,t} - \bar{P}_{H,t-1} = \frac{1}{1-\omega}(\bar{P}_{H,t} - \bar{P}_{H,t-1})$$

Then from Equation (A.13), we obtain

$$\begin{aligned}\delta_{p,t} &= \omega\delta_{p,t-1} + \frac{1}{1-\omega}(\bar{P}_{H,t} - \bar{P}_{H,t-1})^2 - (\bar{P}_{H,t} - \bar{P}_{H,t-1})^2 \\ &= \omega\delta_{p,t-1} + \frac{\omega}{1-\omega}(\bar{P}_{H,t} - \bar{P}_{H,t-1})^2.\end{aligned}$$

Since $\bar{P}_{H,t} = \log P_{H,t} + +o(\|\xi\|)^2$, we finally obtain the relationship between variance of prices to the inflation rate as follows:

$$\delta_{p,t} = \omega\delta_{p,t-1} + \frac{\omega}{1-\omega}\pi_t^2 + o(||\xi||^3).$$

Similarly, price dispersion in the foreign country evolves to

$$\delta^*_{p,t} = \omega^*\delta^*_{p,t-1} + \frac{\omega^*}{1-\omega^*}(\pi^*_t)^2 + o(||\xi||^3).$$

Then, recursively substituting the above equations from period t to infinity and taking the discounted sum of that, we obtain the relationship between price dispersion and the inflation rate:[22]

$$\sum_{t=0}^{\infty}\beta^t\delta_{p,t} = \frac{\omega}{(1-\omega)(1-\omega\beta)}\sum_{t=0}^{\infty}\beta^t\pi_t^2 + t.i.p. + o(||\xi||^3). \tag{A.14}$$
$$\sum_{t=0}^{\infty}\beta^t\delta^*_{p,t} = \frac{\omega^*}{(1-\omega^*)(1-\omega^*\beta)}\sum_{t=0}^{\infty}\beta^t(\pi^*_t)^2 + t.i.p. + o(||\xi||^3). \tag{A.15}$$

Substituting Equations (A.14) and (A.15) into the expected discounted sum of Equation (A.10), we finally obtain the following expression:

$$\begin{aligned}U_t^W \simeq -\frac{U'(C)C}{2}E_0\sum_{t=0}^{\infty}\beta^t\Big\{&(1-\gamma)(1-\phi_n)\kappa(y_t - y_t^n)^2 + \gamma(1-\phi_n^*)\kappa^*(y_t^* - y_t^{*,n})^2 \\ &+ (\sigma-1)(1-\gamma)(1-\phi_n)\gamma(1-\phi_n^*)(y_t - y_t^n)(y_t^* - y_t^{*,n}) + \frac{(1-\gamma)(1-\phi_n)\xi_p}{\zeta}\pi_t^2 \\ &+ \frac{\gamma(1-\phi_n^*)\xi_p}{\zeta^*}\pi_t^{*2}\Big\} + t.i.p. + o(\|a\|^3).\end{aligned}$$

After several simple manipulations, we obtain the central bank's loss function (44) in the main text.

[22] See Woodford (2003) for a detailed derivation of the following equation.

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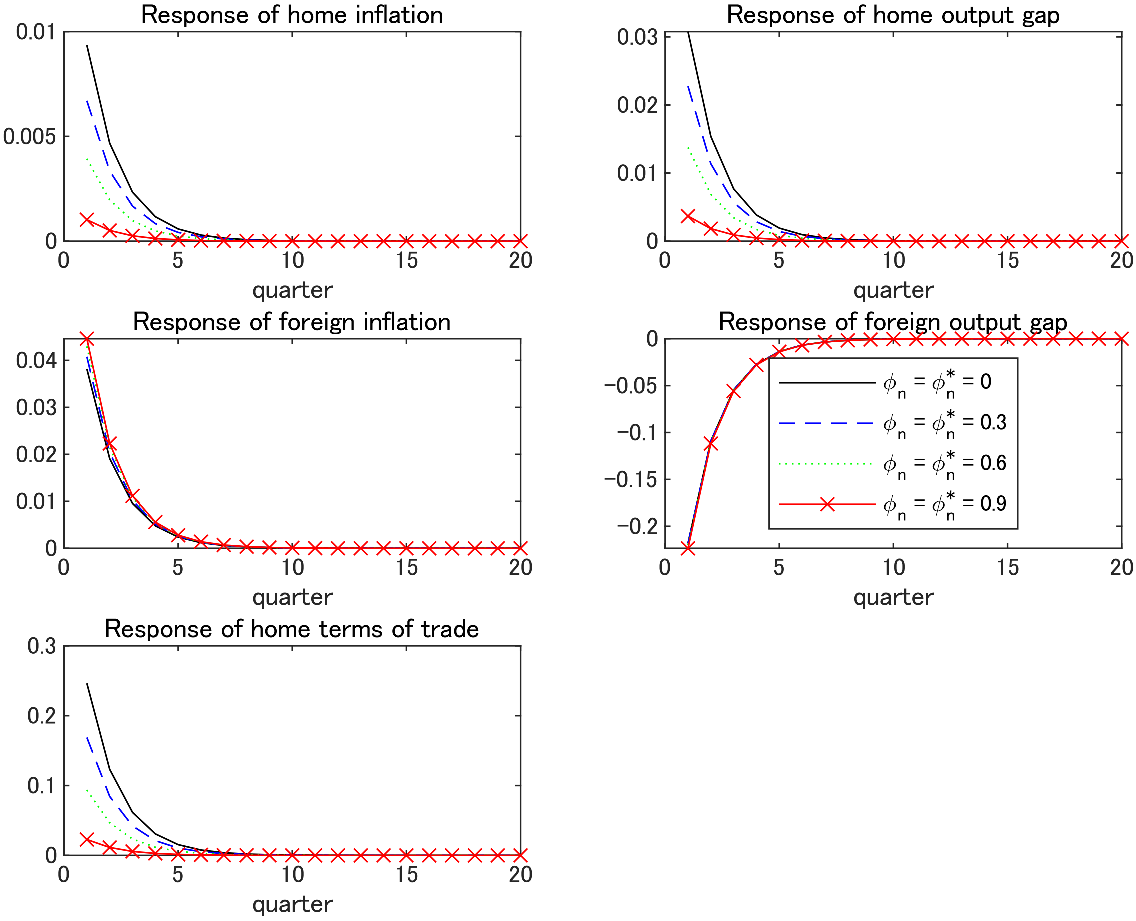

Figure 1: Impulse response to a foreign cost-push shock under discretion

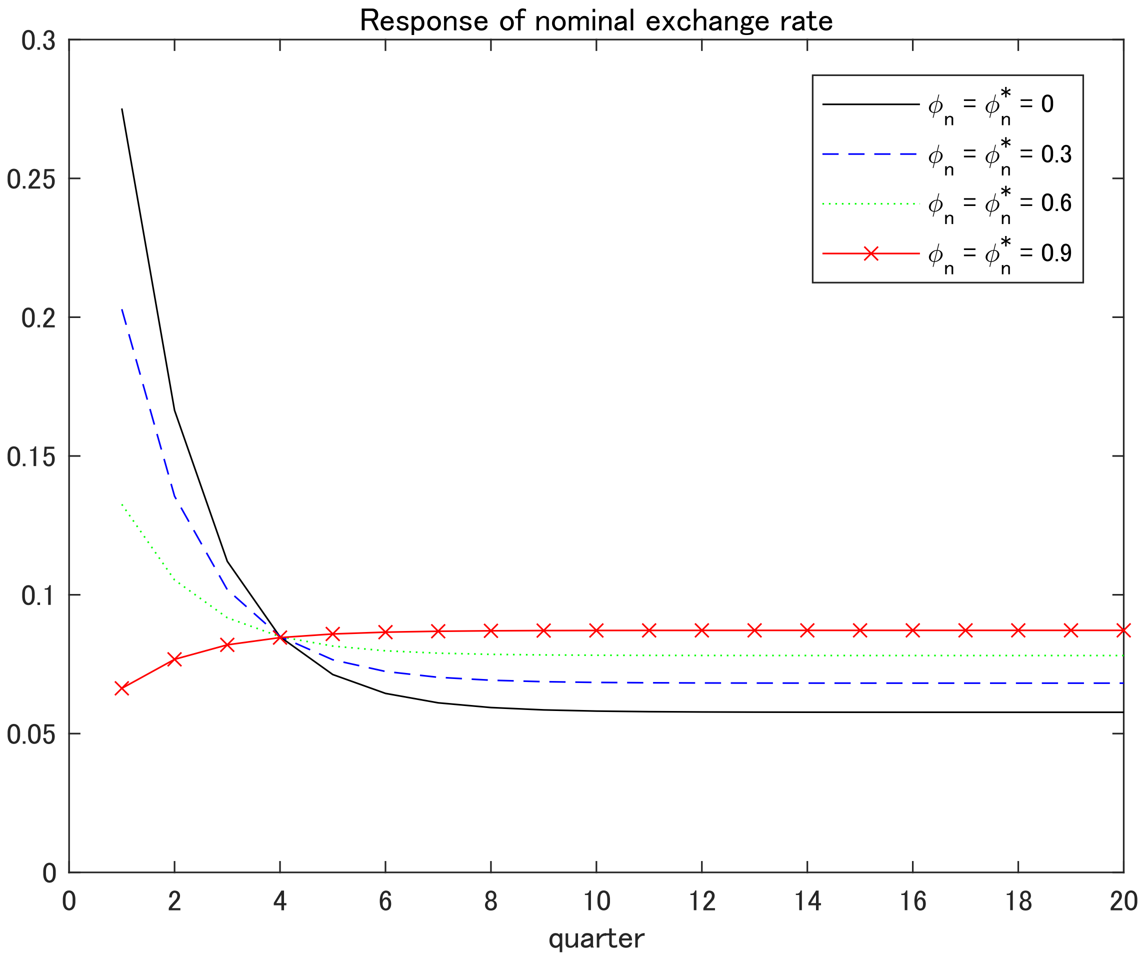

Figure 2: Impulse response to the exchange rate to a foreign cost-push shock under discretion

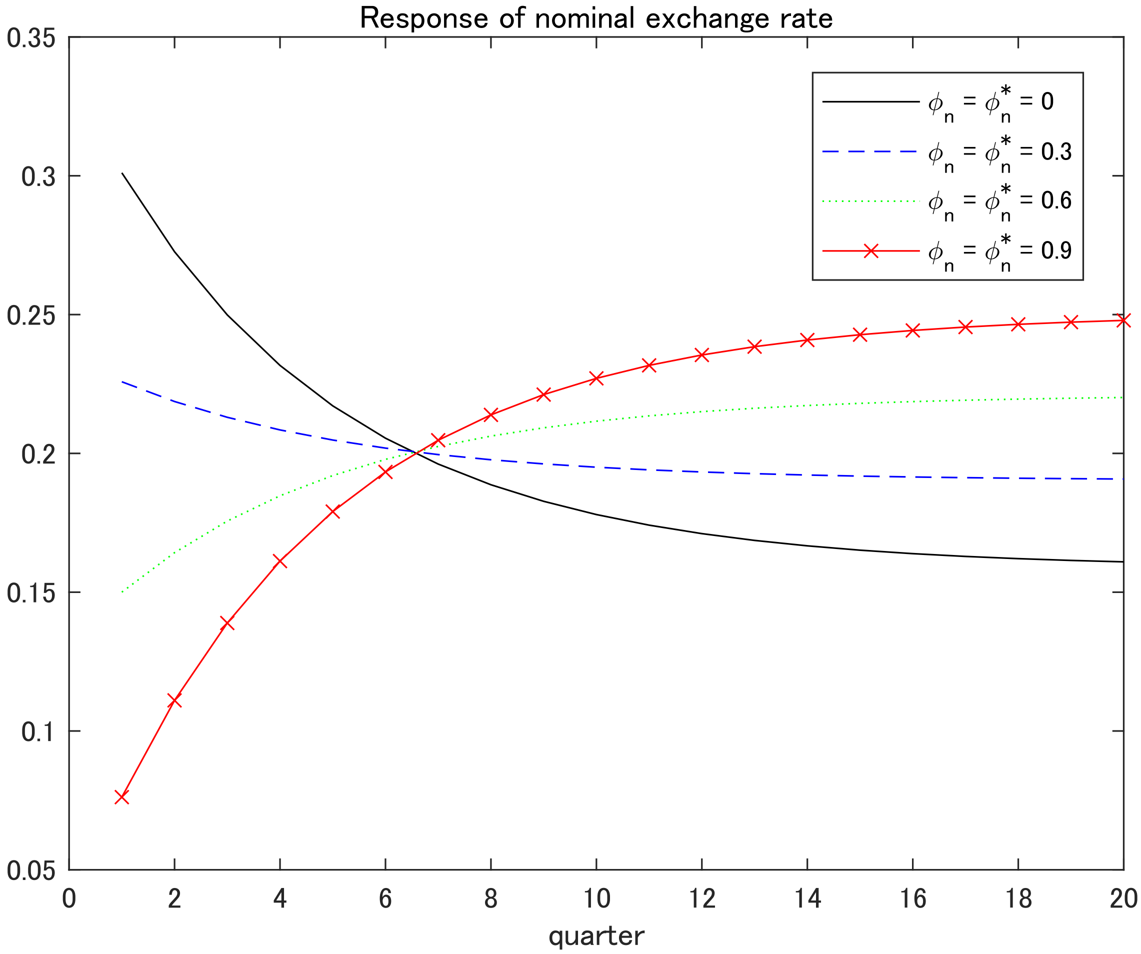

Figure 3: Impulse response of the nominal exchange rate to a foreign cost-push shock under discretion: $\rho = 0.8$

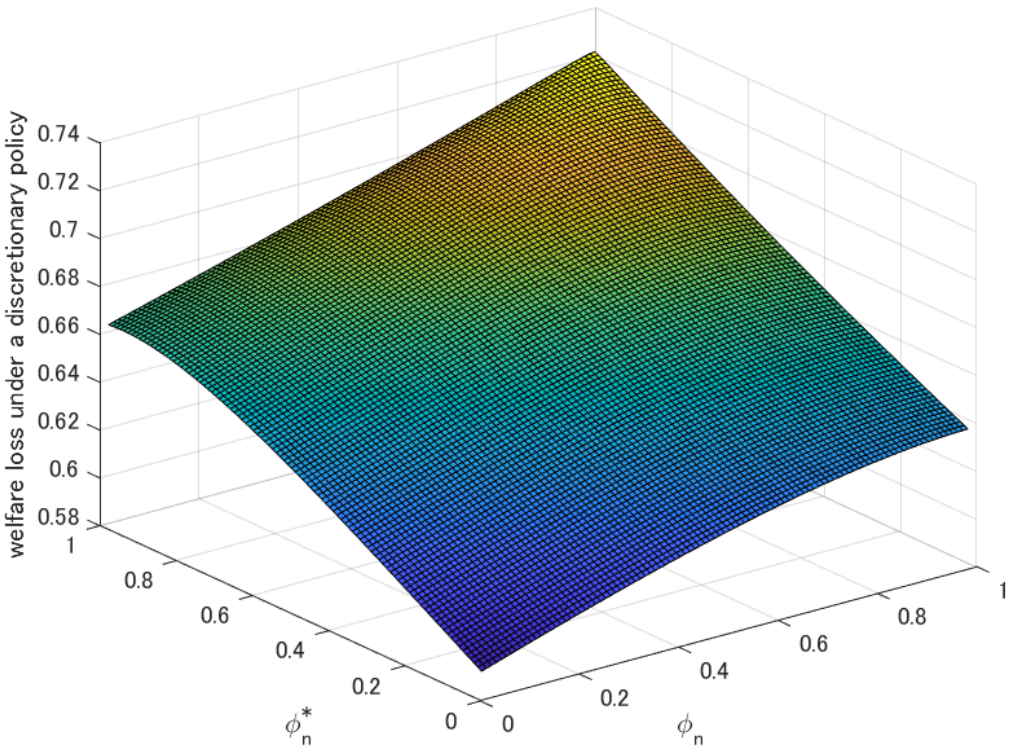

Figure 4: Welfare loss under discretion: home and foreign cost-push shocks

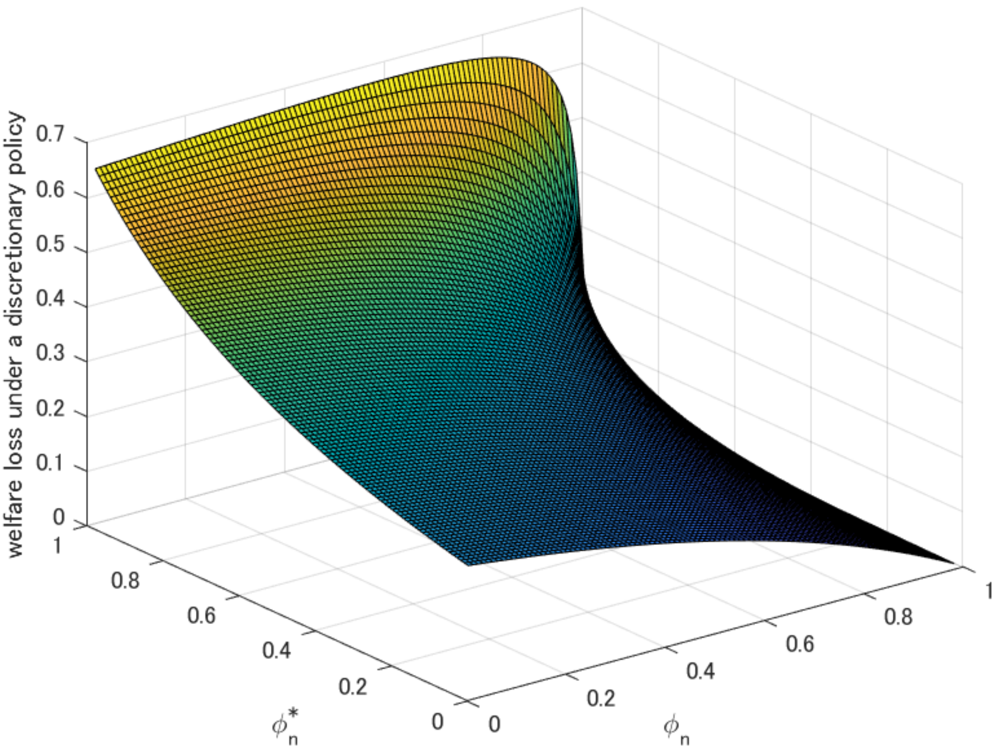

Figure 5: Welfare loss under discretion: only foreign cost-push shocks

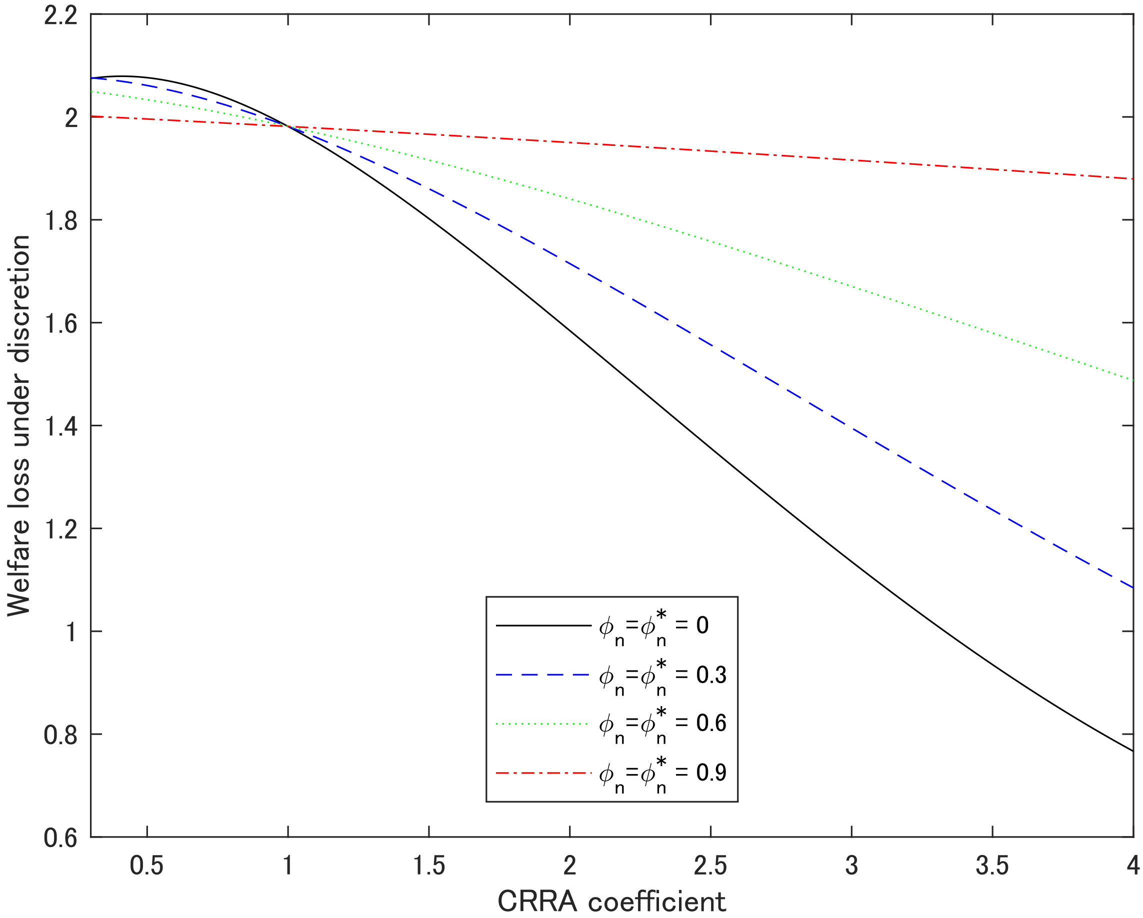

Figure 6: Welfare loss under discretion and CRRA coefficient

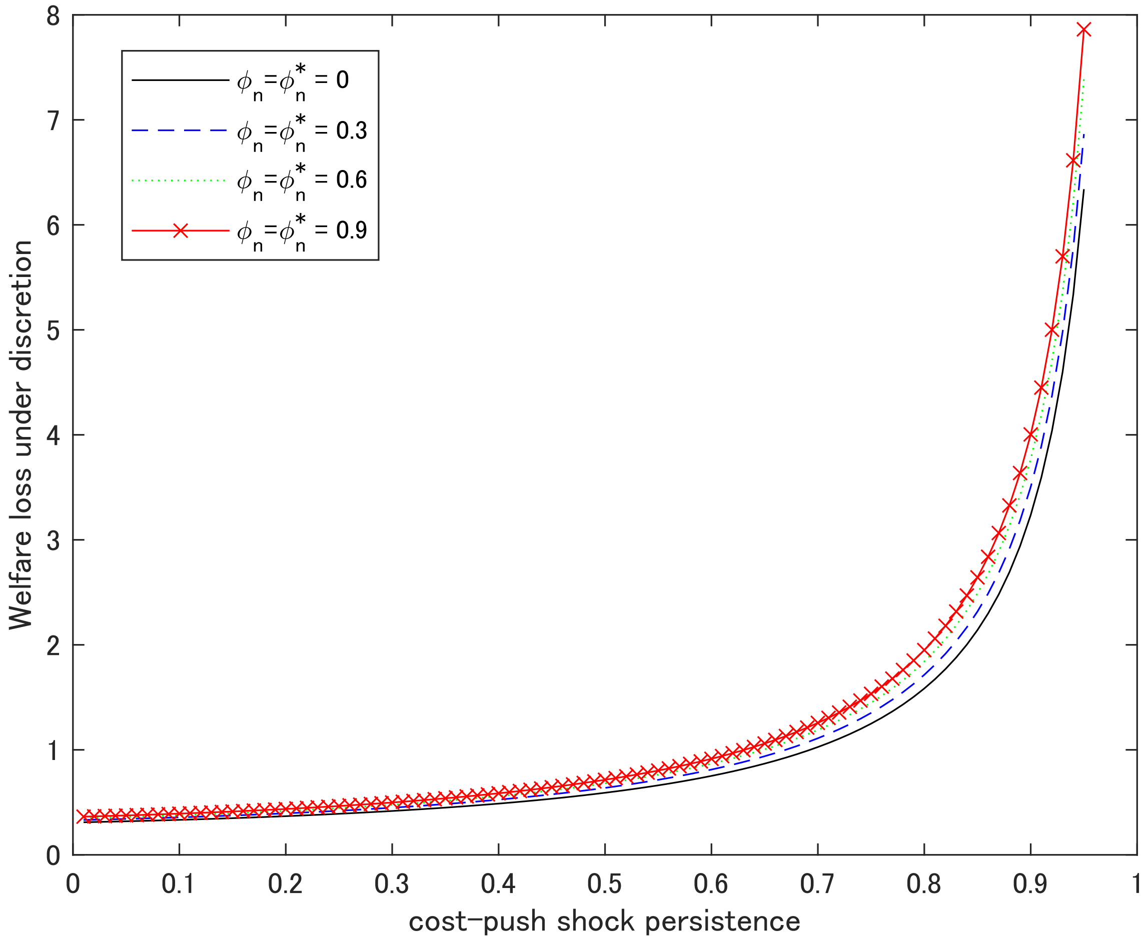

Figure 7: Welfare loss under discretion and persistence of cost-push shock

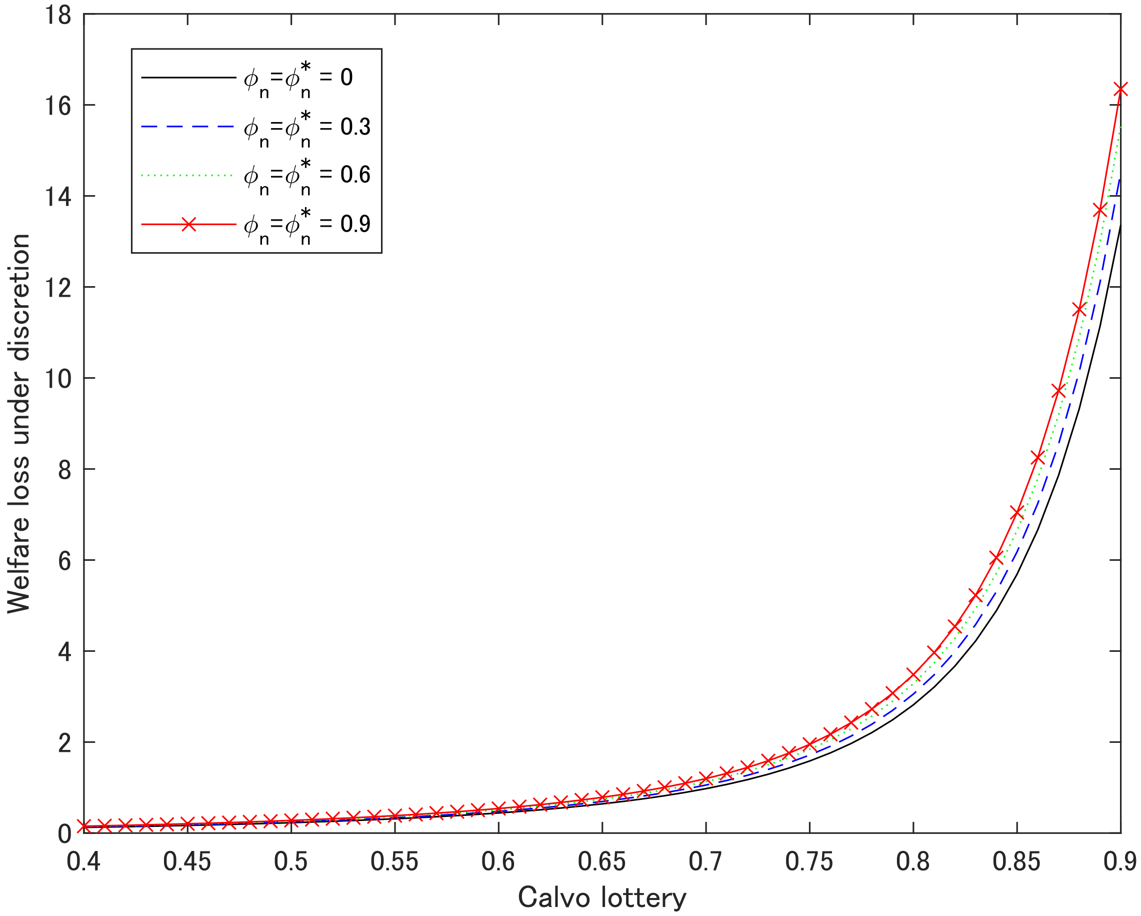

Figure 8: Welfare loss under discretion and degree of Calvo lottery

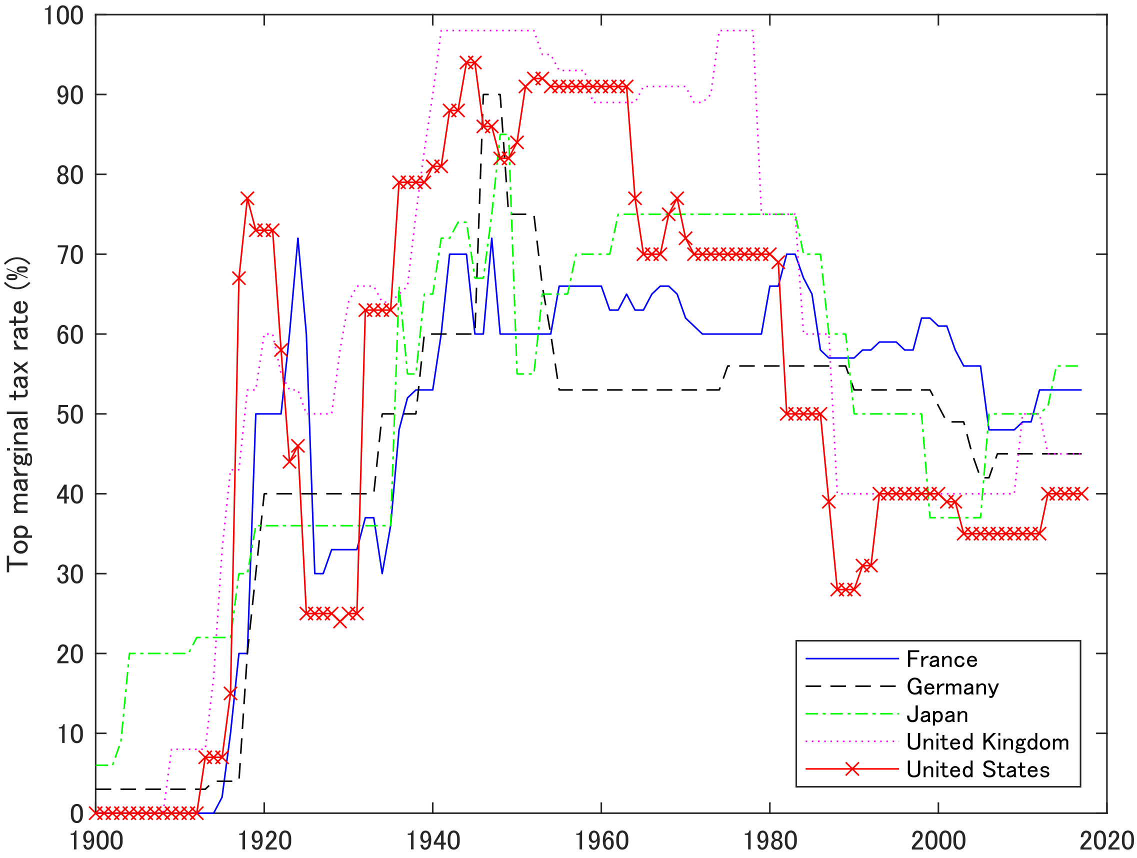

Figure 9: Top income tax rates in rich countries

Source: Alvaredo et al. (2018)