〔共同研究：ことばと論理Ⅳ〕

The Square of Opposition with "most" and "many"[1)]

Shin-ichi SHIMIZU*

§ 1　"All" (and some of its analogues like "every") and "some" are traditionally well-known and well-established quantifiers: and it might as well be reasonable to count "most", "few", and "many" as another set of quantifiers as well. Logical traditions, formal or symbolic, have been enjoying their own "squares of opposition" on the basis of categorical propositions. We claim that another "square of opposition" is formed with respect to a pair of "most" and "many" as well, and it is incorporated into a traditional "square of opposition", which produces a larger "square of opposition". In what follows some of the argumentations will be presented from a logical and modern semantical perspective.

§ 2　Traditional formal logic recognizes four categoricals; universals, existentials, negative universals, and negative existentials. As shown in Figure 1, four logical relations among those categoricals are established; contradiction, contrary, subcontrary, and subalternation.

What modern symbolic logic counts is the "square of opposition" with nothing but contradiction.

Take affirmative universal (1a) for example. With modern symbolic logic in mind, this general proposition is conventionally translated into (1b).

(1) a.　All humans are mortal.

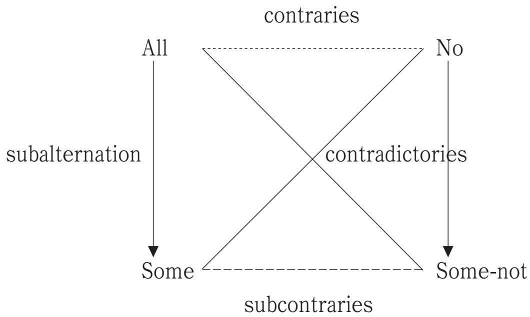
b.　$(\forall x)[H(x) \rightarrow M(x)]$

This move makes it possible to hold (1a) true even when the subject term designates an empty set because of material implication in the scope. This, however, gives rise to a problem. (1b) renders it hard to treat syntactic form and logical representation in an isomorphic fashion, and maintain compositional perspective on semantic interpretation. Most remarkably, the conditional in the logical form (1b) is not to be found in syntactic form (1a) at all.

To solve this problem, Barwise and Cooper (1981) [henceforth B & C] proposes that NP in (2) is to be treated as qauntifier.

(2)　　$[_S [_{NP}$ Q N$] [_{VP}$ ....$]]$

*文学部



(Figure 1)

This means that a quantifier Q is regarded as a set of sets, and is captured to be a binary relation on set A and set B, so that Q_E A B and $Q_E \in \mathcal{P}(\mathcal{P}(E))$ (where E is a domain of objects or entities and $\mathcal{P}(E)$ is a power set of E: the quantifier so characterized is labeled as generalized quantifier Q_E.)

Eijck (1984) and Westerståhl (1985) elaborate upon characteristics of Q_E in the spirit of B & C. It has a property of Extension (Westerståhl uses a different terminology Constancy); given the domain E which includes subsets A and B, and E' which in turn includes E, the effect of binary relation between A and B on E is equivalent to the one on E'. This means that if the domain E extends, say, C, whose property is distinct from those of A and B, is added to E, the binary relation between A and B is not affected, nor is it on the contrary if the domain E' is reduced to E. In other words, Q_E in question does not count the size of the domain.

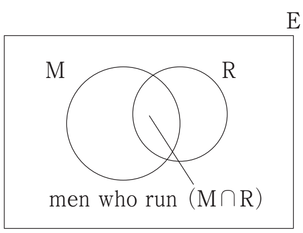
The second property of Q_E is conservativity. Observe the following.

(3) a. Every man runs.

b. Every man is man who runs.

When we see if (3) is true or not in a given model, we focus only on entities which have a property of being man and those which have a property of running, that is, specific subsets in the domain E. Other entities in E are not involved in evaluation of (3). (3a) corresponds to (3b) in interpretation. The following general condition is thus imposed on the determiner of NP.

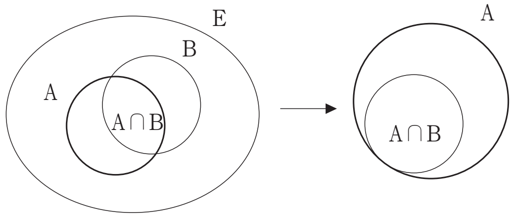
(4) conservativity: If A, B$\in$E then D_E A B $\leftrightarrow$ D_E A$(A \cap B)$



(Figure 2)

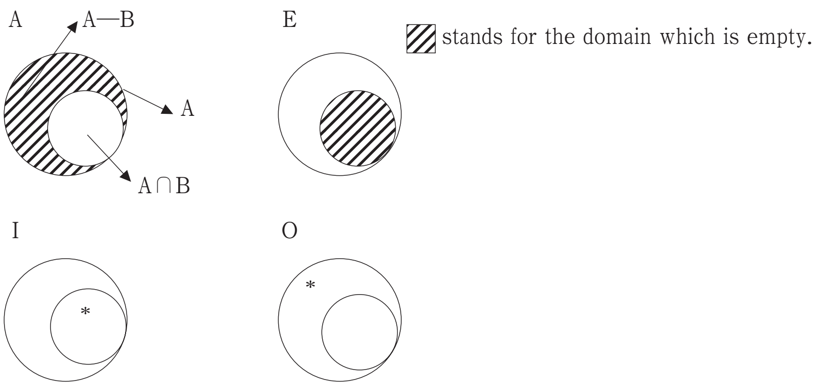
This allows us to show some relations between subsets on E. (3b) or (3a) is shown as a EVERY relation between M and (M∩R); i.e. MR' does not contain any member (where R' stands for the complement of R).

Now combination of extension and conservativity makes it possible to do away with parameter E, and instead "live on" only A (B & C); i.e. nothing but a subset A can be provided to "set a stage" for Q_E (Eijck). The truth of Q_E A B is evaluated only on the basis of A and the intersection A∩B, and with cardinality of the set in mind, only on | A | and | A∩B |. This property is called strong conservativity. This idea might be more intuitively captured via the following Venn diagram Figure 3.



(Figure 3)

If so, our traditional square of opposition is represented as in Figure 4, with the "stage" set in terms of only A.



(Figure 4)

The superset and subset of Figure 3 make it easier for us to have a better grasp of monotonicity. Now observe the following.

(5) a. All men walked.

b. All men walked slowly.

Entailing relations between them are as follows:

(6) a. All men [$_{VP_1}$ walked] $\nRightarrow$ All men [$_{VP_2}$ walked slowly]

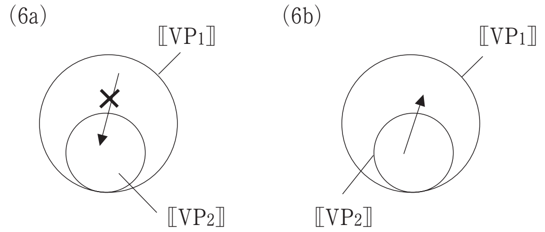
b.　All men [$_{VP_2}$ walked slowly] ⊨ All men[$_{VP_1}$ walked]

The definition of monotone quantifiers is provided in B & C as follows:

(7)　A quantifier Q is *monotone increasing* (mon↑) if $X \in Q$ and $X \subseteq Y \subseteq E$ implies $Y \in Q$ (i.e. for any set $X \in Q$, Q also contains all the supersets of X.)

(8)　Q is *monotone decreasing* (mon↓) if $X \in Q$ and $Y \subseteq X \subseteq E$ implies $Y \in Q$ (i.e. for any set $X \in Q$, Q also contains all the subsets of X.)

Monotonicity of (6a, b) is thus represented as in Figure 5, where ⟦VP_2⟧ (a set of things having a property of walking slowly) is subordinate to ⟦VP_1⟧ (a set of things having a property of walking), and it shows us that universal affirmatives are of upward entailment, i.e. monotonic increasing.



(Figure 5)

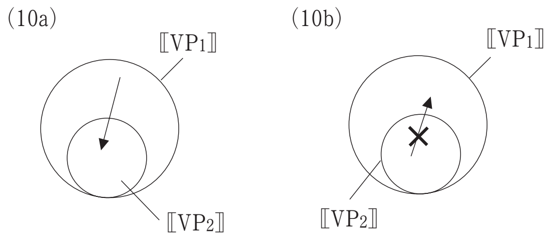
(9)　a.　No girl drinks heavily.

b.　No girl drinks.

(10) a.　No girl [$_{VP_1}$ drinks] ⊨ No girl [$_{VP_2}$ drinks heavily]

b.　No girl [$_{VP_2}$ drinks heavily] ⊭ No girl [$_{VP_1}$ drinks]

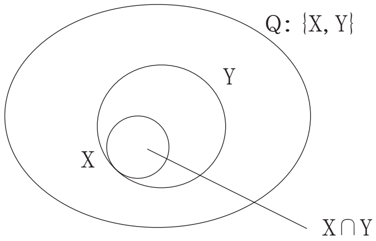
"Drink heavily" is subordinate to "drink". Hence we get the diagram for (9a, b) such as:



(Figure 6)

This indicates that universal negatives are of downward entailment, i.e. monotonic decreasing.

§3　Judgment of entailment of (6) and (10) depends upon only our semantic knowledge. We will see if we can have a more formal test of monotonicity. Again take up B & C's definition (7), and see what it has to say. It says that a necessary and sufficient condition for upward monotonicity is "if X∈Q and X⊆Y, then Y⊆Q". This means that "if X∩Y∈Q, then X∈Q and Y∈Q."



(Figure 7)

And assume that Y=⟦VP$_1$⟧ and X=⟦VP$_2$⟧, and this gives us (11a).

(11) a. NP (VP$_1$ and VP$_2$) ⊨ NP VP$_1$ and NP VP$_2$

b. (NP VP$_1$ or NP VP$_2$) ⊨ NP (VP$_1$ or VP$_2$)

We can also have (11b) for the test. Why (11b)? (11b) holds true iff (12a), a slightly simplified version of B & C's (7), is equivalent to (12b).

(12) a. Q is monotonic increasing iff for all X, Y⊆E: if X∈Q and X⊆Y, then Y∈Q.

b. Q is monotonic increasing iff for all X, Y⊆E: if X∈Q or Y∈Q, then X∪Y∈Q.

Assume that Q satisfies the condition that "if X∈Q and X⊆Y, then Y∈Q" with respect to (12a). And select X or Y such that X∈Q and Y∈Q. Then does "X∪Y∈Q" hold? It is evident that Figure 7 will provide us a positive answer. This means that the implication (12a)→(12b) holds. Now assume that Q satisfies "if X∈Q or Y∈Q, then X∪Y∈Q" with respect to (12b); and select X and Y such that X∈Q and X⊆Y: Then, does "Y∈Q" hold? It does, indeed. We will get the implication (12b)→(12a); hence a mutual implication. (12b) does hold true. This means that we have (11b) if X and Y are substituted for ⟦VP$_1$⟧ and ⟦VP$_2$⟧ respectively. Now see if (11a, b) correctly predict upward monotonicity of, say, "all".

(13) a. All students were singing and dancing.

⊨ All students were singing and all students were dancing.

b. All students were singing or all students were dancing.

⊨ All students were singing or dancing.

(14) a. Some girls were smoking and drinking.

⊨ Some girls were smoking and some girls were drinking.

b. Some girls were smoking or some girls were drinking.

⊨ Some girls were smoking or drinking.

(15) a. Not a single cat was singing and dancing.

⊭ Not a single cat was singing and not a single cat was dancing.

b. Not a single cat was singing or dancing.

$\nvDash$ Not a single cat was singing or not a single cat was dancing.

Entailment of each pair of (13) and (14) shows that quantifiers "*all*" and "*some*" are of upward entailment; whereas, from (14), a quantifier "*not a single* CN" is found to be of downward entailment. Similarly, from (8), we will have (16a, b) as for downward monotonicity.

(16) a. NP (VP_1 or VP_2) $\vDash$ NP VP_1 and NP VP_2

b. (NP VP_1 or NP VP_2) $\vDash$ NP (VP_1 and VP_2)

(16) predicts that "no" is of downward entailment. Observe (17), which shows a sharp distinction from (18) where none reveals valid entailment.

(17) a. No girl prayed or knelt.

$\vDash$ No girl prayed and no girl knelt.

b. No girl prayed or no girl knelt.

$\vDash$ No girl prayed and knelt.

(18) a. All girls prayed or knelt.

$\nvDash$ All girls prayed and all girls knelt.

b. All girls prayed or all girls knelt.

$\nvDash$ All girls prayed and knelt.

So far we have dealt with "all", "some", and "no"; i.e. universal affirmative and negative, and particular affirmative. The question in point is what monotonicity particular negative, or O-proposition entertains. We will return to this question later.

When it comes to the square of opposition, negation plays a crucial role in scrutinizing the nature of categoricals. Q_E we have been discussing is symbolized as:

(19) $Q_E = \{X \subseteq E \mid X \in Q\}$

(19) claims that Q_E is a quantifier which is a set of X which constitutes a subset of the universe E of objects such that the X is a member of Q: Q_E is a set of sets, after all. Now according to B & C's definition, we define the external negation (20) on the basis of (19).

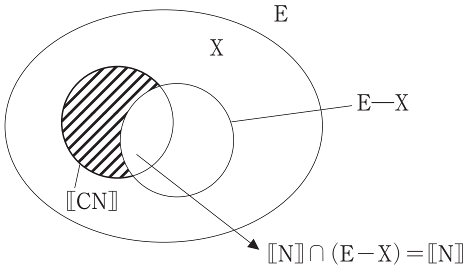
(20) $\neg Q = \{X \subseteq E \mid X \notin Q\}$

In other words, $\neg Q$ is equivalent to a difference $\mathcal{P}(E) - Q$. The second type of negation is internal negation which corresponds a negative particular, and it is defined as in (21).

(21) $Q\neg = \{X \subseteq E \mid (E - X) \in Q\}$

(21) says that the internal negation is a set of X such that $X^C \in Q$ (where X^C stands for the complement of X).

Now let us ask what are the external and internal negations of **all** and **some**, respectively; to put it another way, what are $\neg$**all** and **all**$\neg$, and $\neg$**some** and **some**$\neg$. First we consider $\neg[\![\text{all CN}]\!]$. Given (20), this reduces to $\{X \mid X \notin [\![\text{all CN}]\!]\}$. This gives us a situation like;



(Figure 8)

This diagram shows that thinking in terms of ⟦CN⟧∩X is to claim that if you take any element out of ⟦CN⟧, it never belongs to the set X. This means that ⟦CN⟧$\not\subseteq$X; and therefore ⟦CN⟧∩X ≠⟦CN⟧, which stands for ¬**some**. So we have a series of equivalences like;

¬⟦all CN⟧= {X | X∉⟦all CN⟧} (by (20))

= {X | ⟦CN⟧∩X≠⟦CN⟧}

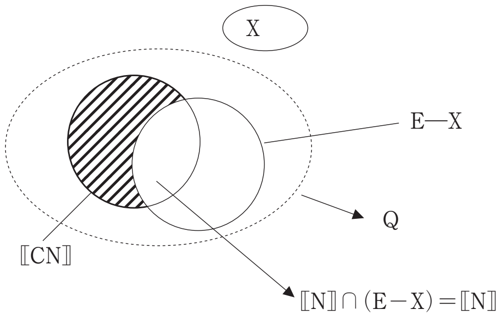
=¬**some**

Now what about the internal negation **all**¬. We get by (21):

⟦all CN⟧ ¬= {X | $\underline{\text{(E−X)∈ ⟦all CN⟧}}$}

Let us think what the underlined description of Q, ⟦all CN⟧ ¬, represents. This gives us a binary relation Q like:

⟦all CN⟧= {⟦CN⟧, E−X}

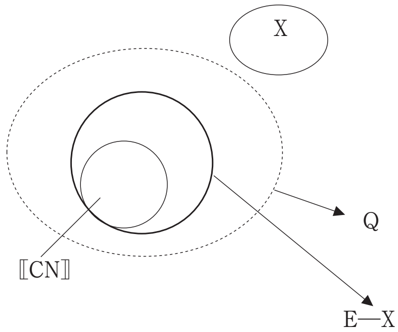


(Figure 9)

Figure 9 is trivially reduced to Figure 10 with ⟦CN⟧⊆ (E−X). Given this, it is evident that ⟦CN⟧∩X=ϕ; this is exactly what ⟦no CN⟧ refers to. This train of reasoning can be thus summarized like;

⟦all CN⟧¬= {X | (E−X)∈ ⟦all CN⟧}

= {X | ⟦CN⟧∩(E−X)=⟦CN⟧}



(Figure 10)

$= \{X \mid \llbracket CN \rrbracket \cap X = \phi\}$

$= \llbracket \text{no CN} \rrbracket$

Similarly, we can be provided with the following train of reasoning as far as ¬**some** is concerned.

$\neg\llbracket \text{some CN} \rrbracket\neg = \{X \mid X \notin \llbracket \text{some CN} \rrbracket\}$ 　　(by (20))

$= \{X \mid \llbracket CN \rrbracket \cap X = \phi\}$

$= \llbracket \text{no CN} \rrbracket$

Finally the internal negation **some**¬ is found to be the same relational effect as ¬**all.**

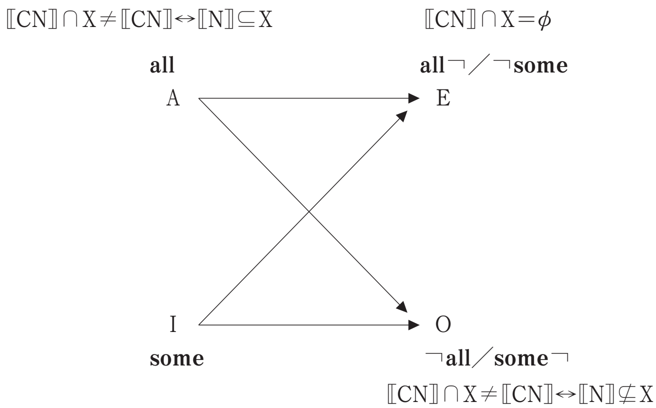
$\llbracket \text{some CN} \rrbracket\neg = \{X \mid (E-X) \in \llbracket \text{some CN} \rrbracket\}$ 　　(by (21))

$= \{X \mid \llbracket CN \rrbracket \cap (E-X) \neq \phi\}$

$= \{X \mid \llbracket CN \rrbracket \cap X \neq \llbracket CN \rrbracket\}$

$= \neg$**all**

It should be recognized that we have established relations of negation, external and internal, among the categoricals A, E, I, and O.



(Figure 11)

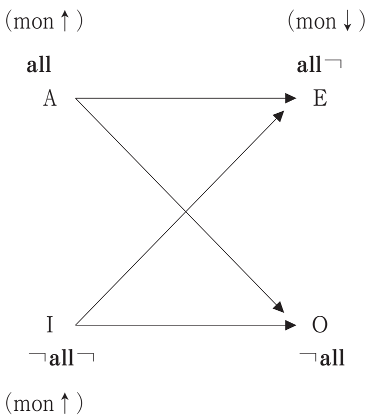
Now let us think of expressing **some** in terms of **all**; that is to say, how to use negations to express **some**. Notice that there is one more possibility of negating a given Q; we could externally negate an internally negated quantified proposition, or vice versa. B & C gives us a formal definition like;

(22) The *dual* of a quantifier Q on E is the quantifier Q* defined by $Q^* = \{X \subseteq E \mid (E-X) \notin (Q\}$, i.e., $Q^* = \sim(Q\sim) = (\sim Q)\sim$.[2)]

It follows from this that we are in a position to place **some** as a dual of **all**, i.e. $(\neg$**all**$)\neg$ or $\neg($**all** $\neg)$.

Nature of negations is very closely related to the directionality of monotonicity. Given the upward entailing nature of Q, the externally and internally negated Q's both have the reverse directionality; and it is the case as well that given the downward entailing nature of Q, they both are of upward entailment. Assume that $Q = \{X \mid X \in Q\}$, and that it is upward entailing. Then what could we say about the monotonic directionality of $\neg Q$? As is shown already, by definition (20), $\neg Q = \{X \mid X \notin Q\}$. Select X and Y such that $X \in \neg Q$ and $Y \subseteq X$. If we assume that $Y \in Q$ since $Y \subseteq X$ and Q is upward entailing, then $X \in Q$, which is inconsistent with the assumption of $X \notin Q$ as with $\neg Q = \{X \mid X \notin Q\}$; and therefore $X \notin Q$ and $Y \notin Q$. This means that $Y \in \neg Q$, and so $\neg Q$ is downward entailing. The internal negation is in order now; and by (21), $Q\neg = \{X \mid (E-X) \in Q\}$. Let us assume that Q is upward entailing, i.e. $X \in Q\neg \wedge Y \subseteq X$. Given this, $(E-X) \in Q$. Since $Y \subseteq X$, it is the case that $(E-X) \subseteq (E-Y)$; hence $(E-Y) \in Q$ since Q is assumed to be upward entailing. This means that $Y \in Q\neg$; and $Q\neg$ is of downward entailment.

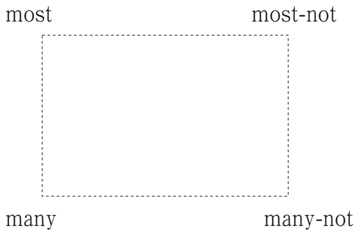
From all above, we can make a claim that the monotonic directionality of **some**, i.e. $\neg$**all**$\neg$, is the same as **all**, which is upward entailing, (mon↑). The internal negation **all**$\neg$ is then downward entailing (mon↓), and the external negation of **all**$\neg$ makes us a dual of **all** ($\neg$**all**$\neg$), which should be in turn (mon↑).



(Figure 12)

It is true of right monotonicity that universal affirmative entails particular affirmative, in that they enjoy the same directionality. Now the negation of **all** makes ¬**all** (mon↓), which assures us that universal negative entails particular negative because of their same directionality (mon↓). From this it follows that the dual of Q shares the same directionality as the one of Q, which assures entailment from universal to particular.

§4. Now with “right-monotonicity” scrutinized for “standard” quantifiers, we are in the position to see how to position “most” and “many” into the square of opposition. First, asking whether the Figure 13 holds or not is in order.



(Figure 13)

Observe the following pair:

(23) a. Many philosophers are liberal.

b. Many philosophers are philosophers who are liberal.

(23a) is derived from (23b) via deletion of redundant “philosophers who are”. (23b) refers to P (a set of things having a property of being philosophers) and P∩L (intersection of P and L, which is a set of things having a property of being liberal), and if either one of the following two descriptions in terms of cardinality is satisfied, conservativity of the quantified expressions with “many” remains true and monotonicity is thus guaranteed to hold good (cf. Westerståhl 1985):

(24) a. $|P \cap L| > k \cdot |P|$ (where k is a constant and $0 < k < 1$)

b. $|P \cap L| > f(|E|)$ (where E is a universe of a given model)

The latter guarantees that fewer than $f(|E|)$ is never “many”. Monotonic directionality of “many” is provided via the above-mentioned tests with the result of (mon↑).

(25) a. Many MLB players run fast and hit hard.

b. ⊨ Many MLB players run fast and many MLB players hit hard.

(26) a. Many philosophers are liberal or many philosophers are stingy.

b. ⊨ Many philosophers are liberal or stingy.

According to Westerståhl (1985), “most” meets logicality; however, it is ambiguous between “more than half” interpretation and “almost all” interpretation. With the former “most”, (27a) has the interpretation (27b), and with the latter “most”, (27c).

(27) a. Most X's are Y's

b. $|X-Y|<|X\cap Y|$, or $\frac{|X\cap Y|}{|X|}>\frac{1}{2}$

c. $|X\cap Y|\geq k\cdot|X|$ (where possibly $k=\frac{9}{10}$)

With respect to monotonicity, "most" also shows us a monotonically increasing property as the following tested sentences show:

(28) a. Most Americans are both pious and conservative.

b. ⊨ Most Americans are pious and most Americans are conservative.

(29) a. Most Americans are pious or most Americans are conservative.

b. ⊨ Most Americans are pious or conservative.

Now that it turned out that "many" and "most" share the same monotonicity, they are both entitled to entailing relations; and then the next question in order is which quantifier is to be sitting on the "upper deck" or "lower deck" of the square of opposition. Our claim is that the Figure 13, where "most" but not "many" sits on the upper deck, is correct.

Keynes (1906) tells us in passing that interestingly enough, we may begin with "few", which should be counted as a member of the set of quantifiers, to shed fresh light upon the problem of where to position "most" in the square, relative to "many". He notices that, in relation to "standard" quantifiers such as "all" and "some", some logicians such as De Morgan and Hamilton treat as quantified propositions "plurative" propositions like;

(30) a. Most S's are P's.

b. Few S's are P's.

and says that "most" is interpreted to have the meaning "at least one more than half" and (30b) is equivalent to (31).

(31) Most S's are not P's.

We could interpret his "most" to be either more or less the same as Westerståhl's "most" with (27b) or the one with (27c) because of liberality of the meaning "at least one more than half". Keynes himself is not definite on this difference. If the latter hold, there could be some proposal in which a quantifier like "almost all" may be stacked up on a quantifier "most", two decks up "many".

Let us begin with examination of (32a) and (32b), an internal negation of (32a).

(32) a. Few philosophers are stingy.

b. Few philosophers are not stingy.

Because of exhaustiveness of "stingy", (32a) provides us with $|P-S|>|P\cap S|$, and (32b) with $|P-S|<|P\cap S|$; which means that there is never the chance that both (32a) and (32b) can not be true at the same time. This corresponds to our "contrariness" in the sense of traditional formal logic.

It turns out that "few" is monotonic decreasing. The following pair of sentences comes from McCawley (1993), who refers to them for discussion of quantifier scope and applicability of Conjunction Reduction.

(33) a. Few rules are both correct and easy to read.

b. Few rules are both correct, and few rules are easy to read.

The derivation of the surface form (33a), involving tough-movement, is not allowed, for we can not regard them as a mutual variants. He attributes this fact to different truth value assigned to each sentence. (33a) is true and (33b) is false, given the situation that "many rules are correct but few of the correct ones are easy to read"; and thus it is impossible for (33a) to entail (33b). This fails in exactly what our test for upward entailment predicts.

Furthermore, Huddleston and Pullum (2002) provides us with a pair of data which indicates the downward entailment in more intuitive sense; it is true that the set of "ignorer of big signs" is a subset of "ignorer of signs", and so "from the fact that (34a) entails (34b)," they say, "we know that *few good drivers* is a downward entailing quantified NP".

(34) a. Few good drivers ignore *signs*.

b. Few good drivers ignore *big signs*.

Our tests, however, find "few" downward entailing on their own ground.

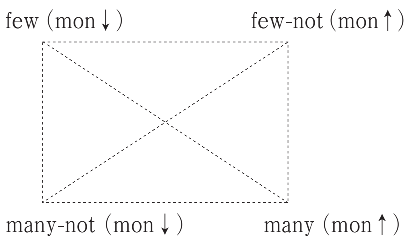
(34) c. Few drivers run slowly or ignore signs.

$\vDash$ Few drivers run slowly and few drivers ignore signs.

d. Few boys smoke or few boys drink.

$\vDash$ Few boys smoke and drink.

This all means that the internal negation of "few", as opposed to "affirmative *few*", as we have shown that a negation, internal or external, will render the monotonicity reversed. It has already turned out that "many" is monotonic increasing and its internal negation monotonic decreasing. Plus "few" shows "contrary" relation with "few-not". This makes it possible to tentatively draw the following square of opposition with "few" sitting on the left "upper deck".



(Figure 14)

Here our concern is with “contradictoriness”. As Declerck (1993) shows, “many” and “few” are used in intensively contrastive manner.

(35) *Many* are called but *few* are chosen.

Truly this, as such, does not necessarily reveal contradiction between “many” and “few”, but it alludes to “contradictoriness” between “many” and “few”. Now observe the following sentences.

(36) a. Not many women work for nonprofit organizations.

b. It is not the case that many women work for nonprofit organizations.

c. Not all people were created equal.

d. It is not the case that all people were created equal.

It is very natural that we take (36a) as external negation (36b) on a par with (36c) vs. (36d); and indeed (36a) has the same interpretation as (37).

(37) Few women work for nonprofit organizations.

Now observe the following;

(38) a. Few merchants are generous.

b. *Not few merchants are generous.

c. Not a few merchants are generous.

Here we just have to take into account English idiosyncrasy. (38b) becomes well-formed if “few” is replaced with “*a few*” in (38c); and thus (38c) results in an external negative variant of (38a).[3] Our observation is that the former contradicts the latter, and this consolidates our picture of contradiction with “many” and “few”.

(39) a. $\neg$**few**$=$**many**

b. **few**$=\neg$**many**

Although we don’t ponder over contradictoriness-bearing negative variants of these quantifiers here, yet what follows will hold (via substitution).

(40) a. $\neg$**few**$\neg=$**many**$\neg$

b. $\neg$**many**$\neg=$**few**$\neg$

Notice that monotonicity also consolidates contradictoriness found in Figure 14. Contradictoriness is guaranteed to hold as a result of externally *negating* a given proposition and negations, as we saw, reverse monotonic directionality in an upside down fashion. This is exactly reflected in what the way in which relevant quantifiers are positioned in Figure 14 represents.

Then we have to ask ourselves why “many” (or “many-not” for that matter) are forced to sit on the “lower deck”. To be definite on this question, “many” and “many-not” must stand in “subcontrary” relation; that is, a sentence with “many” and the one with “many-not” can be both true at the same time. Suppose quantifiers on the upper deck are contraries and they are both false. Plus the categoricals, diagonally positioned across the square, are “contradictory” against each other. Then it must be necessary that both “many” and “many-not” are true, and obviously enough they turn out to be “subcontraries”. This shows that it is considered correct for us to stack up both “few” and a negative variant of *few* on the upper deck, just one storey up from “many” and “many-not”.

One more additional argument for appropriateness of Figure 14 comes from entailing relations. In order for Figure 14 to be correct, (41a-b) and (42a-b) must hold true at the same time.

(41) a. **few** $\vDash$ **many**$\neg$

b. **few**$\neg \vDash$ **many**

(42) a. **many**$\neg \nvDash$ **few**

b. **many** $\nvDash$ **few**$\neg$

Suppose again that “contrariness” holds, i.e. both “few” and “few-not” are false: and then this means “many” and “many-not” are true because of contradiction between “few” and “many-not”, and similarly between “few-not” and “many”. The truth of “many-not”, for example, cannot entail the falsity of “few”, as (42a) indicates.

Now we will set about replacing “few” with “most-not”, and “few-not” with “most” respectively on Figure 14 to reduce it to Figure 13. Interestingly, Figure 13 consolidates itself on an independent ground. As things turns out, “most” is more closely related to “all”, and “many” to existential quantifiers. Now observe that there are some differences in syntactic behavior among these quantifiers.

(43) a. There are *many* students who like baseball.

b. **There are** $\left\{\begin{array}{l}\textbf{a few}\\ \textbf{few}\end{array}\right\}$ **skyscrapers in Kyoto.**

c. There are *some* Japanese who play for MLB.

(44) a. *There are *all* boys playing out in the field.

b. *There are *most* students who distrust LDP.

(45) a. **There are** $\left\{\begin{array}{l}\textbf{more than}\\ \textbf{over}\end{array}\right\}$ **half percent of all students who distrust LDP.**

b. *There are almost all (of the) students who distrust LDP.

The quantifiers in (43a-c) co-occur with *there*-construction and they seem to belong to the same group in the sense that they share existential implication; however, (44) and (45) show that “all” does not turn up in *there*-construction, and “most” resists *there*-insertion. “Most” also shares with “all” a property of not co-occuring with existential construction. “Most” seems to form itself in the same group with “all”, and is plausibly allowed to sit up on the upper deck, one storey down from “all”. As Westerståhl (1985) points out, “most” in “most As are B” can be interpreted as “almost all”, and importantly the proposition with this interpretation is true even when the set designated by the subject term is empty. This is exactly a distinguishing characteristic, that, modern symbolic logicians claim, a standard quantifier “all” has; that is, it does not carry existential implication with it, and this hinders us from *there*-insertion with “all”. Similarly, this is responsible for non-occurrence of “most” in *there*-construction.

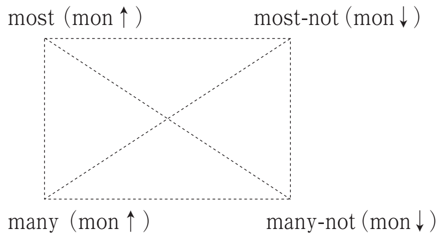
It might be reasonable to presume that “most” in the sense of “few-not” has the “almost all” interpretation. Consider the following:

(46) a. Most players on the team are non-Cubans.

b. Most players on the team are Cubans.

To make a defensible contradictory statement (46a) from (46b), it is necessary that $|P \cap C|$ should be expressly greater than $|P - C|$. Put it another way, under the condition that when $|P \cap C|$ gets closer to ϕ, (46a) should be a defensible contradiction of (46a); otherwise, under the “more than half” interpretation, there is very little difference in cardinality difference between $|P \cap C| > |P - C|$ and $|P \cap C| < |P - C|$, and (46a) is just not good enough to label it a defensible contradiction.

Here we can transform Figure 13 into the following rearranged Figure 15, with affirmatives positioned on the left column.



(Figure 15)

“Most” and its denial, i.e. external negation of “most”, stand in contradictory relation. $|S - P| < |S \cap P|$ is true with “most Ss are P”. As far as “$\neg$(most Ss are P)” is concerned, $|S - P| \not< |S \cap P|$ holds, which means $|S - P| > |S \cap P|$. This is bluntly expressed by (47), and is easily illustrated by a pair of sentences in (48).

(47) Many Ss are not P.

(48) a. It is not the case that most children like hot chocolate.

b. Many children don’t like hot chocolate.

Given that $\neg$**most=many**$\neg$, “most” and “many-not” are contradictory against each other. Notice also that monotonicity directionality is diagonally opposite, and this gives us further supporting evidence that “most” and “many-not” are contradictories.

“Most” and “most-not”, independently of how large or small the set designated by the subject term is, cannot be both true at the same time.

(49) a. Most of his classmates are happy with their results of the final.

b. Most of his classmates are not happy with their results of the final.

(50) a. Most of Americans are satisfied with the result of the last presidential election.

b. Most of Americans are not satisfied with the result of the last presidential election.

Each pair of (49) and (50) cannot be true at the same time, respectively.

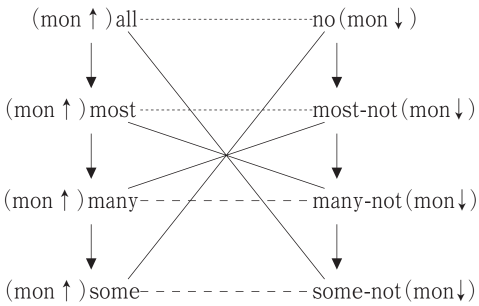
"Most", on the square of Figure 15, entails "many". As is shown in (43a), "many" appears in *there*-construction, and sure it does have existential implication. The same is true of "some". It generally is interpreted as "there is *at least one* x such that ...": and thus "*one or more*". That means that "some" enjoys liberality of interpretation. So does "many", and thus it has the interpretation: "there are *many or more* x's such that ...", which allows "many" to be entailed by "most".

There is another way of verifying that "most", on the square of Figure 15, entails "many". Assume that "most" does not entail "many". This means that the categorical with "most" is true; whereas the one with "many" is false. Since "many" and "most-not" stand in contradictory relation as we have seen, "most-not" falls into truth if "many" is false. Then we come to the conclusion that both "most" and "most-not" are true. As we have seen, and by the nature of square of *opposition* as well, "most" and "most-not" are in "contrary" relation by which it is meant that both of them cannot be simultaneously true: and therefore, by *reductio ad absurdum*, the initial assumption is false that "most" *does not* entail "many". "Most" *does indeed* entail "many".

§5. Conclusion. To integrate or incorporate "most" and "many" into the traditional square of opposition, we have made good use of one of the developments of formal semantics, i.e. monotonicity in the study of generalized quantifiers ranging from "standard" ones like "all" and "some" to "non-standard" ones like "most" and "many". We examined methods of testing monotonic directionality, and validated them.

We have taken notice of Keynes's observation, i.e. an idea of "few" as an equivalent counterpart of "most-not", by way of setting about our task of organizing the traditional square of opposition with "most" and "many" included in there. It turned out to be good and easy access to our reasoning. Not fully complete though it may be, our conclusion is Figure 16 with directionality of monotonicity onto each quantifier. Any quantifier on either column is in opposition to the corresponding counterpart on the other end of line, which is overtly reflected monotonic directionality on each quantifier: and it depends on kind of the line what opposing relation they are in. A successive sequence of quantifiers, in top-to-bottom fashion on either column, is in subalternating relation partly because each of them enjoys the same directionality of monotonicity.

The present discussion should be considered to involve an integral characterization of a very



(Figure 16)

(where “┈┈” stands for “contraries”, “- - -” for “subcontraries”, “——” for “contradictories”, and “——▶” for “subalternation (*or* entailment)”)

small set of quantifiers in a natural language in terms of opposing relations and entailment, and it remains to be seen if a further research can see a possibility of counting out more quantifiers including numerals, proportional quantifiers, and etc..[4)]

NOTES

1) This research was supported by Momoyama Gakuin University under the research project 98KYO119.

2) The symbol “～” used in (22) corresponds to our notation “¬” in what it stands for.

3) An additional argument is made for claiming that an external negation of “many” is an equivalent counterpart of “few”. Jackendoff (1972) provides a following pair of sentences.

(i) a. Many of the arrows didn’t hit the target, but many of them hit it.

b. *Not many of the arrows hit the target, but many of them hit it.

In his argument of negation, (i a) is taken as an example of VP-negation, which corresponds to our internal negation. The status of (i a) indicates that, in the case of internal negation, one conjunct is not regarded as denial of the other conjunct, i.e. the first and second conjuncts are true at the same time. Unacceptability of (i b) indicates that its first and second conjuncts cannot be true simultaneously in a given situation. Jackendoff regards this negation as sentential negation as opposed to VP-negation. We take the former to be external negation. Given our square of opposition, “few” is found to be sitting on the deck, one story up from and diagonally right across “many”, and it follows from this that ¬**many＝few**.

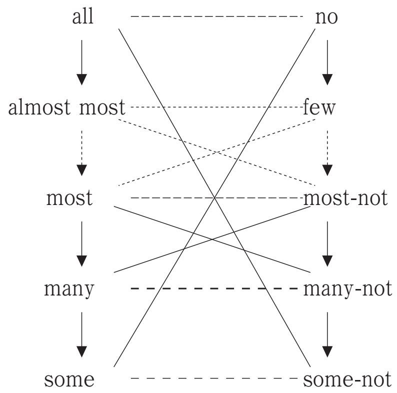
4) We could suggest that there be a possibility of (i) where “almost all” and “most” are treated separately. Westerståhl (1985) points out that ‘most As are B’ under the “more than half” interpretation is false when the subject term is empty. Existential constructions claim that there are As designated by the subject term, and it is thus false when the set is ϕ; and so the “most” must share the same property as existential constructions do. This predicts that there must be some “most” that appear in *there*-construction, but he does not give us any examples. We can provide the following examples indeed:

(i) a. *Most butterflies* that there are deep in the mountains are huge.

b. *Most of the students* that there were in the classroom are ones she had met before.

For as yet unknown reason, they are all relativized “most” NP. If they turns out to have “more than half” interpretation one way or another, there will possibly be some new proposal in which we can divide

"MOST" into two "most's", say, "most" and "almost all", and pair up the latter with "few", resulting in (ii).



(ii)

REFERENCES

Barwise, J. and R. Cooper: 1981, 'Generalized Quantifiers and Natural Language', *Linguistics and Philosophy* 4, 159-219.

Carroll, L.: 1958, *Symbolic Logic and the Game of Logic.* Dover Publications, Inc..

Declerck, R.: 1991, *A Comprehensive Descriptive Grammar of English*, Kaitakusha.

Huddleston R. and G. K. Pullum: 2002, *The Cambridge Grammar of the English Language*, Cambridge University Press.

Jackendoff, R.: 1972, *Semantics Interpretation in Generative Grammar,* MIT Press.

Keenan, E. L. and J. Stavi: 1986, 'A Semantic Characterization of Natural Language Determiners', *Linguistics and Philosophy* 9, 253-326.

Keynes, J. N.: 1906, *Formal Logic* (4th ed.), MaCMillan.

McCawley, J. D.: 1993, *Everything that Linguists have Always to Know about Logic*, The University of Chicago Press.

Partee, B. H., A. ter Meulen, and R. E. Wall: 1993, *Mathematical Methods in Linguistics*, Kluwer Academic Publishers.

Quine W. V.: 1972, *Methods of Logic* (3rd ed.), Holt, Rinehart and Winston, Inc.

Tidman P. and H. Kahane: 1999, *Logic and Philosophy* (8th ed.), Wadworth Publishing Company.

van Eijck, J.: 1985, 'Generalized Quantifiers and Traditional Logic' in van Benthem, J. and A. ter Meulen (eds.), *Generalized Quantifiers in Natural Language*, Foris, Dordrecht, 1-19.

Westerståhl, D.: 'Logical Constants in Quantifier Languages', *Linguistics and Philosophy* 8, 387-413.

Wiggins, D.: '"Most' and 'All"' in Platts M. (ed.), *Reference, Truth and Reality*, Routledge & Kegan Paul, 318-346.

【邦語文献】

ガストン・カゼノヴァ:『ブール代数』 村田 全・小室博昭 共訳(1968). 白水社.

The Square of Opposition with “most” and “many”

Shin-ichi SHIMIZU

Our concern is with constructing a traditional square of opposition into which “most” and “many” are integrated. Our basic position is in favor of traditional formal logic that originated with Aristotle, but the present discussion have taken liberties with recent developments of formal semantics to such an extent that they make contribution to more understanding of what the square of opposition looks like. The concept of directionality of monotonicity and especially our tests contribute to our conclusion. Still our discussion finds crucial insight in Keynes, one of formal logicians of a century ago. We propose as a conclusion a traditional square of opposition in which a near universal and near particular are neatly incorporated into a traditional proto-type square in such a way that they are entirely wrapped up.